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Nonlocal hidden variable theories

EPR "paradox" proposed that quantum mechanics was incomplete. Einstein, Podolsky, Rosen, believed that by making use of "hidden" variables, they could maintain a deterministic & non probablistic universe. Perhaps we didn't know or couldn't know, but with hidden variables, there was an answer at all times.

Bell's theorem shows this is wrong. In other words it's not that there's an answer and we don't or can't know it... it's that there really isn't an answer. But, Bell's theorem only ruled out local hidden variable theories.

Bell's theorem shows that local hidden variable theories & QM necessarily predict different things. These have been tested, and QM wins handily.

☆AN ASIDE: When I say "locality," I mean the property that events do not influence events that are spacelike separated from them.

So that's all well & good, but what of NONLOCAL hidden variable theories? (NLHV theories). Can they provide any insight? There is one way they can... if they could be eliminated, we could establish that local hidden variables were unacceptable not for their locality, but for their realism.

☆☆ ASIDE THE SECOND: "Realism" means that a theory is non probablistic, that reality exists independent of observation. In other words, that "God does not play dice."

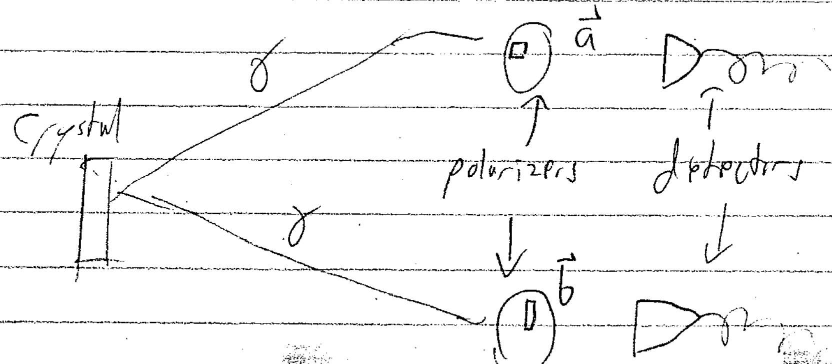
Bohm interpretation

The Bohm interpretation is the most famous NLHV theory. In it, Ψ is a real field, the Ψ-field. It is complex spinor valued & acts on particles nonlocally, via the "quantum potential."

It is totally deterministic, & all particles have definite momentum & position at all times (these are hidden variables). Other observables are not always defined, only when measured do the Ψ-field & position/momentum create them.

Crypto-nonlocal hidden variable theories (CNHV)

Leggett's class of NLHV theories, the "most realistic." Consider a crystal emitting two correlated photons, either one polarized H & one V, or one V & the other H.



How do hidden variable theories describe this?

1. Each pair has unique λ (could be simple, complex, anything).
2. Any process has a stat. ensemble $\rho(\lambda)$
3. $A(\vec{a}, \vec{b}, \lambda, B) = \pm 1$
   $B(\vec{a}, \vec{b}, \lambda, A) = \pm 1$

$$AB = P(\vec{a}, \vec{b}) = \int \rho(\lambda)\, A(\vec{a}, \vec{b}, \lambda, B)\, B(\vec{a}, \vec{b}, \lambda, A)\, d\lambda$$

4. Outcome of A independent of outcome of B.

$A(\vec{a},\vec{b},\lambda,B) = A(\vec{a},\vec{b},\lambda)$

& vice-versa.

5. Outcome of A independent of $\vec{b}$ (polarizer setting)

$A(\vec{a},\vec{b},\lambda) = A(\vec{a},\lambda)$

& vice-versa, since polarizers are set at spacelike separation.

For <u>nonlocal</u> hidden variable theories, we eliminate req. 5 (following Anthony Leggett).
So now, A can depend on $\vec{b}$.

☆☆☆ ASIDE THE THIRD: Why not req. 4? Because if we allowed that, we would be allowing active conspiracy to appear quantum, and could never prove much of anything!

"Wait!" you protest. "A depends on $\vec{b}$?!?" Yes, and many other environmental variables too:

$A(\vec{a},\vec{b},c,d,e,\ldots\lambda)$

But, clearly a single detector system should look the same regardless of its env. So for one detector,

$$\bar{A}(\vec{a},\vec{b},c,d,e\ldots) = \int \rho(\lambda)\, A(\vec{a},\vec{b},c,d,e\ldots\lambda)\, d\lambda = \bar{A}(\vec{a})$$

In particular, for a beam of orientation $\vec{u}$,

$$\bar{A}(\vec{u}\vec{a}) = 2(\vec{u}\cdot\vec{a}) - 1$$

the quantum result. But all experiments are composed of ensembles of pure beams! So:

$$\overline{AB} = \overline{AB}(\vec{a},\vec{b}).$$

Now imagine two photons of orientation $\vec{u}$, & $\vec{v}$ are created.

(I) $\bar{A}(\vec{u},\vec{v},\vec{a},\vec{b}) = \bar{A}(\vec{u},\vec{a}) = 2(\vec{u}\cdot\vec{a})-1$
(II) $\bar{B}(\vec{u},\vec{v},\vec{a},\vec{b}) = \bar{B}(\vec{v},\vec{b}) = 2(\vec{v}\cdot\vec{b})-1$
(III) $\overline{AB}(\vec{u},\vec{v},\vec{a},\vec{b}) = \bar{A}(\vec{u},\vec{a})\,\bar{B}(\vec{v},\vec{b})$.

But now imagine two photons of "unknown" polarization are created.

"Unknown?" you say. "There is no unknown!"

Right you are. They are in a stat. ensemble of polarizations $F(\vec{u},\vec{v})$.

Obviously, I & II still hold. But obviously, III does not necessarily.

Those theories that satisfy I & II but not III are subclass L of CNHV theories. (For general CNHV, you must allow elliptical polarization, a much trickier endeavor).

Leggett's Inequalities

Now we derive Leggett's inequalities, a set of inequalities much like Bell's, for subclass L CNHV theories (for the general case, see his paper).

I follow his derivation very closely.

First, let us define some working terms.

$$\boxed{\begin{aligned} &P(\vec{a},\vec{b}) = \iint F(\vec{u},\vec{v})\,\overline{AB}(\vec{u},\vec{v},\vec{a},\vec{b})\,d\vec{u}\,d\vec{v} \\ &\overline{AB}(\vec{u},\vec{v},\vec{a},\vec{b}) = \int \rho_{\vec{u}\vec{v}}(\lambda)\,A(\vec{a},\vec{b},\lambda)\,B(\vec{a},\vec{b},\lambda)\,d\lambda \\ &\overline{A}(\vec{u},\vec{v},\vec{a},\vec{b}) = \int \rho_{\vec{u}\vec{v}}(\lambda)\,A(\vec{a},\vec{b},\lambda)\,d\lambda = 2(\vec{u}\cdot\vec{a}) - 1 \\ &\overline{B}(\vec{u},\vec{v},\vec{a},\vec{b}) = \int \rho_{\vec{u}\vec{v}}(\lambda)\,B(\vec{a},\vec{b},\lambda)\,d\lambda = 2(\vec{v}\cdot\vec{b}) - 1 \\ &\int \rho_{\vec{u},\vec{v}}(\lambda)\,d\lambda = \iint F(\vec{u},\vec{v})\,d\vec{u}\,d\vec{v} = 1 \end{aligned}}$$

Given $A = \pm 1$, $B = \pm 1$, it is known that:

$$-1 + |\overline{A} + \overline{B}| \leq \overline{AB} \leq 1 - |\overline{A} - \overline{B}|$$

Thus

$$-1 + 2\iint d\vec{u}\,d\vec{v}\,F(\vec{u},\vec{v})\,\left|(\vec{u}\cdot\vec{a})^2 + (\vec{v}\cdot\vec{b})^2 - 1\right| \leq P(\vec{a},\vec{b}) \leq$$

$$1 - 2\iint d\vec{u}\,d\vec{v}\,F(\vec{u},\vec{v})\,\left|(\vec{u}\cdot\vec{a})^2 - (\vec{v}\cdot\vec{b})^2\right|$$

$$\boxed{\begin{aligned} &\text{Define:} \\ &d\vec{u} = \frac{d\theta_u}{2\pi} \qquad d\vec{v} = \frac{d\theta_v}{2\pi} \\ &\xi = \frac{\theta_a + \theta_b}{2} \qquad \phi = \theta_a - \theta_b \\ &\psi = \frac{\theta_u + \theta_v}{2} \qquad \chi = \theta_u - \theta_v \end{aligned}}$$

$$-1 + 2\iint \frac{d\psi}{2\pi}\frac{d\chi}{2\pi}F(\psi,\chi)\,|\cos(2(\xi-\psi))|\,|\cos(\phi-\chi)| \leq P(\xi,\phi) \leq$$

$$1 - 2\iint \frac{d\psi}{2\pi}\frac{d\chi}{2\pi}F(\psi,\chi)\,|\sin(2(\xi-\psi))|\,|\sin(\phi-\chi)|$$

Integrate all parts over ξ, and use:

$$\int \left|\cos 2(\xi - \psi)\right| \frac{d\xi}{2\pi} = \int \left|\sin(2(\xi - \psi))\right| \frac{d\xi}{2\pi} = \frac{2}{\pi}$$

while defining:

$$\boxed{\begin{aligned} \int P(\xi, \phi)\, \frac{d\xi}{2\pi} &\equiv \bar{P}(\phi) \\ \int F(\psi, \chi)\, \frac{d\psi}{2\pi} &= \bar{\rho}(\chi) \end{aligned}}$$

then:

$$-1 + \frac{4}{\pi}\int \frac{d\chi}{2\pi}\, \rho(\chi) \left|\cos(\phi - \chi)\right| \leq \bar{P}(\phi) \leq 1 - \frac{4}{\pi}\int \frac{d\chi}{2\pi}\, \rho(\chi) \left|\sin(\phi - \chi)\right|$$

Finally, using

$$\left|\sin(\phi - \chi)\right| + \left|\sin(\phi' - \chi)\right| \geq \left|\sin(\phi - \phi')\right|$$
$$\left|\cos(\phi - \chi)\right| + \left|\cos(\phi' - \chi)\right| \geq \left|\sin(\phi - \phi')\right|$$
$$\left|\sin(\phi - \chi)\right| + \left|\cos(\phi' - \chi)\right| \geq \left|\cos(\phi - \phi')\right|$$

We arrive at:

$$\boxed{\begin{aligned} \left|\bar{P}(\phi) + \bar{P}(\phi')\right| &\leq 2 - \frac{4}{\pi}\left|\sin(\phi - \phi')\right| \\ \left|\bar{P}(\phi) - \bar{P}(\phi')\right| &\leq 2 - \frac{4}{\pi}\left|\cos(\phi - \phi')\right| \end{aligned}}$$

THE LEGGETT INEQUALITIES

Compare to the quantum mechanical prediction of
$P(\phi) = \cos(2\phi)$

at $\phi = 0$, $\phi' = 18.8°$,

$$|P(\phi) + P(\phi')| \leq 1.5897$$

but

$$|P_{QM}(\phi) + P_{QM}(\phi')| = 1.7923$$

a clear violation! So CNHVs conflict with QM (at least, subclass L CNHVs).

Experiments

This can be confirmed experimentally, and has been. The first was Gröblacher et al. in 2007, achieving a violation of 3.2 std. devs.

This was confirmed by a group in Singapore who achieved 17 std. devs. violation. (Branciard et al.).

Most recently, the original group has achieved 80 std. devs. violation in a new experiment (Gröblacher et al.).