The deuteron, n-p scattering, and the strong force

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Phys 215C Final Presentation

Outline

- What's cool about the deuteron?
- What can it tell us about the effective strength of the nuclear potential?
- Quick review of s-wave “zero” energy scattering
- What do we get from applying the above to n-p scattering? (oops! spin dependence...)
- Is there really no singlet “deuteron”?
- The current state of matters

What's cool about the deuteron?

- It is *the only* bound state of two nucleons.
  - To be proven later!
- It has no excited states!
- It is a spin triplet state!
- It probes the strong force between nuclei!
- It would likely NOT exist if it weren't for quantum mechanics!
  - Okay, that's probably true about a lot of things... let's see what I mean, anyway.

Just how bound is it, really?

Schrodinger says:

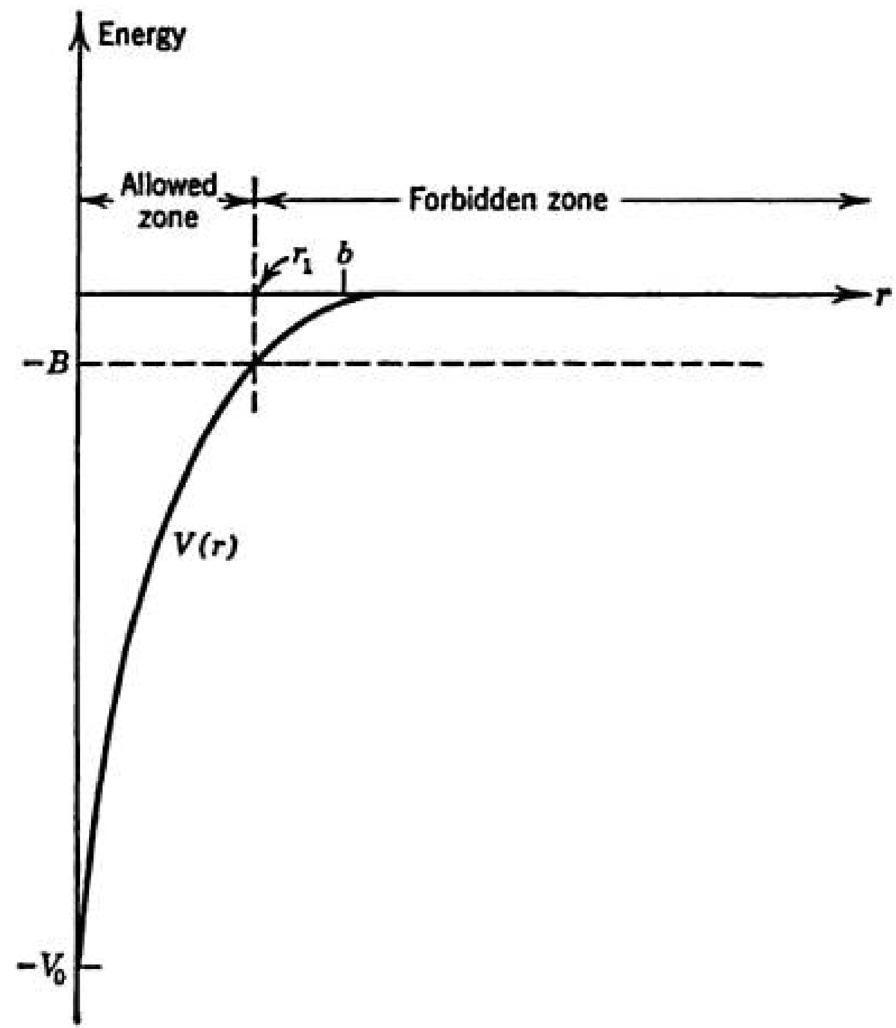
$$\frac{d^2 u}{dr^2} + \kappa^2(r)u(r) = 0$$

where $\kappa = \frac{\pm 1}{\hbar}[M(E - V(r))]^{1/2}$.

Then $\Psi_{out} \propto \exp(-r/R)$, with

$R = \hbar (MB)^{1/2}$.

This gives $R = 4.31 \times 10^{-13}$ cm.



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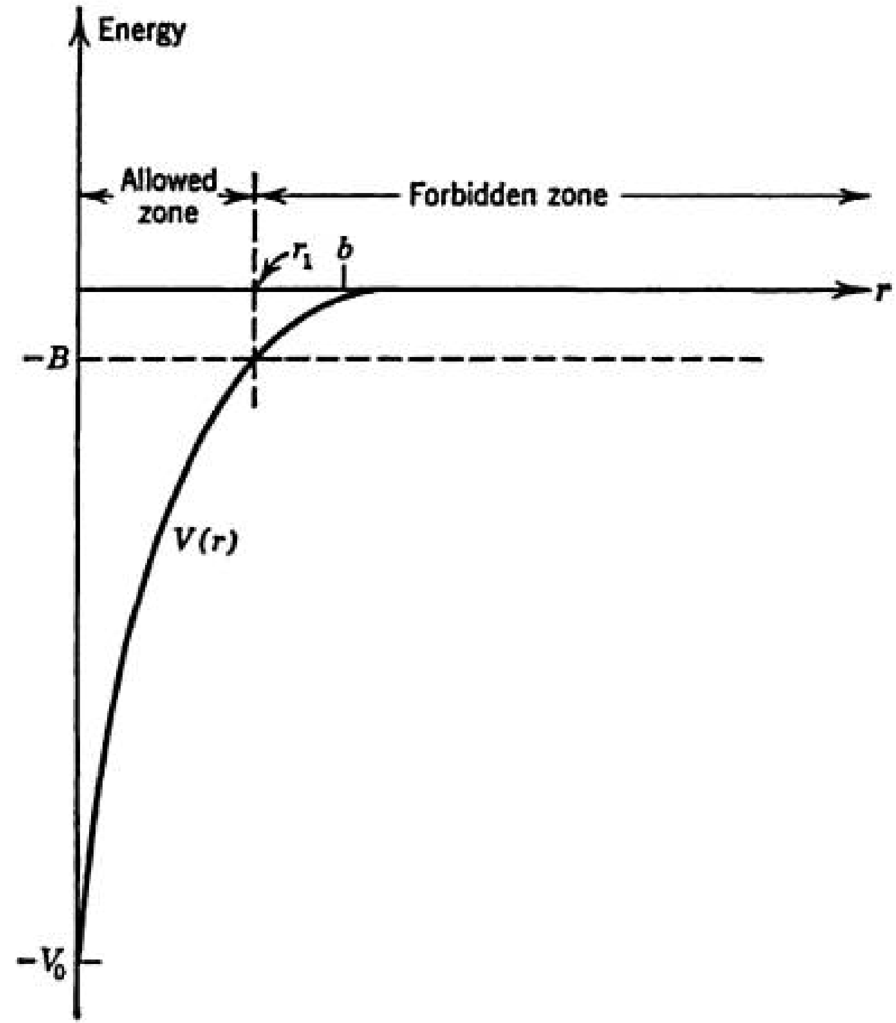

but b = 2 x 10^-13 cm!

What about the depth of the potential?

Assume square well potential. Then:

$$u(r)=A\sin(Kr),\ K=\frac{1}{\hbar}[M(V_0-B)]^{1/2}\ \text{for } r<b$$

$$u(r)=B\exp(-\kappa r),\ \kappa=\frac{1}{\hbar}(MB)^{1/2}\ \text{for } r>b.$$

Match at the boundary:

$$\tan(Kb)=-\left(\frac{V_0-B}{B}\right)^{1/2}.$$

$$\rightarrow V_0 b^2=\left(\frac{\pi}{2}\right)^2\frac{\hbar^2}{M}\sim 1\ \text{MeV barn}$$

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But where did b come from? Scattering experiments, of course!

S-wave "zero" energy scattering

- Recall the lowest term in the partial wave expansion for f(θ):

$$f_0 = \frac{1}{k} \exp^{i\delta_0} \sin(\delta_0)$$

- In "zero" energy scattering we not only claim this is the dominant term, but we also extrapolate it down to k=0.
- Then the scattering length, a, becomes a useful thing to consider:

$$a = -\lim_{k \to 0} \frac{\tan(\delta_0)}{k}$$

- Note that as k-> 0, f must remain finite → δ must shrink as well.

Scattering Length Intuition

- First, we see that: $\lim_{k \to 0} f(\theta) \to -a$
- Then naturally, $\sigma = 4\pi a^2$
- And, outside the range of the potential, in free space, $u(r) \sim (r-a)$, a straight line crossing r-axis at r=a!
- Finally, the sign of a is crucial!

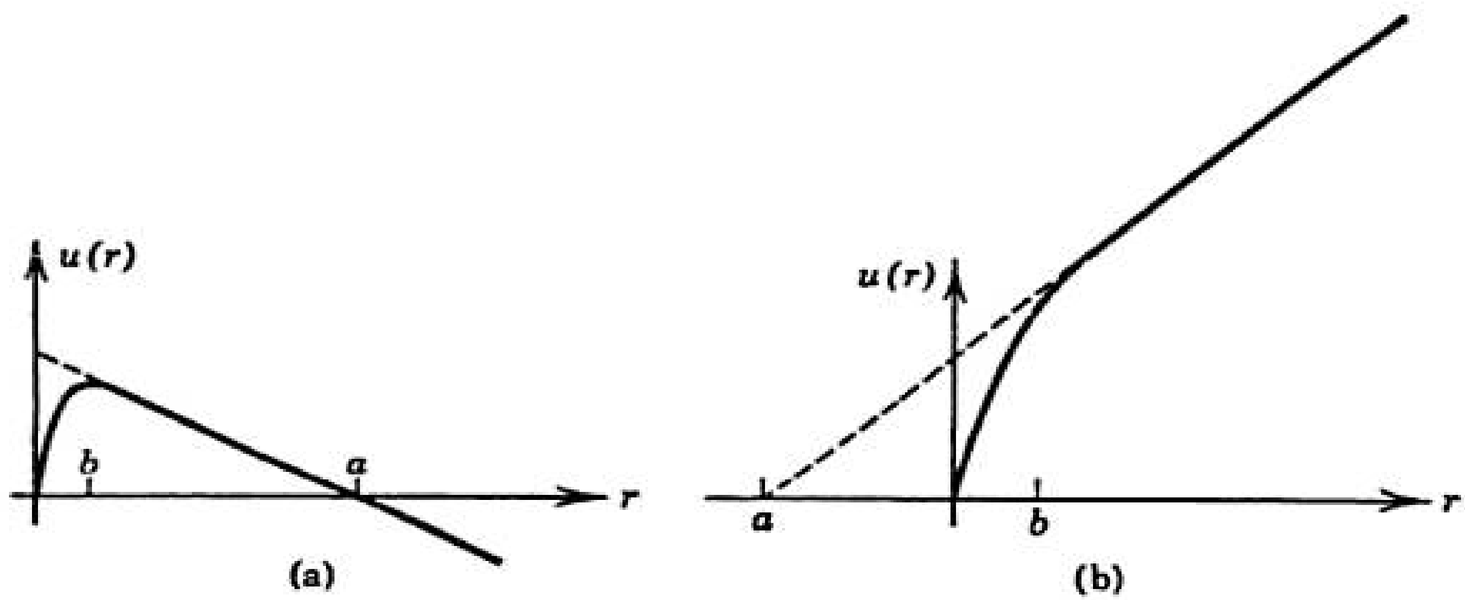

FIG. 3.4. (a) A positive scattering length a implies that a bound state exists (compare Fig. 2.2a). (b) A negative scattering length a implies that the system has no bound state close to zero energy.

Now let's apply all this to n-p scattering...

As before consider a spherical square well.

This time, however, we are scattering particles of $E > 0$. Then:

$$u(r) = B\sin(K'r) \text{ for } r < b$$

$$u(r) = C\sin(kr + \delta_0) \text{ for } r > b$$

where, $K' = \frac{1}{\hbar}[M(V_0 + E)]^{1/2}$, $k = \frac{1}{\hbar}[ME]^{1/2}$.

Matching at interface, we have:

$$K'\cot(K'b) = k\cot(kb + \delta_0).$$

Assumptions: $\delta_0 \sim ak \gg kb$; $V_0 \gg B \gg E$.

$$\rightarrow k\cot(\delta_0) = \frac{-1}{\hbar}[MB]^{1/2}$$

And the cross-section is....

$$\sigma(\theta) = \frac{1}{k^2}\sin^2\delta_0 = \frac{1}{k^2}\frac{1}{1+\cot^2(\delta_0)} \approx \frac{\hbar^2}{MB}$$

$$\rightarrow \sigma_{tot} = 2.3\,\text{barns!!}$$

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Experiment: 20 barns!!!

What's going on?

- E.P. Wigner, 1933: nuclear potential must be **spin-dependent**!
- Triplets must see different well depth than singlets, but in scattering experiment spin was not controlled
- Therefore instead write:

$$\sigma_{tot} = \frac{3}{4}\sigma_t + \frac{1}{4}\sigma_s$$

- This yields: $54 < \sigma_s < 75$ (barns)

Singlet Deuterons?

- We have shown so far that there must at least be a quantitative difference in the effect of nuclear potential on triplets and singlets.
- We also mentioned that no singlet deuterons have been found. Can we at least semi-rigorously prove this must be so?

YES: scattering off parahydrogen

- Introduce projection operators:

$$\pi_s = \frac{1}{4}\left[1 - (\sigma_n \cdot \sigma_p)\right]$$

$$\pi_t = \frac{1}{4}\left[3 + (\sigma_n \cdot \sigma_p)\right]$$

- Then a becomes an operator and splits up into:

$$a_{eff} = a_s \pi_s + a_t \pi_t$$

- Applying this operator to the parahydrogen state, we can solve for the expected cross-section.

Moment of Truth...

$$\sigma_{para} = \frac{16}{9} 16\pi [\frac{3}{4} a_t + \frac{1}{4} a_s]^2$$

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$$\boldsymbol{\sigma}_{\mathbf{para}} = \frac{\mathbf{16}}{\mathbf{9}} \mathbf{16}\boldsymbol{\pi}\left[\frac{\mathbf{3}}{\mathbf{4}}\mathbf{a}_{\mathbf{t}} + \frac{\mathbf{1}}{\mathbf{4}}\mathbf{a}_{\mathbf{s}}\right]^{\mathbf{2}}$$

$$\sigma_{\mathrm{para}} = 77\,\mathrm{barn\ \ if\ \ a_s} > 0$$
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$$\sigma_{para,expt} = 4 \text{ barns!}$$

Current knowledge...

states accessible only to the neutron–proton system. For both cases, when the spins of the two nucleons are coupled to give a total spin $S = 0$ (see Appendix C) the nucleons only experience a central potential.

In Fig. 3.2, the central potential for the anti-symmetric states with $S = 0$ is denoted by V_{C0}. The central potential for symmetric states differs from this, and is not shown.

When the spins couple to $S = 1$ there are four contributions to these potentials, which are then each of the form

$$V(r) = V_{C1}(r) + V_T(r)\Omega_T + V_{SO}(r)\Omega_{SO} + V_{SO2}(r)\Omega_{SO2},$$

where

$$\begin{aligned} \Omega_T &= 3\frac{(\boldsymbol{\sigma}_1 \cdot \mathbf{r})(\boldsymbol{\sigma}_2 \cdot \mathbf{r})}{r^2} - \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 \\ \hbar\Omega_{SO} &= (\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2) \cdot \mathbf{L} \\ \hbar^2\Omega_{SO2} &= (\boldsymbol{\sigma}_1 \cdot \mathbf{L})(\boldsymbol{\sigma}_2 \cdot \mathbf{L}) + (\boldsymbol{\sigma}_2 \cdot \mathbf{L})(\boldsymbol{\sigma}_1 \cdot \mathbf{L}). \end{aligned} \tag{3.4}$$

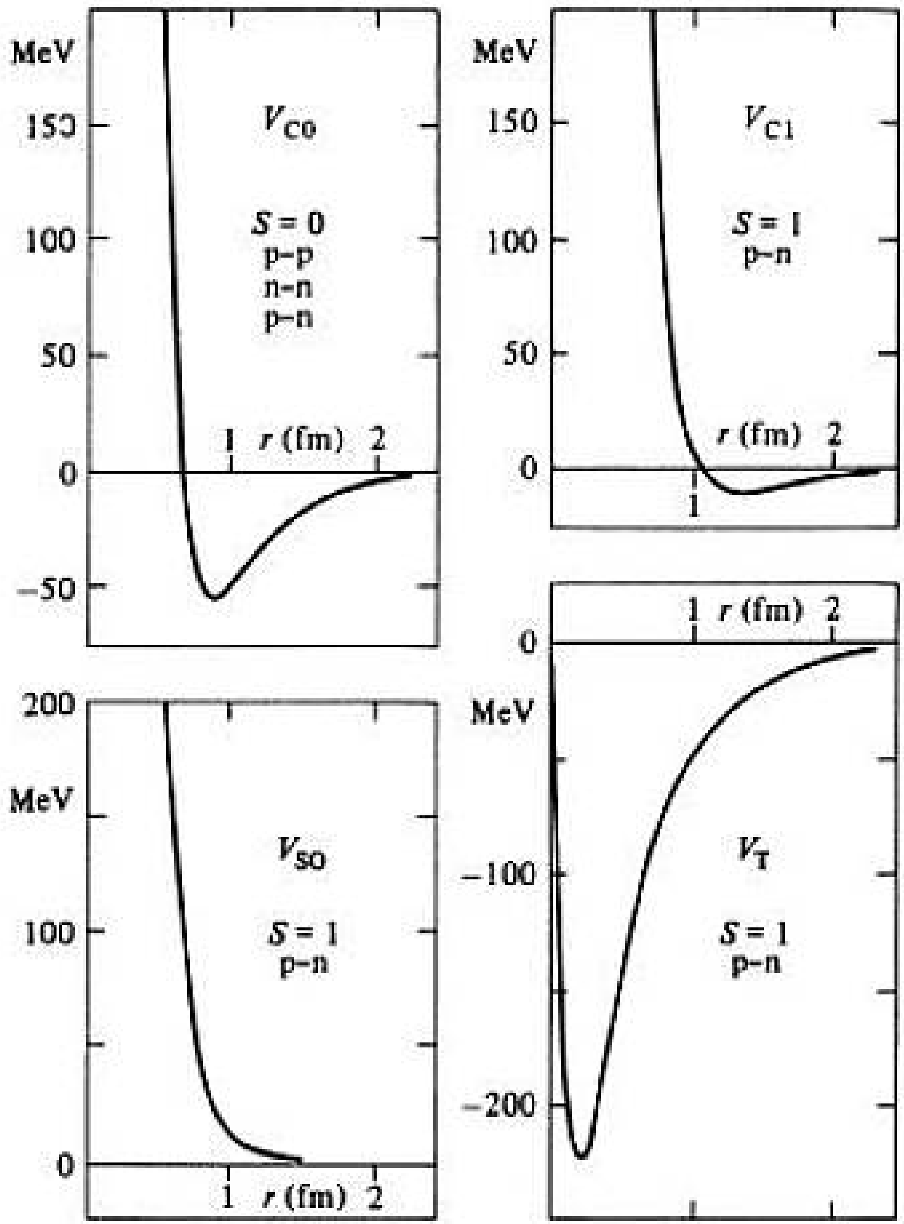

Fig. 3.2 The most important components of the 'Paris potential'. (After Lacombe, M. *et al.* (1980), *Phys. Rev.* **C21**, 861.)

Summary

- Deuteron is barely bound
- Minimal potential well for binding it is 30 MeV
- Strong interaction between nuclei is spin-dependent
- It's spin-dependent enough to actually exhibit a qualitative difference: no singlet bound state can exist.

Thanks!

- Prof. Blaes' lecture notes
- Blatt and Weisskopf, *Theoretical Nuclear Physics*
- *MIT* Applied Nuclear Physics Lec notes (2004)
- Greenwood and Cottingham, *An Introduction to Nuclear Physics*