Vortex Dissipation in Type I Superconducting Films

by

Hunter Y. McDaniel

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Advisor: Dr. John M. Martinis

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The dissertation of Hunter Y. McDaniel is approved:

______________________________________________________________________

John M. Martinis                      Date

______________________________________________________________________

Omer M. Blaes                        Date

University of California, Santa Barbara

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Abstract

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The effect of trapped magnetic flux which forms vortices in thin superconducting lines is measured, analyzed and modeled. In particular the electrical behavior of these vortices in coplanar waveguide (CPW) resonators is considered at microwave frequencies. The experimental method is described in detail. Sputtered Aluminum and epitaxial Rhenium metals on a sapphire substrate CPWs are tested.

It is found that vortices increase the loss tangent $\delta$ and decrease the resonance frequency. These effects are explored in detail and two models are considered to explain them. The effect in epitaxial Rhenium is found to be potentially understood with a lumped element model where the vortices contribute an added normal state resistance and inductance to the transmission line. The behavior of sputtered Aluminum differs from Rhenium and may be better described by a vortex vibration model.
Acknowledgments

I would like to start by thanking the Physics department for providing me with the means to learn so much about the beautiful world in which we live. Without naming all of my professors let me just say that each and every one of you taught me something that has contributed to this work. Your insight has given me the confidence to complete this effort. I would like to thank the administration for giving me the chance to do this double major and for putting up with all my petitions over the years. Thank you Kerri O’Conner and Jean Dill for guiding me along the path to approval and credit for this work towards my degree. Thanks to David Cannell and Chris Takacs for giving me the chance to begin doing research in UCSB Physics and for encouraging my move into Dr. Martinis’ group. I would like to especially thank the members of Dr. Martinis’ quantum computing group. I know I was a handful at times, but you were patient and I am most grateful. Nadav Katz, Robert McDermott, Matthias Stephen, Markus Ansmann, Radek Bialzeck, Erik Lucero and Ken Cooper thank you so much for your time, knowledge and countless efforts. I would also like to thank you guys for the blood, sweat and tears you put into fabricating resonators. To Matthew Neeley who was crucial to this work and thank you for helping with the set up and orchestration of the experiment. Matthew’s software made all of this possible in a one year time span and collaboration with him really made this experiment what it is. Thanks is deserved to many others whom there is not space to mention, but you know who you are and I thank you. To the electron vortices, thanks for giving me a measurable effect that I could make a thesis out of.

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Chapter 1

Introduction

Background

Superconductivity has been the subject of much research since it was discovered in 1911. A superconducting material allows the free flow of charge without resistance enabling electrons to maintain their kinetic energy (motion) for long durations of time. This occurs in some metals below a critical temperature, $T_c$, typically less than 10K. Aluminum has a $T_c$ of 1.2K and Rhenium has a $T_c$ of 1.6K and lead has a $T_c$ around 7.2K, for example. Generally, when a magnetic field is applied to a superconducting material, electrons flow in screening currents around the edge of the material to cancel the changing magnetic flux (Lenz’s law). This Meissner effect expels the applied magnetic field event at DC so that inside the material the magnetic field is always zero. This expulsion occurs up to a critical screening current which occurs at a critical applied magnetic field, $H_{c_1}$, where the superconductivity of the material breaks down and flux penetrates into the material.

A second type of superconductivity was discovered in alloys years later, where the magnetic field expulsion wasn’t complete. In these type II superconductors when a magnetic field above the lower critical field, $H_{c_2}$, is applied some penetrates through, usually around defect sites, in magnetic flux tubes or electron vortices. These flux tubes are also called electron vortices because around the trapped flux electrons flow in small current loops which maintains the trapped flux. These regions of trapped flux contain the quantum unit of magnetic flux called the flux quantum given by $1.1$. As the magnetic field is increased more vortices form until the upper critical field is reached, $H_{c_2}$, when the superconductivity breaks down.

$$\Phi_0 = \frac{h}{2e} = 2.067833636 \times 10^{-15} \text{ Wb} \quad (1.1)$$

It was known that vortices form in type II superconductors above a certain critical field. For type I, in 2004 Stan et al. conclusively demonstrated with direct imaging that vortices could form in type I superconducting films if a strong enough magnetic was present when the material was cooled down below $T_c$. Stan et al showed that the number

- 1 -
of vortices in a strip of material (wire) is proportional to the magnetic field beyond a critical value, which scales as the inverse square of line width $W$.\footnote{1}

**Figure 1.1:** STM images of vortices in 10 µm (a) and 100 µm (b) lead wires. The 10 µm strip was cooled in an 85 µT field and appears lighter because of Meissner expulsion, still many vortices (dark spots) are visible. The 100 µm strip was cooled in a 5.3 µT field. Both images are 140 µT full scale and 145 µm wide. This image was taken directly from the paper by Stan et al.\footnote{1}

**Figure 1.2:** Number of vortices versus cool down field for 1.6 µm (a), 10 µm (b) and 100 µm (c) lead strips. Beyond the critical field $B_m$ denoted by the arrows, the number is proportional to the field. This image was taken directly from the paper by Stan et al.\footnote{1}
In the above equations, $N$ is the number of vortices on a wire with area $A$ when cooled down at field $B$ and $B_m$ is the critical cool down field where vortices will begin to form. These vortices are generally unwanted and usually the condensed matter physicist goes to great lengths to keep them out of the experiment with magnetic shielding or field canceling coils.

In recent years many new and novel applications have arisen by exploiting superconductivity in circuits such as in quantum computing. In particular in quantum computation it is important that the delicate quantum states are preserved as long as possible and so energy loss mechanisms must be minimized\(^2\). Understanding loss mechanisms and phase error can be very important especially in a quantum computer where the state is fragile and is needed to be preserved as long as possible. Some work has been done to understand the effects of vortices in type II superconductors and will be addressed in chapter 4, but little research focused on the effects in type I superconductors.

**Goals**

Since the most common element in a circuit is the wiring and the parameters for vortex formation in this geometry have already been documented, the next step is to look at their effects. A common frequency range for superconducting circuits is in the Microwave band, and typically microwaves are used to characterize materials. So the first goal is to determine what effect vortices have in thin-film type I superconducting wire at microwave frequencies.

With an understanding of dissipation generated by the vortices it will be important to understand how these effects arise. So the next goal will be to look at possible models for the vortex physics at microwave frequencies. These models will include material parameters so it will be important to look at different materials to determine how the model might change in different superconductors.
Chapter 2
The Experiment

Microwave Resonance

A common way to characterize electronic materials at microwave frequencies is with a resonator. There are different geometries for a microwave resonator but the common quality is a Lorentzian transmission curve on resonance. The bandwidth at half max (-3dB transmission point) of this curve is proportional to the loss tangent of the resonator which can be directly related to material losses given other known loss mechanisms like a resistive load. More commonly talked about is the inverse of the loss tangent, called the Q or quality factor of the resonator.

\[
\text{loss tangent: } \delta = \frac{BW_{3dB}}{f_0} \quad (2.1)
\]

\[
\text{quality factor: } Q = \frac{1}{\delta} \quad (2.2)
\]

Materials which do not allow efficient transmission of energy will have a high loss tangent or low Q resonance. The quality factor also describes the time scale of energy loss in a resonator and is roughly equal to the oscillation time scale divided by the energy decay time scale. A high Q resonator may resonate with a supplied energy for many more cycles than a low Q resonator.

Typically a resonator is characterized by it’s frequency response, this can be the physical position of a mass on a spring, the pressure in an air cavity or the voltage transmitted through a transmission line to name a few. The response of an resonator is a complex valued function with amplitude and phase information, the curve may look as follows:
Figure 2.1: Typical magnitude and phase response of a resonator near its resonance frequency $f_0$. The 3 dB bandwidth is the range of frequencies where 50% or more of maximum power transmission occurs. Right on resonance 0 dB or 100% of the maximum transmission occurs for an ideal resonator. The transmitted phase is 0 degrees much lower than $f_0$, -90 degrees at $f_0$ and -180 degrees for frequencies much greater than $f_0$.

The geometry of the resonator is chosen so as to limit the unknowns. For example, to test a specific dielectric you would choose a lumped element LC resonator and use a large parallel plate capacitor (with the dielectric of interest). Since it had already been determined how the number of vortices varied with line width, a co-planar waveguide (CPW) transmission line resonator was chosen. A CPW is essentially a two dimensional structures with a wire line in between conducting planes on top of a dielectric substrate.
This type of resonator was especially easy to fabricate, having only 1 layer on top of the substrate. However, it does not couple as strongly to the dielectric as other designs which is known to contribute to loss at low temperatures\(^2\). A transmission line resonator achieves fundamental resonance at a frequency which gives one half a wavelength across the line and higher order resonances at integer multiples thereof. The frequency dependence of the loss can be measured by looking at the harmonic modes of the CPW. With this geometry the only location vortices can have a significant effect will be right on the transmission line where their effect will show up clearly in the loss tangent. A small but limited effect of flux trapping in the ground plane was discovered with early resonator designs. Layouts included a perforated ground plane which greatly reduces the effective width of the ground plane and therefore the number of vortices that can form making the effect negligible.
In order to couple a resonator to the outside world transmission lines must be used. As is typical, 50 $\Omega$ characteristic impedance lines were used. If the resonator was attached directly to the 50 $\Omega$ line it would be heavily loaded to the extent that the Q would be entirely dependent on the lines and not the qualities of the resonator. As will be discussed in more detail in the theory section, coupling capacitors were used to effectively isolate the resonator and transform the input and output impedance. Our fabricated resonators are made with coupling capacitors ranging from .5 fF to 10 fF where the choice depends on how high the intrinsic Q of the resonator is. It was found that 2 fF coupling capacitors were ideal for the order of the Qs measured.

**Taking the Data**

![Agilent network analyzer](image)

*Figure 2.3: The Agilent network analyzer used, port one is on the left, port two the right.*

We measure the transmission curve as the $S_{21}$ S-Parameter using an Agilent N5230A network analyzer (NA). The $S_{21}$ parameter gives magnitude and phase information about the outgoing relative to the incoming microwaves, as explained in more detail in appendix A. For an ideal resonator on resonance, one would expect the magnitude of $S_{21}$ be 0dB (100% transmission) the phase to be -90 degrees (see figure 2.1). The data is taken by first locating the resonance by sweeping the frequency over larger ranges and locating the peak in transmission, then fine tuning the sweep to be on the resonance. The $S_{21}$ sweep data is imported to a computer via GIPB software and a best fit is used to find the closest Lorentzian match.
Figure 2.4: Screenshot of the data retrieval and fitting software. The data shown is a high Q resonance, the top two plots are the raw data and the bottom two are calibrated. Note the perfect circle on the bottom right.

The data from the NA is a complex function of frequency that, when plotted on the real and imaginary axes, should form a circle. The raw data will not form a circle due to stray coupling, electrical length of the transmission line and other frequency dependent artifacts that need to be calibrated out. The transmission lines leading to and from the resonator and amplifiers are calibrated by dividing the data by S21 measured from a short. Following the calibration, a fit is attempted by the data retrieval software to ensure a sufficient set of data was taken. In this way it is possible to determine if the sweep can be fit closely to a Lorentzian immediately when the data is taken rather than later during the full analysis.
Figure 2.5: Magnitude and phase data from a low-Q response, with the Lorentzian fit. The white points and green fit are the magnitude of $S_{21}$ with the axis on the left. The red points and blue fit are the phase of $S_{21}$ with the axis on the right.

Figure 2.6: Polar plot of the same data in figure 2.5 with the fit. Note how the calibrated forms a circle that nearly closes on itself.
The Lorentzian that describes the S21 response is given by equation 2.1. A least squares fit for the amplitude $A$, phase $\phi$, quality factor $Q$ and resonance frequency $\omega_0$ was used.

$$S_{21} = \frac{Ae^{i\phi}}{1 + 2iQ\frac{\omega - \omega_0}{\omega_0}}$$ (2.3)

**Cooling the Resonator**

The resonator is cooled to below the critical temperature by a Janis adiabatic demagnetization refrigerator (ADR), which reaches a base temperature of about 75 mK. The main stage is brought to and kept at 3.4 K by a closed-cycle helium pulse-tube cooler. A magnetic ferric ammonium alum (FAA) salt and gadolinium-gallium garnet (GGG) crystal are thermally coupled with the base stage. The salt and garnet are kept in thermal equilibrium with main stage before the experiment begins. The experiment begins by magnetizing the salt slowly with an internal Niobium-Titanium superconducting coil up to a highly magnetized state while still in equilibrium with the main stage at 3.4 K. At this point the magnet has 9 A of current in the coil and the salt is in a 4 Tesla field, this step is called the “mag up.” To ensure the salt is fully magnetized and at 3.4 K we wait 30 min at this 9 A field. Then a mechanical heat switch is disconnected bringing the salt out of thermal contact with the 3.4 K stage and the current in the coil is slowly brought back down to zero. This step is called the “mag down.” During this step the salt goes from a completely magnetized state to a demagnetized state which effectively increases the degrees of freedom. Since temperature is essentially just energy per degree of freedom (equipartition theorem) and the isolated stage has a constant energy, the temperature falls from 3.4 K to the base temperature of 75 mK. The decreased temperature of the salt leads to heat flow out of the base stage and into the salt, thereby reducing the sample temperature. The base temperature will slowly rise as thermal energy creeps in, so it is monitored closely with a ruthenium oxide thermometer.

There are a few reasons why such low temperatures are necessary. The most obvious reason is of course to have the material superconducting, which occurs below 1.2 K for aluminum and 1.6 K for Rhenium. The other reason is related to the two fluid model of free charge in superconductors. In this model a superconductor is made of two
fluids, one being paired electrons (cooper pairs) and the other is made up of electrons in their ‘normal’ low energy state. As the number of normal electrons rise with the temperature as it approaches $T_c$, increased dissipation is expected. It was observed that as the base temperature rose, the resonator $Q$ decreased. To minimize the temperature dependence on the data the base temperature was kept as cold as possible, typically below 100 mK.

Figure 2.7: The Janis ADR with field coil attached (green and copper outside cylinder). Microwaves enter and leave through ports on top of the unit. The resonator sits in the base stage roughly in the middle of the field coil.

The resonator is housed in the base stage and thermally coupled to the salt. Since data is taken for various magnetic fields present during the mag down, an additional coil is needed to provide a variable field for the resonator when it goes superconducting. A simple exterior toroid coil connected to a power supply was used. It was found to generate a uniform 267 $\mu$T field per mA of current. Prior to the mag down step the desired field is set by the second exterior coil. It is important to note that the salt and
superconducting magnet are magnetically shielded from the outside, and more specifically from the resonator by a high permeability shield. The magnetic field was measured with a magnetometer to ensure magnetic isolation of the large fields generated by the superconducting coil magnet. Henceforth, any mention of a coil magnet and/or it’s current will refer only to the exterior coil which generates the field in which the resonator sits unless explicitly mentioned.

Especially for low Q (high loss) measurements, the transmitted power can become quite small due, in part, to attenuation along the microwave path into and out of the ADR. The network analyzer has trouble measuring these incoming low power signals which can be of the order of the thermal noise generated at room temp by the network analyzer. Therefore, amplification is needed before the microwaves return to the network analyzer. A HEMT (high electron mobility transistor) cryogenic amplifier (placed in the 3.4 K) stage was used primarily with two optional exterior amplifiers. The extra amplification only changes the magnitude of the S21 response and not loss tangent or other parameters of interest. To make sure their effects were neglected, the S21 parameters were divided out accordingly during the calibration of the data. The cryogenic amp is especially useful since it has very little thermal noise. For more about the cryogenic amplifier see appendix B.
The experiment involves taking the S21 data at various cool down fields to vary the number of vortices. The entire mag up and mag down cycles from base to 3.4 K and back takes about 2 hours. It is a more practical to only go up enough to bring the resonator out of the superconducting state (about 1.2 K for Aluminum and 1.6 K for Rhenium), change the field and go back down to the base temperature. This is easily achieved with the ADR since the temperature can be modulated with the superconducting magnet. A mag up to around 4 Amps is sufficient to bring the resonator well above the critical temperature where the cool down field can be changed before a mag back down. This saves about 1.5 hours per data set but as mentioned, the base temperature does rise slowly. After 3-4 measurements the base temperature will be above 100 mK and begin to rise more rapidly with time. This has been determined to be a good time to fully mag up and down to bring the base temperature back down to 75 mK.

This process results in a time scale of about 45 min to 1 hour per cool down field data point on average. This is actually very fast considering the time scales for cooling down a dilution refrigerator, another common instrument. An ADR is ideal for this experiment.

*Figure 2.8: Power profile along the microwave path and overall schematic.*
Chapter 3

Data

Data was taken on two CPW resonators; both had 5 µm wide lines, both had 2 fF coupling capacitors and were both made on sapphire substrates. The difference was in the material used for the waveguide and the way it was fabricated; one was aluminum made with a sputtering process, the other was rhenium made with epitaxial growth.

Two major effects were observed for the resonance with the introduction of vortices into the CPW. The first effect was a severe broadening of the resonance with field which implies a dissipative loss due to the introduction of a vortex. The second effect was a downward shifting of the resonance frequency which implies a phase change due to the introduction of a vortex. Figure 3.1 compares the magnitude of S21 for a rhenium resonator at two frequencies.

Figure 3.1: The magnitude of S21 is plotted for zero and 226 µT applied cooldown fields. The resonance shifts lower and de-Q’s (increases in loss) with the increased field due to the presence of vortices. Recall that the loss goes as the width at half max (-3dB from peak).
In figures 3.2 and 3.3 the loss tangent is plotted versus field for the first and second order resonances ($f_2 \approx 2f_1$)

![Graph showing loss tangent vs. cooling field for aluminum CPW, 5 μm, 5 fF data and linear fits.]

Figure 3.2: Aluminum resonator data and linear fits.
In summary it was found that the loss increased linearly with the field which is known from Stan et al to increase linearly with number of vortices. This implies a linear relationship between the loss tangent and the number of vortices in both Aluminum and Rhenium. Each vortex adds some consistent loss to the transmission line. Also, the critical field where the resonator starts to de-Q was approximately the same for both resonators at around ±85 μT. This agrees with relation 1.2 which gives a critical field of 82.7 μT for a 5 µm wide line. On the first order resonance the loss tangent for Aluminum increased approximately 15 times faster with the field than Rhenium while on the second resonance this ratio was about 3.9 times as fast with the field. There was a significant difference in the way the two materials were effected by increased cool down field at the second order resonance relative to the first. The loss tangent for the second order Aluminum CPW resonance changed about half as fast with filed as the first order resonance. While the loss tangent for the second order Rhenium resonance was effected

Figure 3.3: Rhenium resonator data and linear fits.
by the cool down field approximately 1.6 times as severely as the first. In other words
the losses in Aluminum were more significant at the lower resonance while the losses in
the Rhenium were more significant at the higher resonance. This has major implications
on what loss mechanisms function in the two materials. The slopes are point averaged
and summarized in the chart below. If define the slope the fits as $S$:

$$\frac{\Delta \delta}{\Delta B} = S$$  \hspace{1cm} (3.1)

<table>
<thead>
<tr>
<th>Material</th>
<th>$S_1$ at $\omega_1$ (T$^{-1}$)</th>
<th>$S_2$ at $\omega_2$ (T$^{-1}$)</th>
<th>$S_2/S_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum</td>
<td>15.11</td>
<td>6.867</td>
<td>0.455</td>
</tr>
<tr>
<td>Rhenium</td>
<td>1.063</td>
<td>1.656</td>
<td>1.56</td>
</tr>
<tr>
<td>$S(Al)/S(Re)$</td>
<td>14.2</td>
<td>4.15</td>
<td></td>
</tr>
</tbody>
</table>

*Table 3.1: The aluminum slope data includes 4 points left of zero field and 4 points to the
right. The rhenium data includes 4 points left and 13 right of zero. The data is
summarized with point-averaged slopes.*

The second effect was a shifting of the resonance to lower frequencies with the
relative shift $\Delta \omega/\omega_0$ being proportional to the field. The resonance shift data mirrored the
loss tangent data with respect to the critical field, again supporting equation (1.2).
Relative Resonance Shift vs. Cooling Field - Aluminum CPW, 5µm, 5fF

data: 6.0 GHz
right fit: 9.9237e-006 µT⁻¹
left fit: 1.2328e-005 µT⁻¹

data: 12.0 GHz
right fit: 5.7333e-006 µT⁻¹
left fit: 5.5988e-006 µT⁻¹

Figure 3.4: Aluminum resonance shift data
Figure 3.5: Rhenium resonance shift data.

The resonance shifted more rapidly with field for the aluminum resonator at its first resonance than the rhenium. Data given in table 3.2. The second resonance however, shifted at nearly the same rate for both materials. More interestingly was the difference in how the materials responded at the higher resonance. The aluminum resonator shifted less rapidly with field at its higher resonance having about half the shift versus field slope. The rhenium resonator has a nearly identical slope at both resonances, implying the shift was independent of frequency.

\[
\frac{\Delta \left( \frac{\Delta \omega_0}{\omega_0} \right)}{\Delta B} = C \quad (3.2)
\]
The number of points used in the resonance shift and loss tangent data is different for a few reasons. The loss tangent data was found to be more sensitive to the base temperature. The intrinsic loss tangent was found to increases slowly as the ADR warmed up. The temperature was recorded with each data point taken. Later on the points taken above approximately 110 mK were thrown out. It was also clear that a few points still seemed to be way off the trend. It was realized that it might be necessary to wait for the resonator to come into thermal equilibrium with the base stage before data should be taken. The mag up/down cycle brings the base stage from 75 mK to 3.4 K then back down and we had been assuming the resonator was at the same temperature as the base stage. An effort was made to ensure good thermal coupling with the resonator was made on the mount, but these realizations came after much data was taken. Rather than throw out all the data and start anew (recall the 45 min – 1 hr time scale per point) some points were excluded for obviously being taken at a higher resonator temperature. Following the realization that there may be a delay in when we read the base temperature and when the resonator actually reached that temp, we waited 5-10 min after reaching the base temp before taking data. Multiple measurements were taken to ensure the loss tangent was not changing when the S21 sweep was taken.

The resonance shift did not appear to have the same temperature dependence as loss tangent. This effect was consistent throughout all the data points. As can be seen in the plots it was surprisingly linear over a wide range of points. This may be due in part to the ease of determining the resonance frequency from a data set, it is just the frequency where the magnitude of S21 is greatest. Whereas getting loss tangent is a bit more tricky and involves best fitting a curve with multiple parameters, which was more difficult as loss increased.

Using the transmission line theory in chapter 4 and the results from Stan et al it is possible to convert the loss tangent and resonance shift data directly to a series resistance
and inductance per vortex. This step requires only some accepted transmission line formulas and the combination of formulas. The results of these calculations are the following expressions (see chapter 4 for more on deriving these equations):

\[ R_{sv} = \frac{\partial (R_\ell)}{\partial N} = \frac{2n\pi \Phi_0 Z_0 S}{A} \]

\[ L_{sv} = \frac{\partial (L_\ell)}{\partial N} = \frac{2n\pi \Phi_0 Z_0 C}{\omega_0 A} \]

<table>
<thead>
<tr>
<th>Material</th>
<th>R_{sv1} (mΩ)</th>
<th>R_{sv2} (mΩ)</th>
<th>L_{sv1} (fH)</th>
<th>L_{sv2} (fH)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum</td>
<td>0.196</td>
<td>0.178</td>
<td>22.7</td>
<td>25.1</td>
</tr>
<tr>
<td>Rhenium</td>
<td>0.0138</td>
<td>0.0430</td>
<td>14.6</td>
<td>28.2</td>
</tr>
<tr>
<td>R,L_{sv}(Al)/R,L_{sv}(Re)</td>
<td>14.2</td>
<td>4.14</td>
<td>1.55</td>
<td>0.890</td>
</tr>
</tbody>
</table>

Table 3.3: The series resistance and inductance per vortex at the first and second order resonances for aluminum rhenium are given. These values were calculated from equations 3.3 and 3.4 using S and C from tables 3.1 and 3.2 as well as other known quantities.
Chapter 4

Theory

Two models are considered for how these observed effects could be caused by the vortices. The first involves treating the vortex as a simple lumped element with an effective resistance and inductance. This model is simplistic but reasonable since vortices have been shown to have resistive material\(^3\) in their core which the current wants to avoid. The extra distance that the current must travel can be thought of as an added inductance since any length of metal has an inductance. The core is in parallel with the superconducting material which has it’s own inductance that shunts some of the current into the resistive material, leading to loss. In terms of the data, an added series resistance would increase the loss tangent and an added series inductance would decrease the resonance frequency. The second model\(^3\) takes into account vortex movement and involves treating the vortex as an oscillating mass in a fluid. This also seems reasonable since fluid drag is a non-conservative force which dissipates energy by heat much like friction. Any extra power dissipated would increase the loss tangent so this idea is plausible in the context of the data.

Transmission Line Theory

Before going into the vortex model, further some consideration must be made in relating loss tangent to line losses for a transmission line. It can be shown that on resonance a transmission line behaves in an identical way to a parallel lumped element (RLC) resonator with equivalent components. Since the Q of an RLC resonator is known\(^4\) this gives a way of relating the transmission line parameters to loss tangent.
Figure 4.1: A transmission line resonator behaves like a lumped element resonator on
resonance with some equivalent capacitance, inductance and resistance.

\[ R_{eq} = \frac{Z_0}{\alpha \ell} \] (4.1)

\[ C_{eq} = \frac{\pi}{2\alpha \ell Z_0} \] (4.2)

\[ L_{eq} = \frac{n \alpha_0^2 C_{eq}}{2Z_0} = \frac{n^2 \pi \omega_0}{C_{eq}} \] (4.3)

\[ n \omega_0 = \frac{1}{\sqrt{L_{eq} C_{eq}}} \] (4.4)

The important transmission line parameters here are \( \alpha \) (loss in nepers/length), \( Z_0 = 50 \ \Omega \) (characteristic impedance), \( n \) (resonance order) and \( \ell \) (line length). These are properties of all transmission lines not just CPW geometries. So then the loss tangent is related to these parameters through the equivalent elements.

\[ Q_p = \frac{R_p}{X_p} = \frac{n\pi}{2\alpha \ell} \] (4.5)

\[ \delta = \frac{2\alpha \ell}{n\pi} \] (4.6)

So as one would expect, the loss tangent is proportional to \( \alpha \), the loss in the line. The Q and loss tangent above assume there is no loading, or a very high impedance load on the input and output. This unloaded Q is not what is measured, but it can be backed out if the external Q is known via the inverse law above. The external Q is simple to figure out because it only involves a coupling capacitor on each end followed by a 50 \( \Omega \) transmission line.
would normally be (non zero) plus an added term for the vortex. The added term is the just the series inductance per vortex, so $L_v = L_{sv}$. Solving for the resistance per vortex $R_v$ you get:

$$R_{square} \approx R_v \approx \frac{\omega^2 (\xi L_{\ell})^2}{R_{sv}}$$ (4.18)

The inductance per length of a strip of metal depends on two length scales, one is line width $w$ and the other I will just call $b$ (it is not important). Lets consider the width to effectively decrease by $d$ in the vicinity of the vortex. The change in inductance per length can then be related to this parameter $d$:

$$L_{\ell} = \mu_0 \ell n \left( \frac{w}{b} \right)$$ (4.19)

$$\Delta L_{\ell} = \mu_0 \ell n \left( \frac{w-d}{w} \right) \approx \frac{\mu_0 d}{w}$$ (4.20)

So the change in inductance in this region is just equation 4.20 times the distance over which this change occurs. If we take this decrease in width to be the diameter of the vortex then the distance over which the inductance change occurs is just $d$. We find the change in inductance due to the vortex is:

$$L_{sv} = \frac{\mu_0 d^2}{w}$$ (4.21)

$$d = \sqrt{\frac{wL_{sv}}{\mu_0}}$$ (4.22)

In this way we may estimate the size of the vortex from the resonance shift using the lumped element model.

It may also be of interest to relate this model to the loss tangent and resonance shift directly. To relate the lumped element model to loss tangent and resonance the series inductance and resistance per vortex must be converted to per length quantities. If we imagine $N$ vortices on the CPW and replace the resonance frequency as an integer multiple of the first resonance, then:

$$R_{\ell} \approx \frac{N(n_0 \omega_0)^2 \xi (\xi L_{\ell})^2}{\ell R_v}$$ (4.23)

$$L_{\ell} \approx \frac{NL_{sv}}{\ell} + L_{\ell}$$ (4.24)
Which when we combine with equations 4.1 and 4.15 we get a relationship between loss tangent and resonance

\[ \delta = N \frac{n_0 \omega_0^2 \left( \frac{\delta L}{L} \right)^2}{\pi Z_0 R_v} \]  

(4.25)

\[ \omega_0 = \frac{n \pi Z_0}{NL_v + L_v \ell} \]  

(4.26)

\[ - \frac{\Delta \omega_0}{\omega_0} = \frac{\omega_0^{N=0} - \omega_0}{\omega_0^{N=0}} = \frac{NL_v}{NL_v + L_v \ell} = N \frac{L_v}{\ell L_v} \]  

(4.27)

The final step is to combine these equations with 1.3 to get the magnetic field dependence:

\[ \delta = \frac{n_0 A \omega_0^2 \left( \frac{\delta L}{L} \right)^2}{2 \pi \varphi_0 Z_0 R_v} \]  

(4.28)

\[ - \frac{\Delta \omega_0}{\omega_0} = \frac{AL_v}{2 \pi \varphi_0 \ell L_v} \]  

(4.29)

Features of the lumped element model include a loss tangent and relative decrease in resonance frequency which is proportional to the number of vortices and therefore to the cool down field. It is important to note that the loss tangent slope \( S \) (3.1) is proportional to the order of resonance \( n \). The relative resonance shift versus field slope \( C \) is predicted to be independent of the resonance order.

**Vortex Vibration Model**

In the vibrating vortex vibration model the vortices are considered to be dampened oscillators, like a mass on a spring in a viscous fluid. Effectively the vortices are held in place by a pinning force which acts like a restoring spring. The surrounding superconducting metal acts like a fluid through which the vortices are driven to vibrate in by the microwaves. The vortices together act like an energy sink as they vibrate back and forth in the viscous fluid at the drive frequency. Each individual vortex is then governed by the damped harmonic oscillator differential equation 4.24 with a damping term proportional to the flow viscosity. In the equations below \( D \) is the vortex diameter, \( \eta \) is the flow viscosity, \( m \) is the effective mass of the vortex and \( k \) is the effective spring constant for the pinning potential.

\[ m \ddot{x} + \gamma \dot{x} + kx = F \]  

(4.30)
\[ \gamma \propto \frac{D\eta}{m} \]  

This model was first investigated by Gittleman and Rosenblum\(^6\) in the 1960’s for Type II superconductors who calculated a flow viscosity and pinning potential given by 4.26 and 4.27 in cgs units. This leads to a power dissipated per unit volume that should have the form given by equation 4.27.

\[ \eta = \frac{\phi_0 H_{c2}}{c^2 \rho_n} = \text{const.} \]  

\[ k = \frac{2\pi \alpha_c \sqrt{\phi_0}}{c \sqrt{H_0}} \]  

\[ P(\omega) = \frac{J_0^2 \phi_0 H_{c2} \eta^2 \omega^2}{2\varepsilon^2 [\omega^2 \eta^2 + (\omega^2 m - k)^2]} \]

The details of the constants in this equation will not be discussed in detail here but may be found in the Gittleman’s paper\(^6\). Generally speaking, \(\phi_0\) is the flux quantum, \(H_{c2}\) is the upper critical field (only applies to type II superconductors), \(\rho_n\) is the normal state resistivity, \(\alpha_c\) is a measure of the flux pinning defects, \(H_0\) is the magnetic field perpendicular to the metal film, \(J_0\) is the magnitude of current density and \(c\) is the speed of light. Gittleman found that at microwave frequencies the pinning is weak so \(\alpha_c\) and therefore \(k\) are effectively zero. Furthermore, the effective mass of the vortices was calculated by Suhl\(^7\) and is so small it can be neglected. With these considerations in mind the frequency dependence drops out of the power per unit volume equation, an important distinction between the lumped element and vibrating vortex models. Note the power dissipated is proportional to the magnetic field \(H_0\) and so the loss tangent that is also proportional to field. This would imply the same slope for both the first and second order resonances since the frequency dependence dropped out.

\[ \delta = \frac{1}{Q} = \frac{\Delta T_{\text{oscillation}}}{\Delta T_{\text{energy-decay}}} \propto \frac{P(\omega)}{n \omega_0} \propto \frac{(B - B_m)}{n} \]  

\[ \omega_0^2 = k/m - \frac{\gamma^2}{4} \]

The general solution to the damped oscillator as opposed to the undamped involves a shifted resonance for each individual vortex, but this doesn’t tell us anything about the resonance of the macroscopic resonator in which they sit. It is important to recognize the difference between the vortex resonance frequency \(\omega_0^2 = k/m - \gamma^2 / 4\) and the CPW resonance frequency given by 4.15. The vortex vibration model treats the
vortices as dissipative oscillators which have individual which does shift lower when you add dampening, but it doesn’t make sense to think about the vortices as existing without damping. So in that respect there really is no shift in their resonance frequency. It is subtle but non trivial to deduce how the overall resonance should change in this model. One weakness of the vortex vibration model is that is doesn’t provide any phase information about how the resonance should shift.
Chapter 5

Results

The data agrees with previous work by Stan\(^1\) which suggested a critical field that was dependent on the inverse square of line width. The critical field was around the 85 \(\mu\)T for the 5 \(\mu\)m lines primarily used as predicted. There was a slight variation between the samples which is likely due to fabrication inconsistencies. Other measurements taken on CPW’s of smaller and larger width lines even further backed this up.

The loss tangent and resonance frequency varied with field in the same manner as the number of vortices in Stan’s work\(^1\). Figure 1.2 looks quite similar to figures 4.2 through 4.5; beyond the critical field these quantities are proportional to the cool down field. This suggests a linear relationship between loss tangent and the number of vortices as well as resonance shift and the number of vortices. This is agrees with both models.

When the second resonance is considered the metals behaved differently. In particular the Rhenium agreed with the lumped element model more closely with the slope increasing by a factor of about 1.6 for the second resonance. Recall the lumped element model predicts the loss tangent slopes \(S\) be proportional to the order of resonance, so a factor of 2 is predicted. \(S\) for aluminum actually decreased by a factor of about .5 which agrees well with the vortex vibration model. Recall that the vortex vibration model predicts a slope which is inversely proportional to the order of resonance, so a factor of .5 is predicted.

Using the model and the data a resistance per vortex can be backed out from the lumped element model. Some material parameters must be known, they are summarized in the table below. The resistance per square was calculated assuming a 100 nm metal thickness using equation 5.1.

<table>
<thead>
<tr>
<th></th>
<th>Coherence Length (\zeta) (nm)</th>
<th>Normal State Resistively (\rho) ((\Omega)m)</th>
<th>Resistance per Square ((\Omega))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum</td>
<td>1600</td>
<td>(2.65 \times 10^{-8})</td>
<td>.265</td>
</tr>
<tr>
<td>Rhenium</td>
<td>2000*</td>
<td>(18 \times 10^{-8})</td>
<td>1.8</td>
</tr>
</tbody>
</table>

*Approximate value.

Table 4.1: Coherence lengths from BCS theory\(^3\), room temperature normal state resistivity\(^3\) and resistance per square for aluminum and rhenium.
\[ R_{\text{square}} = \rho \frac{L}{Lt} = \rho \frac{t}{t} \]  

(5.1)

The resistance per square is provided as a basis of comparison. One would expect an upper limit for the vortex resistance to be resistance per square at room temperature.

The vortex resistances calculated from the loss tangent slopes and equation 4.23 are provided in the following table:

<table>
<thead>
<tr>
<th>Material</th>
<th>( R_v ) at ( \omega_1 ) (( \Omega ))</th>
<th>( R_v ) at ( \omega_2 ) (( \Omega ))</th>
<th>( R_{\text{square}} ) (( \Omega ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum</td>
<td>0.056</td>
<td>0.027</td>
<td>0.265</td>
</tr>
<tr>
<td>Rhenium</td>
<td>0.11</td>
<td>0.128</td>
<td>1.8</td>
</tr>
<tr>
<td>Ratio of Re to Al</td>
<td>1.8</td>
<td>4.7</td>
<td>6.8</td>
</tr>
</tbody>
</table>

Table 4.2: Summary of the predicted vortex resistances by the lumped element model with the resistance per square available for comparison. Note the predicted values are only one order of magnitude smaller than the resistance per square. The ratio is provided also as a comparison, the rhenium had a higher vortex resistance in each case as would be expected from the higher material resistivity.

As far as the relative resonance shift, both the Aluminum and Rhenium had a linear dependence on cool down field past the critical field which agrees with the lumped element model. The Rhenium agreed exactly with the lumped element model in that the first and second order resonances both had the same slope of shift versus field, note figure 3.5. Aluminum again did not agree with the lumped element model with a smaller slope at the second order resonance by a factor of about 0.6. Recall that there was no prediction on the resonance shift by the vortex vibration model. The vortex inductances are calculated using the frequency shift slopes and equation 4.24 in the following table:

<table>
<thead>
<tr>
<th>Material</th>
<th>( L_v ) at ( \omega_1 ) (( fH ))</th>
<th>( L_v ) at ( \omega_2 ) (( fH ))</th>
<th>( L_\zeta ) (( fH ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum</td>
<td>22.7</td>
<td>25.1</td>
<td>642</td>
</tr>
<tr>
<td>Rhenium</td>
<td>14.6</td>
<td>28.2</td>
<td>780</td>
</tr>
<tr>
<td>Ratio of Re to Al</td>
<td>1.55</td>
<td>0.890</td>
<td>1.25</td>
</tr>
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</table>

Table 4.3: Summary of the predicted vortex inductances by the lumped element model. The parallel line inductance is provided as a basis of comparison and to justify the assumption earlier that the vortex resistance was much smaller.

From equation 4.22 and these inductances per vortex the following decreases in effective line widths were calculated. The line width is known to be 5 \( \mu \)m. The decreases calculated are much smaller than the coherence length of the materials.
suggesting either the decrease in effective width is not the only contributing factor to the resonance shift or the vortex is larger than the coherence length.

<table>
<thead>
<tr>
<th>Material</th>
<th>$d$ at $\omega_1$ (μm)</th>
<th>$d$ at $\omega_2$ (μm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum</td>
<td>0.116</td>
<td>0.316</td>
</tr>
<tr>
<td>Rhenium</td>
<td>0.241</td>
<td>0.335</td>
</tr>
<tr>
<td>Average</td>
<td>0.252</td>
<td></td>
</tr>
</tbody>
</table>

*Table 4.4: Summary of the effective decrease in line widths for aluminum and rhenium at each resonance. The average decrease in effective width is about .25 μm.*

In summary, the loss tangent data agreed with both models and the Stan paper\(^1\) by being proportional to the applied field beyond the critical field. When the second order resonances were considered for both loss tangent and resonance shift Rhenium fit quite well with the lumped element model. The only deviation was in the ratio of the loss tangent slopes being 1.6 when 2 was predicted. When the second order resonances of Aluminum were considered it agreed well with the vortex vibration model. The resistance per vortex calculated for both materials was about one order of magnitude lower than the resistance per square of the materials at room temperature and significantly, the aluminum had a lower vortex resistance. The inductances per vortex calculated form the lumped element model were on the order of femtohenrys and were similar for both materials. In both cases the vortex inductances were much significantly smaller than the line inductance determined from the inductance per length and the characteristic size of the vortex, justifying the assumption made earlier.
Chapter 6

Conclusion

Two effects of vortex introduction were observed and quantified in type I superconducting wires. First, power is dissipated linearly with the number of vortices beyond the critical field as can be seen in the loss tangent data. As was determined in conjunction with Stan’s paper, any magnetic field above the critical field that is perpendicular to the plane of the wire that is present during the superconducting phase transition is going to result in unwanted losses in the wires. This is especially important in Qubits where the goal is to maintain a quantum state as long as possible. If vortices are present the state will decay faster in a way that should be proportional to the magnetic field because of this power dissipated. Second, there is a phase shift associated with the vortices that can effectively be thought of as an increase in the inductance of the line.

Two models were presented with different levels of agreement from the two materials. The rhenium resonator was made with an epitaxial growth process that resulted in a high quality metal lattice, it is likely that there are very few defects in the rhenium. Since vortices are known to form and pin where defects are in materials, the rhenium would be expected to produce more ideal results. The aluminum resonator was fabricated with a sputtering process which tends to lead to more defects. Since the nature of the defects and what effects they might have was not incorporated into the lumped element model, it can be expected that aluminum would deviate from the models’ prediction further than rhenium. The vortex loss data for Aluminum agreed well with the vortex vibration model having the right scaling with resonance order.

The lumped element model was used to calculate vortex resistance values that were only one order of magnitude below the resistances per square. There are many reasons why these values could be off. The vortex was assumed to be of the order of the coherence length, which clearly is an estimate. One would not expect there to be a hard boundary between the normal state metal core and the surrounding superconducting material. The estimation for the size of the vortex from the effective decrease in line width was much smaller than the coherence length, the actual value may be somewhere in between. It is likely that there is a more continuous boundary with average size the
coherence length. In addition, the resistance of a circle of metal would be less than a square. If we used the area for an estimate for how much less we would find the resistance per circle was 20% less than the resistance per square (a square with sides equal to the diameter of the circle). Also, the resistance per square was calculated using the room temperature resistivity but the resistivity of metals increases with temperature approximately linearly at ‘warm’ temperatures. The ‘normal state’ resistivity of aluminum and rhenium at sub 1K temperatures is the residual resistivity which depends on the material properties and thermal history so it is difficult to estimate. The lumped element model has value not only for order of magnitude estimates but also for a qualitative understanding of the vortex effects. The more complex vortex vibration model seems to have promise but as currently developed is inadequate for describing the effects observed in the CPWs aside from the linear dependence of power dissipated on magnetic field. Part of the theory’s shortcoming may lie in the fact that it was developed for type II superconductors and I have tried to use it to understand type I superconductors.

There were certainly other sources of error in the experiment that may have led to inaccuracies. As mentioned, many of the early data points had problems with temperature. They were taken with the base temperature too warm or without waiting for the resonator to come into equilibrium with the base stage. It was realized the early strategy was flawed when the loss tangent would have curvature and appear to be rising nonlinearly with field. After cycling the ADR down to the lowest base temp possible the loss tangent would resume the linear trend. As soon as the 100 mK max base temperature rule was imposed and a 5 min wait period was set before taking data this issue went away. The issue went away but at the cost of time, so there was a trade off between the speed of data taking and the error. Another problem was that in the early stages of this work there was not a good way to get the loss tangent of an S21 sweep quickly after the data had been taken. A days worth of data would be taken then analyzed later. This presented problems because sometimes a sweep, especially the low Q sweeps, would be so bad that a lorentzian could not be fit. So software was developed to enable a quick fit and calculation of loss tangent right when the data was taken. This also helped spot bad data points due to temperature issues right when they occurred. For example, it
was possible with this software to continuously check the Q until it was no longer changing and that would be the appropriate time to save the data and move on to the next point.

This work could be expanded upon in various ways. This most obvious and important priority would be to do the experiment on other materials. Aluminum and rhenium were chosen out of convenience with the resonators already fabricated and available. Another big improvement would be testing the same materials that had been deposited or grown differently. Sputtered rhenium and epitaxial aluminum would be a conclusive test to see if some of the effects are process dependent as I suspect the are. In addition, only the first and second order resonances were tested but if it were possible higher order resonances could confirm the order dependence of the lumped element theory. This would be difficult because of high frequency considerations. The amplifiers used in this experiment were only good up to around the second resonance (12 GHz) so they would not work for third order measurements. Clearly more points would give more accurate data for aluminum and rhenium, so that would be good as well.

As had been suspected, electron vortices can have unwanted effects on superconducting circuits and in general should be avoided. By using magnetic shielding and thin wires their energy dissipative and phase altering effects can be avoided. On the other hand there may be new and novel uses for the added inductive and resistive properties of electron vortices and they may be implemented as circuit elements much like one would use an inductor or resistor in an amplifier. There is much work to be done in this exciting frontier of physics.
Appendix A

S-Parameters

The S-Parameter matrix describes the relationships between incoming and outgoing wave voltages for a two port network. $S_{21}$ is the only parameter used in this experiment and it gives the ratio of the voltage coming out of port B to the voltage going into port A when the powers are properly matched to that outgoing voltage at port A is zero.

\[
\begin{bmatrix}
V_B^{\text{in}} \\
V_B^{\text{out}}
\end{bmatrix} =
\begin{bmatrix}
S_{11} & S_{12} \\
S_{21} & S_{22}
\end{bmatrix}
\begin{bmatrix}
V_A^{\text{in}} \\
V_A^{\text{out}}
\end{bmatrix}
\quad (A1)
\]

\[
V_B^{\text{i}} = S_{21} V_A^{+} + S_{22} V_A^{-}
\quad (A2)
\]

\[
V_A^{\text{out}} = 0 \Rightarrow S_{21} = \frac{V_B^{\text{out}}}{V_A^{\text{in}}}
\quad (A3)
\]

The S-Parameters are complex valued and include both amplitude and phase information. $S_{21}$ is commonly used because it gives the voltage or power gain for a properly matched network. A properly matched network is one where the input and output impedances are equal so as to maximize the power transfer. When a port is properly matched there are no reflected waves hence the outgoing voltage is zero.

\[
\frac{P_{\text{out}}}{P_{\text{in}}} = 10 \log_{10} |S_{21}|
\quad (A4)
\]
Appendix B

Cryogenic HEMT Low Noise Microwave Amplifier Data Sheet

CITCRYO1-12A

Features
RF Frequency: 1-12 GHz
Gain @ 11K: 35.5 dB ±0.5dB (2-12 GHz), typical
Noise temperature @ 11 K: < 6 K, typical average
Noise figure @ 11 K: < 0.09 dB, typical average
IRL: > 15 dB (4.5-12 GHz), typical
ORL: > 20 dB (2.5-12 GHz), typical
Operating temperature: 4.2 K - 320 K
DC power @ 11 K: 1.0 Vdc at 22 mA

Description
The CITCRYO1-12A is a cryogenic, low noise, broadband amplifier. In its standard configuration it comes with female SMA connectors on the RF-input and output and a 4-pin 2 mm pitch header for the DC. The amplifier requires three separate DC-voltages.

Performance Characteristics (T<sub>a</sub>=11K)

<table>
<thead>
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<th>Parameter</th>
<th>Min</th>
<th>Typ</th>
<th>Max</th>
<th>Unit</th>
</tr>
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<tbody>
<tr>
<td>Linear Gain</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>1 GHz</td>
<td>31.5</td>
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<td>12 GHz</td>
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<td>dB</td>
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<td>Noise Temp</td>
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<td>K</td>
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<td>9.0</td>
<td>K</td>
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<table>
<thead>
<tr>
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<th>Min</th>
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<td>IRL</td>
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<td>0.5</td>
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<td>7 GHz</td>
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<td>12 GHz</td>
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<td>9</td>
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<td></td>
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<td>7 GHz</td>
<td>15.0</td>
<td>25.0</td>
<td>dB</td>
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<tr>
<td>12 GHz</td>
<td>15.0</td>
<td>24.0</td>
<td>dB</td>
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</tbody>
</table>

0.5-11GHz LNA #78D at 12.8K
MMIC WBA13 47C3MA CT11 425X-0665, Bias: Vd=1.2V, Id=20.15mA , Vg1=2.83V, Vg2=2.83V
Date: OCT-12-2005
### Appendix C

#### CPW Resonator Design Parameters

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<table>
<thead>
<tr>
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<td>$C_c$</td>
<td>0.5-10 fF</td>
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<td>$C_f$</td>
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<td>$L_f$</td>
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Bibliography


