Phys 110C: Problems for HW 4

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April 20, 2008

1 HW4 1: Loop Antenna

A plane electromagnetic wave travels through vacuum. The electric field of the wave is given by:

$$\vec{E}(\vec{r}, t) = \hat{y} E_0 e^{i(kz = \omega t)}. \tag{1}$$

A circular loop of radius a, with N turns, and resistance R is located with its center at the origin. The loop is oriented so that a diameter lies along the z-axis, and the normal to the plane of the loop makes an angle θ with respect to the y-axis. Find the emf induced in the loop as a function of time. Assume that $a << \lambda$. (Why?)

2 HW4 2: Energy Density inside a conductor

Find the ratio of the time average of magnetic energy density, to the time average of electric energy density, $\langle W_m \rangle / \langle W_e \rangle$, for an electromagnetic wave inside a conductor. Then find approximate expressons for this ratio for the limiting cases of an insulator and a good conductor.

3 HW4 3: Ion Plasma Frequency

An ionized gas contains movable ions as well as electrons. Why are we able to neglect the contribution of the ions to the conductivity, and hence to the plasma frequency, and thus to the effective dielectric constant of a plasma?

4 HW4 4: Refraction at a plasma surface

Find the index of refraction of a plasma with electron density n. An electromagnetic wave of frequency $\omega = 2\omega_p$ travels through vacuum until it is incident on the surface of the plasma, at an angle of incidence θ. Find the angle of refraction. Find the angle of incidence for which total "internal" reflection takes place.

Note that, for a plasma, total "internal" reflection takes place for electromagnetic waves shining from vacuum onto the plasma; whereas usually we think of total "internal" reflection as taking place for light within the material shining onto a surface to vacuum. Why should this be the case?

For fun: Can you find a material that would show total "internal" reflection, like the plasma, for visible light?

5 HW4 5: Lorentz Invariance of the Spacetime Interval

Consider the spacetime interval, given by $d^\mu d_\mu = (x^\mu - y^\mu)(x_\mu - y_\mu)$. Show that this is invariant under Lorentz transformations. In 4-vector notation, show that $d^\mu d_\mu = d^{\nu'} d_{\nu'}$, where $x^{\nu'} = \Lambda^{\nu'}{}_{\mu} x^\mu$, and $x_{\nu'} = \Lambda^{\mu}{}_{\nu'} x_\mu$, and likewise for y.

Of course, you can raise or lower indices (such as μ) using the metric $g_{\mu\nu}$ (sometimes denoted $\eta_{\mu\nu}$).

6 HW4 6: Lorentz Invariance of the Wave Equation

The wave equation for a scalar function ψ states that

$$\frac{\partial^2 \psi}{\partial x^2} = \frac{1}{v_\phi^2} \frac{\partial^2 \psi}{\partial t^2} \tag{2}$$

where v_ϕ is the phase velocity of the wave. Assume here that $v_\phi = c$.

a) The Lorentz tranformation from frame S to frame S' can be expressed as two functions, $x'(x, ct)$ and $ct'(x, ct)$. By direct calculation, using the chain rule, show that the wave equation in the primed coordinate frame takes the same form as the wave equation in the unprimed frame.

b) Now consider this problem from the standpoint of 4-vectors. Differentiation with respect to the coordinates x^μ is a covariant 4-vector, often written ∂_μ:

$$\partial_\mu \equiv \frac{\partial}{\partial x^\mu} = \left(\frac{\partial}{\partial ct}, \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right). \tag{3}$$

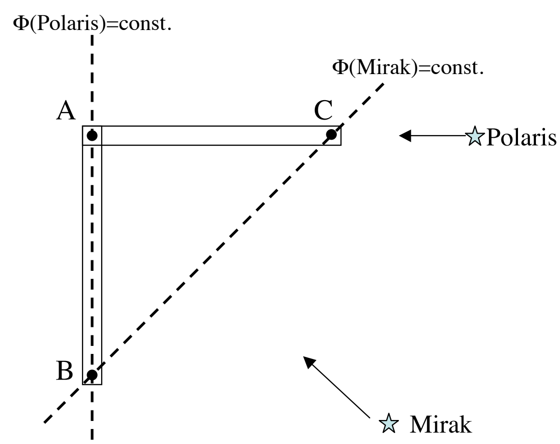
Of course, you can raise the index μ using the metric $g_{\mu\nu}$ (sometimes denoted $\eta_{\mu\nu}$).

Find an expression for the wave equation Eq. 2, using 4-vectors.

Show that the wave equation remains invariant under Lorentz transformations. (Hint: Apply HW4-6).

7 HW4 7: Relativistic Aberration

An astronomer on Earth uses an interferometer to measure the angle between the Pole star, Polaris, and the star Mirak. The astronomer observes that light from Polaris has the same phase at points A and B. On the other hand, light from Mirak has the same phase at points A and C. The distance between points A and B is L; and the distance between points B and C is also L. The two segments are at right angles. The astronomer concludes that the angle between Polaris and Mirak is 45°.



A starship travels past the astronomer at speed $v = 0.8$ c. The starship travels directly toward Polaris. Find the distances between the points in the reference frame of the starship. Use these distances to find the angle between Polaris and Mirak, in the frame of the starship.

Note: This effect is known to astronomers as aberration. It is not difficult to observe, for example from the motions of the Earth, and provided one of the first measurements of the speed of light.