Phys 110C: Problems for HW 7

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1 HW7 1: Poynting Vector for a Standing Wave

Find the Poynting vector for a standing wave, as given by Problem 1 on Midterm 1. You may wish look up the magnetic field for the standing wave, as given in the solution to the midterm, http://www.physics.ucsb.edu/∼phys110C/Midterm_Solutions.pdf .

2 HW7 2: Laplacian Fact

Prove this useful fact:

$$\frac{1}{r^2}\frac{\partial}{\partial r}r^2\frac{\partial}{\partial r}\ \Psi \quad = \quad \frac{1}{r}\frac{\partial^2}{\partial r^2}r\ \Psi \tag{1}$$

The first of these is the radial part of the Laplacian in spherical coordinates, as given on the inside front cover of your book. The second is an alternative form that was used in class to find the impulse-response function (Green's function) for the Helmholtz Equation.

3 HW7 3: Electromagnetic Pulse

A nuclear explosion high above the atmosphere releases lots of X-rays and gamma rays. These strike electrons in the atmosphere and drive them downward out of their atoms. The electrons then execute cyclotron orbits, creating an oscillating current. A similar effect, but larger, is thought take place whenever a gamma-ray burst takes place near the Earth. These are infrequent, and probably aren't dramatic enough to appear in geological or fossil records. This problem explores a simple model of an electromagnetic pulse.

Suppose that the plane $z = 0$ contains a sheet current $\vec{K}(t)$ that is the same at all positions, but varies in time, with components:

$$\vec{K}(x,y,t) = 0, \quad t < 0 \tag{2}$$
$$= K_0 \sin(\omega t)\hat{y}, \quad t > 0 \tag{3}$$

There is no net charge anywhere.

a) An observer resides at height z_Q on the z-axis above the $z = 0$ plane. The observer measures the fields at time t_q. What is the retarded time t_R, for a location (x, y) in the $z = 0$ plane? At what location(s) is the retarded time $t_R = 0$?

b) Write an integral for the vector potential $\vec{A}$ in terms of the sheet current at the retarded time, integrated over the $z = 0$ plane. Evaluate this integral. (Hint: You may find it useful to recall that

$$\int_0^R u\,du\,\frac{f'(\sqrt{A+u^2})}{\sqrt{A+u^2}} = f(\sqrt{A+R^2}) - f(\sqrt{A}). \tag{4}$$

where f' is the derivative of f.

c) Find the electric and magnetic fields for $z > 0$, $t > 0$.

4 Problems from Griffiths

10.12, 10.13, 10.14, 10.25