Quasar Proper Motions and Low-Frequency Gravitational Waves

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ABSTRACT

We report observational upper limits on the mass-energy of the cosmological gravitational-wave background, from limits on proper motions of quasars. Gravitational waves with periods longer than the time span of observations produce a simple pattern of apparent proper motions over the sky, composed primarily of second-order transverse vector spherical harmonics. A fit of such harmonics to measured motions yields a 95%-confidence limit on the mass-energy of gravitational waves with frequencies $\nu < 2 \times 10^{-9}$ Hz, of $< 0.11h^{-2}$ times the closure density of the universe.

Subject headings: Cosmology: Gravitational Radiation, Gravitation - Techniques: Interferometry

1. INTRODUCTION

Like variations in refractive index, the changes in spacetime metric produced by gravitational waves alter optical path lengths. Light propagation through gravitational waves preserves sources' surface brightness and total intensity, to first order in the wave amplitude (Bondi, Pirani, & Robinson 1959, Zipoy 1966, Penrose 1966); but it can produce oscillations in apparent position, at the period of the gravitational wave (Fakir 1994, Durrer 1994, Bar-Kana 1994, Pyne et al. 1996, Kaiser & Jaffe 1996). Over time intervals much shorter then a gravitational wave period, these deflections cause a characteristic pattern of apparent motions in the plane of the sky (in astronomical parlance, proper motions) (Pyne et al. 1996). In this Letter we set stringent upper limits on the energy density of such waves, using measurements of the proper motions of quasars.

Although detected only indirectly to date, most cosmologists believe gravitational waves are commonplace. Very low-frequency gravitational waves arise naturally in inflationary cosmologies (Rubakov & Sazhin, & Veryashkin 1982, Fabbri & Pollock 1983) and dominate the mass-energy of the universe in some of them(Grishchuk 1993). Other possible sources of such a background include phase transitions in the early universe, networks of cosmic strings, and collisions of bubbles (Vilenkin 1981, Hogan 1986).

Various observations set direct or indirect observational constraints on the spectrum of low-frequency gravitational radiation. These constraints are often expressed in terms of $\Omega_G W$, the ratio of the mass-energy density of the gravitational waves to that required to close the universe. Some observational constraints are sensitive to a narrow range of frequencies, and are best expressed in terms of the logarithmic derivative $d\Omega_{GW}/d\ln\nu$ evaluated at some frequency ν. Timing of pulsars sets observational limits for periods as great as the span of observations (Backer & Hellings 1986). For the extremely stable millisecond pulsars the limit corresponds to $d\Omega_{GW}/d\ln\nu < 10^{-8}$, at frequency

$\nu = 4.4 \times 10^{-9}$ Hz (Kaspi, Taylor, & Ryba 1994). After taking into account the energy carried away by gravitational waves and the effects of galactocentric acceleration, orbital periods of binary pulsars are sensitive to gravitational waves with periods as great as the light travel time from the pulser. Current limits from such data sets a limit of $\Omega_{GW} h^2 < 0.04$ for $10^{-11} < \nu < 4.4 \times 10^{-9}$ Hz, and of $\Omega_{GW} h^2 < 0.5$ for $10^{-12} < \nu < 10^{-11}$ Hz (Bertotti, Carr, & Rees 1983, Taylor & Weisberg 1989, Thorsett & Dewey 1996). Here, the normalized Hubble constant is $h = H_0/100$ km s^{-1}. These measurements are sensitive to gravitational waves with periods as great as the light travel time from the pulsar. A cosmological background of gravitational waves at the epoch of recombination can produce anisotropies of the cosmic background radiation (Linder 1988a, Krauss & White 1992). For gravitational-wave spectra typical of inflation, the anisotropy detected by COBE yields a limit of $d\Omega_{GW}/d\ln\nu \leq 10^{-11}$, with sensitivity of the measurement concentrated near $\lambda \approx 2$ Gpc, or $\nu \approx 10^{-17}$ Hz (Bar-Kana 1994). Few constraints have been set in the frequecy range $10^{-17} < \nu < 10^{-12}$ Hz Linder (1988b) found that a gravitational-wave background in this frequency range would affect the galaxy-galaxy correlation function, but robust calculations would require knowledge of the "true" correlation function, observable only in the absence of such a background.

In contrast to these techniques, proper motions of extragalactic radio sources are sensitive to waves of arbitrarily long wavelength, and are independent of the spectrum and source of gravitational-wave radiation. Under the assumption that the spectrum of gravitational waves is stochastic, the squared proper motion of distant sources, suitably averaged over the sky, is proportional to the energy density of the waves, at frequencies from the inverse of the period of observation to the Hubble time.

In a previous paper(Pyne et al. 1996) we discuss the pattern of apparent proper motions of distant radio sources, produced by a gravitational wave. We use the assumption that the wavelength is short compared to the distance to the source, and the adiabatic approximation that the wavelength is less than the Hubble length; however, because the observed effect is local to the Earth, these assumptions can probably be relaxed without much change in the result. The best present measurements of proper motions attain accuracies of a few microarcseconds (μas) per year (Eubanks et al. 1996) corresponding to observational limits on the energy density of gravitational waves of about that required to close the universe.

2. THEORETICAL BACKGROUND

Very-long baseline interferometry (VLBI) measures positions of radio sources by measuring the difference in arrival times of their signals at antennas in different geographical locations (Shapiro 1976). The interferometrist assumes that the observations are made in a locally Minkowski reference frame (allowing for the orbital acceleration of the Earth and the general-relativistic light-bending of the Sun and planets), and so interprets these observations in the Gaussian normal reference frame. The delay T between arrival times measures the projection of the unit vector pointing toward the source onto the spacelike baseline vector that connects the antennas. Measurement of the delay for many sources on several baselines allows solution for both source positions and lengths and orientations of the baselines.

Pyne et al. (1996) describe the effect of a gravitational wave on a VLB interferometer: the wave produces variations in delay T, which are interpreted as variations in source position. A gravitational wave traveling toward $+z$, with the "+" polarization, produces metric perturbations $h\cos(pt)(\hat{\mathbf{x}}\hat{\mathbf{x}} - \hat{\mathbf{y}}\hat{\mathbf{y}})$ in the background coordinate reference frame, where h is the dimensionless strain of the wave, p is its angular frequency, and t is time. In the interferometrist's Gaussian normal frame, the observed proper motion $\vec{\mu}$ of a radio source at position (θ, ϕ) will be:

$$\vec{\mu} = \frac{hp}{2} \sin(p\eta) \sin\theta \left(\cos 2\phi \hat{\theta} - \sin 2\phi \hat{\phi} \right) \tag{1}$$

Here, θ measures angle from $+z$, the direction of propagation of the wave, and ϕ measures azimuthal angle around it, from the x-axis; the associated unit vectors on the sky are $\hat{\theta}$ and $\hat{\phi}$. Proper time in the Gaussian normal frame is η. We take h to be real, and allow the origin of time η to express the phase of the wave. Fig. 1 shows the pattern of proper motions that this gravitational wave produces.

The properties of this pattern of proper motions are not simple under rotation or superposition. However, the transverse vector spherical harmonics $\mathbf{Y}^{(\mathrm{E,M})}_{\ell,m}$ form an orthonormal basis for vector fields on a sphere, with well-understood behavior under rotation and superposition. Expanded in such harmonics, Eq. (1) takes the form:

$$\begin{aligned} \vec{\mu} = ph \quad & \sin(p\eta) \\ \times\Big\{ \quad & +\frac{\sqrt{5\pi}}{6}[\mathbf{Y}^{\mathrm{E}}_{2,+2} + \mathbf{Y}^{\mathrm{E}}_{2,-2} - \mathbf{Y}^{\mathrm{M}}_{2,+2} + \mathbf{Y}^{\mathrm{M}}_{2,-2}] \\ & -\frac{\sqrt{70\pi}}{60}[\mathbf{Y}^{\mathrm{E}}_{3,+2} + \mathbf{Y}^{\mathrm{E}}_{3,-2} - \mathbf{Y}^{\mathrm{M}}_{3,+2} + \mathbf{Y}^{\mathrm{M}}_{3,-2}] + \ldots \Big\}. \end{aligned} \tag{2}$$

Here we use convention of Mathews (1962,1981) for transverse vector spherical harmonics. These fall into 2 categories, with one family, commonly denoted "poloidal", "potential", or "electric" pointing down the gradients of scalar spherical harmonics; and the other, known as "toroidal", "stream", or "magnetic" pointing perpendicular to their gradients. We denote these categories as "E" and "M", respectively (Mathews 1962, Mathews 1981), with the notation (E, M) meaning "E or M". Note that our $\mathbf{Y}^{\mathrm{M}}_{\ell,m}$ is the $\mathbf{X}_{\ell,m}$ of Jackson (1975). The normalization condition is:

$$\begin{aligned} 4\pi\langle \mathbf{Y}^{\mathrm{E}}_{\ell,m} \mathbf{Y}^{\mathrm{E}*}_{i,j} \rangle = \quad & 4\pi\langle \mathbf{Y}^{\mathrm{M}}_{\ell,m} \mathbf{Y}^{\mathrm{M}*}_{i,j} \rangle = \delta_{\ell i}\delta_{mj} \\ \langle \mathbf{Y}^{\mathrm{E}}_{\ell,m} \mathbf{Y}^{\mathrm{M}*}_{i,j} \rangle = \quad & 0. \end{aligned} \tag{3}$$

Here $\langle ... \rangle$ denotes an average over the sky.

Because the $\mathbf{Y}^{(\mathrm{E,M})}_{\ell,m}$ form an orthonormal basis for vector fields on a sphere, they can represent proper motions due to a single wave, or any spectrum of waves. We use the expansion

$$\vec{\mu} = \sum_{\ell,m} \left[a^{\mathrm{E}}_{\ell,m} \mathbf{Y}^{\mathrm{E}}_{\ell,m} + a^{\mathrm{M}}_{\ell,m} \mathbf{Y}^{\mathrm{M}}_{\ell,m} \right] \tag{4}$$

to define the coefficients $a^{\mathrm{E}}_{\ell,m}$ and $a^{\mathrm{M}}_{\ell,m}$ in terms of the proper motion $\vec{\mu}$, observed over a small fraction of a gravitational wave period. The $a^{(\mathrm{E,M})}_{\ell,m}$ must satisfy (Mathews 1981):

$$a^{\mathrm{E}}_{\ell,+m} = (-1)^m a^{\mathrm{E}*}_{\ell,-m} \quad a^{\mathrm{M}}_{\ell,+m} = (-1)^{m+1} a^{\mathrm{M}*}_{\ell,-m} \tag{5}$$

because $\vec{\mu}$ is real.

The squared proper motion averaged over the sky is related to the energy density of the wave T_{GW} by

$$\langle \mu^2 \rangle = \frac{1}{12} p^2 h^2 = \frac{8\pi}{3} \frac{G}{c^2} T_{GW}, \tag{6}$$

Here $\langle .. \rangle$ includes an average over many wave periods, as well as over the sky. Expansion of the mean square proper motion in vector spherical harmonics shows that 5/6 of the mean square motion resides in the $\ell = 2$ harmonics:

$$\frac{1}{4\pi} \sum_m \left(|a^{\mathrm{E}}_{2,m}|^2 + |a^{\mathrm{M}}_{2,m}|^2 \right) = \frac{5}{6} \langle \mu^2 \rangle. \tag{7}$$

Because the summed, squared moduli of the $a^{(\mathrm{E,M})}_{\ell,m}$ remain constant under rotation for for each ℓ, and for E and M harmonics separately, Eq. (7) holds for a gravitational wave propagating in any direction with any polarization.

If the phases, amplitudes and directions of the many superposed waves in the spectrum are random at the Earth, as is to be expected for a cosmological background of gravitational waves, the random-phase approximation shows that Eqs. (6) and (7) will hold statistically, if $p^2 h^2$ is replaced by the integral $\int p^2 h(p)^2 d^3p$, and if the average of the squared proper motion over time and sky is replaced by an average over an ensemble of gravitational-wave spectra (e.g. Goodman 1985). Here $h(p)^2$ is the spectral density of the square of the dimensionless strain. Thus, Eq. (7) shows that 5/6 of the mean square proper motion due to the low-frequency gravitational wave background appears in second-order transverse spherical harmonics, and E and M harmonics contribute equally. We can express the energy density in such a stochastic background of gravitational waves as Ω_{GW}, its ratio to the closure density of the universe, $T_{CL} = 3c^2 H_0^2/8\pi G$; and we can relate Ω_{GW} to proper motion in second-order transverse harmonics:

$$\Omega_{GW} = \frac{1}{H_0^2}\langle \mu^2 \rangle = \frac{6}{5}\frac{1}{4\pi}\frac{1}{H_0^2}\sum_{(\mathrm{E,M})m} |a_{2,m}^{(\mathrm{E,M})}|^2 \qquad (8)$$

We seek to measure the $a_{2,m}^{(\mathrm{E,M})}$ and to thus measure, or set limits on, Ω_{GW}.

3. OBSERVATIONS AND DATA REDUCTION

We searched for the pattern expected for gravitational waves in proper motions of extragalactic radio sources. We estimated second-order spherical harmonics from a proper-motion solution using the US Naval Observatory comprehensive database of VLBI observations. This contains most of the suitable astrometric and geodetic observations of extragalactic radio sources made with the Mark III and compatible VLBI systemsClark et al. 1985. Eubanks and Matsakis Eubanks et al. 1996 describe the data analysis in detail; here we present a brief overview.

The database comprises 1,469,793 observations of delay T and its time derivative $\dot{T}$ with different baselines and sources. The observations were made between 3 August 1979 and 13 February 1996, with more than 70% after 1990. Proper motions were determined for the 499 sources observed at more than 1 epoch. In fitting for the coefficients of transverse spherical harmonics we used the 323 sources known to be extragalactic by their measured redshifts. Their mean redshift is 1.22.

Reduction of this data to source positions and proper motions follows standard procedures for astrometric and geodetic VLBI data (e.g. Clark et al. 1985). We removed the best models for earth orientation, tidal deformation, and location within the solar system; atmospheric propagation; and solar-system effects of general and special relativity; before fitting for proper motion. Simultaneous observations at 2 frequencies allowed calibration of the propagation delay through the ionosphere. The solutions for proper motions also included $\approx 400,000$ "nuisance" parameters, including those describing positions of sources and stations; clock behavior at different stations; optical path length though the atmosphere in a vertical direction and its horizontal gradients; unmodeled motions of the Earth, including variations in rotation, polar motion, nutation, and precession; and gravitational deflection of light by the Sun Eubanks et al. 1996. These nuisance parameters are chosen to have little covariance with source and station positions, and thus to proper motions. After the fit, the weighted root-mean-square of the scatters of the residuals in delay and in delay rate were 31 ps and 93 fs s^{-1}, respectively.

Effects of source structure and variations in atmospheric path length are among the expected systematic errors. The sources are highly energetic neighborhoods of active galactic nuclei. The well-known motions of jets from such sources can mimic proper motion, particularly in cases where the stationary core component is weak or absent (e.g. Guirado et al. 1995). Such motions are expected to be many μas yr^{-1}, but should not be correlated over the sky, so that they increase noise in the measurements but do not bias lower-order transverse spherical harmonics. Atmospheric path length is removed via models incorporating contemporaneous meteorological data, and the residual path length and its horizontal gradients are parametrized and estimated directly from the VLBI solution. Remaining atmospheric delay errors might show angular dependences that could mimic those expected for gravitational waves.

We then fit transverse spherical harmonics to the measured proper motions. The fit minimized χ^2, the summed, squared differences of proper motions and the motions modeled by Eqs. (4) and (5). These differences were weighted by the expected errors, given by the standard errors from the fit to delay and delay rate, scaled by a factor of 1.35, and with an assumed error added in quadrature (in each component of proper motion) of 30 μas yr^{-1}. This reweighting serves to make the χ^2 per degree of freedom about 1, as expected; it accounts, in particular, for effects of variations in source structure. Comparison of fits to subsets of the data suggested this scaling and addition in quadrature. Fits to subsets of the data did not change the fitted values significantly, although the standard errors increased.

The fit included coefficients for $\ell = 1$ transverse spherical harmonics, as well as the $\ell = 2$ harmonics characteristic of gravitational waves. The M $\ell = 1$ harmonics correspond to a rotation, of no physical significance, as it is not separable from the Earth's rotation; the E $\ell = 1$ harmonics correspond to acceleration not included in models used for data reduction. This acceleration is an interesting cosmological parameter: it is sensitive to the galactocentric acceleration of the Solar System, as well as any acceleration of the Milky Way relative to distant quasars.

The measured values of the coefficients of second-order transverse spherical harmonics are statistically indistinguishable from noise. The residual χ^2 to the fit was 717, distributed among 627 degrees of freedom. Table 1 summarizes results of the fit, and Fig. 1 shows the results for the $\ell = 2$ harmonics in graphical form. The fitted coefficients are combinations of $a^{(\mathrm{E,M})}_{\ell,m}$ that satisfy Eq. (5) to produce real motions. The mild migration of sources away from the ecliptic in Fig. 1 may reflect effects of the Sun on atmospheric propagation.

4. RESULTS

A particular value of Ω_{GW} corresponds to an ensemble of possible gravitational-wave spectra and coefficients $a_{2,m}^{(\mathrm{E,M})}$. Eq. (8) relates Ω_{GW} to the mean over the ensemble of the summed, squared coefficients. The measurement errors given in the last column of Table 1 also contribute to the measured $a_{2,m}^{(\mathrm{E,M})}$. We take these contributions into account to find the value of Ω_{GW} that yields summed, squared coefficients less than those we observe for only 5% of gravitational spectra, and adopt this as our upper limit. This procedure includes effects of sample or cosmic variance. With 95% confidence, we find that $\Omega_{GW} < 0.11\ h^{-2}$, where $H_0 = h100$ km s^{-1}. This limit holds for a stochastic spectrum of gravitational waves integrated over all frequencies less than half the inverse of the 10-yr span of the observations, or about 2×10^{-9} Hz.

Considerable improvement in this limit should be possible over the next decade. The measurement accuracy for proper motions is proportional to the duration of observations to the 3/2 power. Because most data were acquired in the last 5 to 10 years, an additional decade of observing should improve the bound on Ω_{GW} by a factor of 3 to 8. Moreover, we can reduce effects of source structure by choosing additional sources with little structure, from among the thousands observable astrometrically with the Very Long Baseline Array; and by using models derived from images of the sources to correct for effects of structure directly. Together these improvements may be expected to reduce the formal errors of measurements of Ω_{GW} by a factor of 10 to 100, within the next 10 years.

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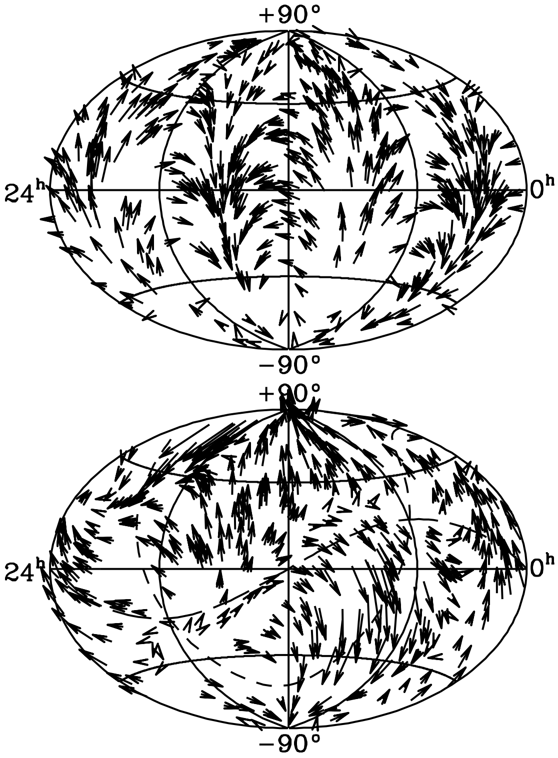

Fig. 1.— Upper: Proper motions expected for a single gravitational wave. The metric perturbation is $h\cos(p(cz-t))(\hat{\mathbf{x}}\hat{\mathbf{x}}-\hat{\mathbf{y}}\hat{\mathbf{y}})$, with $\hat{\mathbf{z}}$ toward declination 90° and $\hat{\mathbf{x}}$ toward right ascension $0^{\rm h}$. Lower: Fitted coefficients of the second-order ($\ell = 2$) transverse spherical harmonics, displayed as proper motions at locations of sources with measured proper motions. Arrow lengths in degrees equal proper motion in μas yr^{-1}. Coefficients are not shown for $\ell \neq 2$. Curves show the ecliptic (long dashes) and galactic plane (short dashes). The fitted coefficients are not statistically significant, so that the observed pattern of motions is consistent with filtered noise.

Table 1. Observed Coefficients of Transverse Harmonics

Parameter	Fitted Value (μas yr^{-1})	Standard Error (μas yr^{-1})
Acceleration[a]		
$\ddot{x}$	1.9	6.1
$\ddot{y}$	5.4	6.2
$\ddot{z}$	7.5	5.6
Second-Order Transverse Vector Spherical Harmonics[b]		
${a^{\mathrm{E}}}_{2,0}$	12.1	16.6
$\frac{1}{\sqrt{2}}({a^{\mathrm{E}}}_{2,+1} - {a^{\mathrm{E}}}_{2,-1})$	-13.6	14.9
$\frac{i}{\sqrt{2}}({a^{\mathrm{E}}}_{2,+1} + {a^{\mathrm{E}}}_{2,-1})$	-21.7	15.6
$\frac{1}{\sqrt{2}}({a^{\mathrm{E}}}_{2,+2} + {a^{\mathrm{E}}}_{2,-2})$	4.2	12.5
$\frac{i}{\sqrt{2}}({a^{\mathrm{E}}}_{2,+2} - {a^{\mathrm{E}}}_{2,-2})$	-0.6	13.3
$i{a^{\mathrm{M}}}_{2,0}$	1.7	14.4
$\frac{i}{\sqrt{2}}({a^{\mathrm{M}}}_{2,+1} - {a^{\mathrm{M}}}_{2,-1})$	17.8	15.0
$\frac{1}{\sqrt{2}}({a^{\mathrm{M}}}_{2,+1} + {a^{\mathrm{M}}}_{2,-1})$	-28.2	15.3
$\frac{i}{\sqrt{2}}({a^{\mathrm{M}}}_{2,+2} + {a^{\mathrm{M}}}_{2,-2})$	-17.6	13.9
$\frac{1}{\sqrt{2}}({a^{\mathrm{M}}}_{2,+2} - {a^{\mathrm{M}}}_{2,-2})$	10.7	15.0

[a] Effects of acceleration of the solar system barycenter relative to the observed extragalactic radio sources corresponds to first-order ($\ell = 1$) harmonics. Rotations also correspond to first-order harmonics; our fits included these, but we do not report them as they cannot be separated from the Earth's rotation.

[b] Second-order ($\ell = 2$) harmonics correspond to effects of low-frequency gravitational waves. Otherwise identical coefficients $a_{\ell,m}^{(\mathrm{E,M})}$ with opposite sign of m have been combined to reflect the fact that the fitted motions are purely real.