

Homework 3

1. In the proof of the Fundamental Theorem of Algebra we introduced a smooth map $f : \mathbf{S}^2 \rightarrow \mathbf{S}^2$, defined by

$$f(x) = \begin{cases} x & \text{if } x = (0, 0, 1) \\ h^{-1}Ph(x) & \text{otherwise} \end{cases}$$

Here, $h : \mathbf{S}^2 - \{(0, 0, 1)\} \rightarrow \mathbf{R}^2$ is a stereographic projection from the “north pole” $(0, 0, 1)$, and $P(z)$ is a polynomial in one complex variable that we treat as a map from the complex plane $\mathbf{C} \cong \mathbf{R}^2$ to itself. Prove that $f(x)$ is a smooth map in the neighborhood of $x = (0, 0, 1)$.

2. Guillemin & Pollack §1.5: problem 2
3. Guillemin & Pollack §1.5: problem 8
4. Guillemin & Pollack §1.5: problem 10