Phys 110C: Problems for HW 1

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1 HW1 Problem 1

Stress-Energy Tensor The Maxwell stress tensor $\overleftrightarrow{T}$ is the *negative of the* spatial part of the electromagnetic stress-energy 4-tensor, $T^{\mu\nu}$. In other words, the elements of the Maxwell stress tensor give $-T^{ij}$ where i or $j = 1, 2, 3$.

The rest of $T^{\mu\nu}$ is given by the Poynting vector $\vec{S}$, and the energy density in EM fields U, as follows: The time-space parts of the stress-energy 4-tensor are the components of the Poynting vector: $T^{0i} = T^{i0} = S_i$ for $i = 1, 2, 3$. The time-time element is the energy density of electric and magnetic fields: $T^{00} = U = \frac{\epsilon_0}{2}|E|^2 + \frac{1}{2\mu_0}|B|^2$. Note that the stress-energy tensor is always symmetric.

The current-density 4-vector $(\rho, \vec{J})$ is the source of the electromagnetic field. Similarly, the stress-energy 4-tensor is the source of the gravitational field.

a) Find the dimensional factor of c^N required to correct the units of the Poynting vector to those of the Maxwell stress tensor (for those who use MKS units, which are different for length and time). Is an analogous factor of c^M required to correct the dimensions of the T^{00} term to those of the T^{ij} terms? Explain.

b) Find the stress-energy 4-tensor for a point charge $+q$ at rest at the origin, evaluated at the event $x^\mu = (0, 0, 0, Z)$. (Note: $x^\mu = (x^0, x^2, x^3) = (ct, x, y, z)$.

c) Use Lorentz transformations to boost your result from part b, to a frame in which the point charge is traveling toward $+\hat{x}$, at $\beta = v/c$. Explain the nonzero Poynting vector, in this frame.

(Possibly useful checks: Note that this particular Lorentz boost will transform the 4-velocity of the charge from $\eta^\mu = (1, 0, 0, 0)$ to $(\gamma, \beta\gamma, 0, 0)$. Also as a check, $T^{\mu\nu}$ must be symmetric in all frames.)

2 Problems from Griffiths

7.58, 8.1, 8.2, 8.4, 8.5, 8.9