Berry Phase and Anyons in the Quantized Hall effect

Physics 215 Final Project

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6/11/2009

Contents

Aharonov-Bohm Effect (Review)

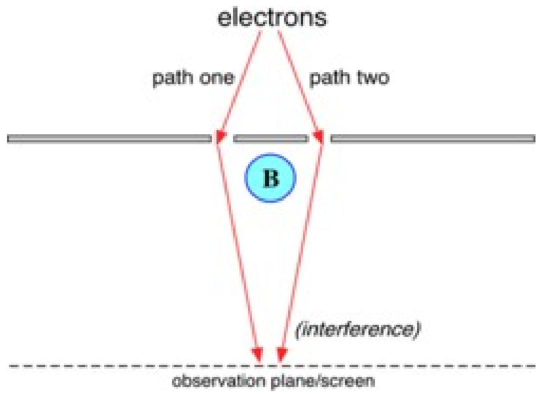
- For a closed loop, the phase factor is proportional to the magnetic flux enclosed

$$\exp\left(\frac{iq}{\hbar c}\oint \vec{A}\cdot d\vec{l}\right) = \exp\left(\frac{iq}{\hbar c}\int \vec{B}\cdot d\vec{S}\right)$$

$$= \exp\left(i2\pi \frac{q}{hc}\Phi_B\right)$$

- Define the flux quantum as $\Phi_0 = \frac{hc}{|e|}$, assume $q = -\nu e$
- The phase factor can be written as

$$\exp\left(-i2\pi\nu \frac{\Phi_B}{\Phi_0}\right)$$



Berry's phase (Review)

- Suppose the Hamiltonian is parametrized by $\{R_1(t), R_2(t), \ldots, R_k(t)\}$
- With non-degenerate eigenstates $|n(R_i)\rangle$
- If the parameters vary slowly with time, we have proved in class that the wave function evolves as

$$\left|n(\vec{R}(t))\right\rangle = e^{i\gamma_n} e^{-\frac{i}{\hbar}\int_0^t E_n(\vec{R}(t'))dt'} \left|n(\vec{R}(0))\right\rangle$$

- Where Berry's phase is

$$\gamma_n(t) = i\int_0^t \left\langle n(\vec{R}(t')) \middle| \frac{\partial}{\partial t'} \middle| n(\vec{R}(t')) \right\rangle dt'$$

Fractional Quantum Hall Effect (Introduction)

- Many electrons+interaction

$$H = \sum_j \left[\frac{1}{2m} (-i\hbar\nabla_j - \frac{e}{c} A_j)^2 + V(\vec{r}_j) \right] + \sum_{j<k} \frac{e^2}{|\vec{r}_j - \vec{r}_k|}$$

- Usually the electrons are confined to a disk of area A pierced by magnetic flux BA
- It can be shown that there are $N = (2k+1)\frac{\Phi_B}{\Phi_0}$ electrons at maximum at lowest Laudau level
- Let z=x+iy. Wave function is

$$\psi_{2k+1} = \prod_{i>j} (z_i - z_j)^{2k+1}$$

Laughlin State

- Wavefunction of the quasihole state

$$\left|\psi(t)\right\rangle = \prod_i (\eta(t) - z_i)\psi_{2k+1}$$

- η is the location of the quasi-hole
- It describes electrons in a strong magnetic field in the lowest Laudau level
- It's the non-degenerate ground state of a repulsive Hamiltonian
- There's a gap between ground state and excited state –(Berry's phase)

Berry's Phase in a Fractional Quantum Hall System

- Consider the Laughlin Quasihole

  $$\left|\psi(t)\right\rangle = \prod_i (\eta(t) - z_i)\psi_{2k+1}$$

- Let's all pretend to agree that this is the wavefunction of a quasihole in the fractional quantum Hall system
- Take the location of the quasi-hole to move slowly around some loop as a function of t
- Want to calculate

  $$\gamma_n = i\int_0^T \left\langle \psi(t) \middle| \frac{d}{dt} \middle| \psi(t) \right\rangle dt$$

Berry's Phase in a Fractional Quantum Hall System

- $$\left\langle \psi(t) \middle| \frac{d}{dt} \middle| \psi(t) \right\rangle = \int \frac{d^2 z}{\eta(t) - z} \langle \psi(t) | \hat{\rho}(z) | \psi(t) \rangle$$
- So the Berry's phase can be expressed as

$$\gamma_n = i \oint d\eta \int d^2 z \frac{\langle \rho(z) \rangle}{\eta - z}$$

- Use the previous fact that $N = (2k+1)\dfrac{\Phi_B}{\Phi_0}$
- It can be further written as

$$\gamma_n = -\frac{2\pi}{2k+1} \frac{\Phi_B}{\Phi_0}$$

Berry's Phase in a Fractional Quantum Hall System

$$\gamma_n = -\frac{2\pi}{2k+1}\frac{\Phi_B}{\Phi_0}$$

- Compare to our previous result $\exp(-i2\pi\nu\frac{\Phi_B}{\Phi_0})$
- This is the phase of a particle of charge

$-\nu e = -e/(2k+1)$!

Berry's Phase in a Fractional Quantum Hall System

- Now consider a multi-quasihole wavefunction
  $$\left|\psi^{a,b}(t)\right\rangle = \prod_i (\eta_a(t) - z_i)(\eta_b - z_i)\psi_{2k+1}$$
- If we carry quasihole a around a closed loop of radius R and b is inside the loop
- N becomes N-1/(2k+1)
- The Berry's phase becomes
  $$\gamma_n = -\frac{2\pi}{2k+1}\frac{\Phi_B}{\Phi_0} + \frac{2\pi}{2k+1}$$

Anyon

- The additional phase factor $\dfrac{2\pi}{2k+1}$
- If we interchange two quasi-particles, the phase factor is $\dfrac{\pi}{2k+1}$, this is just the factor of interchanging two anyons!
- Two identical particles in two dimensions

  $\psi(\vec{r}_1, \vec{r}_2) \rightarrow e^{i\theta}\psi(\vec{r}_1, \vec{r}_2)$
- $\theta = 0, \pi$ Correspond to bosons and fermions
- We have found out anyons in a fractional quantum Hall system!

Conclusion

- We have shown the statistics of quasiparticles in a fractional quantum Hall system
- The interchange of quasiparticles obeys the anyon statistic, which is one of the few observable models for anyon

References

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5. Fractional Statistics in the Quantum Hall Effect, R.B. Laughlin (1989)

The End!

Thank you!

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