Delayed Choice Quantum Eraser

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1 Introduction

Quantum mechanics has a reputation of being strange, and rightfully so. Schrödinger's equation can be a complicated mess, but in the end, it's still a deterministic equation. Most of the really strange aspects pop out of measurement and entanglement. The former manifests itself in the classic double slit experiment. Whether something behaves as a point particle or a wave depends on how we measure it. Measuring "where" the particle is gives us information regarding the particle properties of the object, but kills the interference pattern that can only emerge from a wave. The story doesn't end there, as Scully and Drühl pointed out in 1982 that it is actually possible to "erase" information. The original proposed experiment involves 3 level atoms, and is quite complicated. In 2000, Walborn, Cunha, Pádua, and Monker carried out an optical analog of Scully, Englert, and Walther's 1991 proposed experiment. This experiment is simpler, and clearly demonstrates the point without complicated equations. The spoiler of course, is that one can't actually erase information that is already known, but can only erase information that *could* be known.

2 Idea of the Experiment

One starts with a standard double slit experiment for photons. One gets fringes, as expected.

Next, one puts a quarter wave plate (QWP) with the slow axis at $+\pi/4$ over one of the holes, and a QWP with the fast axis at $+\pi/4$ over the other hole. An x polarized photon that enters will exit one hole left circularly polarized, and the other right circularly polarized (the opposite happens for y polarized photons). Since these polarizations are orthogonal, they cannot interfere. As expected, the interference pattern dies. Heuristically, we can say that one could gain information about which hole the photon came out of by knowing its original and final polarizations. The interference vanishes even if such information is not requested from the system. In some sense, quantum mechanics doesn't care about what we *do* know, but it does care about what we *could* know.

So far this is nothing a certain quantum mysticism video or zombie Sakurai couldn't tell us. As a digression, we can prematurely throw in a quantum eraser if we wanted to: we bombard the QWPs with x polarized photons, and an x polarizer arbitrarily far from the double slits can erase the "which path" information before detection at the screen. If we run the experiment, the fringes come back. However, this requires a direct interaction with the signal photon, which generates the interfernece fringes or lack thereof. We can make this strange case even stranger by introducing entanglement. In fact, I claim that we can erase the "which path" information after the signal photon has already been measured.

To get entanglement, we want a β barium borate (BBO) crystal. When a photon hits BBO, 2 entangled photons of a lower wavelength come out. One (the "signal" photon) will go to the double slit apparatus as usual, and the other ("idler" photon) will go elsewhere. To prevent spurious counts, we will introduce a coincidence counter: when we get one idler photon and its corresponding signal photon, we take a data point. Else, we assume it's background noise. So far, we still haven't gained or lost any information, so we still have no fringes.

Next we introduce the quantum eraser. It turns out that we can erase the "which way" information by measuring the polarization of the idler photon. Thus, a $+\pi/4$ filter will be our quantum eraser. Indeed, with the eraser in place, the interference fringes come back.

The final step is to delay the quantum eraser. Since it doesn't matter the order in which we measure the signal and idler photons, we can delay the polarization measurement of the idler photon arbitrarily. In principle then, we can send N photons into a BBO, measure N signal photons, and send N idler photons to Andromeda. Assuming low loss in interstellar space (mostly vacuum), we can dispense with the coincidence counter. As soon as our friends in Andromeda measure through a $+\pi/4$ filter the N idler photons sent to them, an interference pattern appears on earth. If they don't make such a measurement, then no interference appears. One might complain about the filter killing off half the idler photons, but it turns out that the idler photons polarized along $-\pi/4$ also generate an interference fringe. We can now send signals instantaneously... which violates causality...

The obvious reason for this is because I pulled a fast one (two, actually). While it is true that both $\pm\pi/4$ polarizations of the idler photon results in an interference pattern from the signal photon, the two interference patterns are offset such that their sum is equivalent to no interference. This is why the coincidence counter is critical to this experiment, and not just for discarding "spurious counts"; we need to know which fringe a signal photon belongs to.

3 Quick Optics Review

Before grinding through the kets, let's start with a quick optics review, so we know exactly what those QWPs are doing. At the same time, we can get some kets defined. I will use the following definitions

$|x\rangle$ = x polarized, $|y\rangle$ = y polarized

$|\leftarrow\rangle$ = left circularly polarized = $\frac{|x\rangle+i|y\rangle}{\sqrt{2}}$

$|\rightarrow\rangle$ = right circularly polarized = $\frac{|x\rangle-i|y\rangle}{\sqrt{2}}$

$|+\rangle = \frac{|x\rangle+|y\rangle}{\sqrt{2}}$, $|-\rangle = \frac{|x\rangle-|y\rangle}{\sqrt{2}}$

The Jones matrix for a QWP with the slow axis and fast axis on the x axis are, respectively,

$$e^{i\pi/4}\begin{bmatrix} 1 & 0 \\ 0 & -i \end{bmatrix}, \text{ and } e^{i\pi/4}\begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$$

Since the polarizers in the experiment are at $+\pi/4$, we can't use the above matrices. Luckily, finding the required Jones matrices using what we have is not hard. First, we rotate our coordinates by $+\pi/4$, which rotates the incoming light by $-\pi/4$, and which places the slow (fast) axis of the polarizer on the x axis. Since our polarizers are now oriented conveniently, we can now use the above matrices to act on the light. Finally, we need to rotate by $-\pi/4$ to get our original coordinates. The composite matrix we want is then

$$\begin{aligned}
&\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} e^{i\pi/4}\begin{bmatrix} 1 & 0 \\ 0 & \mp i \end{bmatrix}\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \\
&= \frac{e^{i\pi/4}}{2}\begin{bmatrix} 1 & \pm i \\ 1 & \mp i \end{bmatrix}\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \\
&= \frac{e^{i\pi/4}}{2}\begin{bmatrix} 1 \mp i & 1 \pm i \\ 1 \pm i & 1 \mp i \end{bmatrix} \\
&= \frac{e^{i\pi/4}}{\sqrt{2}}\begin{bmatrix} e^{\mp i\pi/4} & e^{\pm i\pi/4} \\ e^{\pm i\pi/4} & e^{\mp i\pi/4} \end{bmatrix}
\end{aligned} \tag{1}$$

What the polarizers do to x and y polarized light can then be summarized into the following table.

	slow axis at $+\pi/4$	fast axis at $+\pi/4$
x	$\vert\leftarrow\rangle$	$i\vert\rightarrow\rangle$
y	$i\vert\rightarrow\rangle$	$\vert\leftarrow\rangle$

4 Math

The state will be described with linear combinations of the following ket:

$$|\text{idler polarization}\rangle \otimes |\text{signal polarization}\rangle \otimes |\text{signal position}\rangle \tag{2}$$

As usual, the tensor product symbols will subsequently be dropped. To simplify things, I will label the signal photon's position by which slit it enters: $|L\rangle$ for the left slit, and $|R\rangle$ for the other. Without the QWPs, the state ket is (I believe that the BBO spits out a "singlet" state, but it doesn't matter)

$$(|x\rangle|y\rangle - |y\rangle|x\rangle)(|L\rangle + |R\rangle)/\sqrt{4} \tag{3}$$

Given what we know about QWPs in §3, the state kets immediately after the QWPs is

$$\{|x\rangle(i|\rightarrow\rangle|L\rangle + |\leftarrow\rangle|R\rangle) - |y\rangle(|\leftarrow\rangle|L\rangle + i|\rightarrow\rangle|R\rangle)\}/\sqrt{4} \tag{4}$$

Using $|x\rangle = \frac{|+\rangle+|-\rangle}{\sqrt{2}}$, $|y\rangle = \frac{|+\rangle-|-\rangle}{\sqrt{2}}$ to rearrange into the "eraser" basis,

$$
\begin{aligned}
=&\{|+\rangle(i|\rightarrow\rangle|L\rangle+|\leftarrow\rangle|R\rangle-|\leftarrow\rangle|L\rangle-i|\rightarrow\rangle|R\rangle)\\
&+|-\rangle(i|\rightarrow\rangle|L\rangle+|\leftarrow\rangle|R\rangle+|\leftarrow\rangle|L\rangle+i|\rightarrow\rangle|R\rangle)\}/\sqrt{8}\\
\\
=&\{|+\rangle(i|\rightarrow\rangle-|\leftarrow\rangle)(|L\rangle-|R\rangle)\\
&+|-\rangle(i|\rightarrow\rangle+|\leftarrow\rangle)(|L\rangle+|R\rangle)\}/\sqrt{8}\\
\\
=&\{|+\rangle\sqrt{2}(i-1)|-\rangle(|L\rangle-|R\rangle)\\
&+|-\rangle\sqrt{2}(i+1)|+\rangle(|L\rangle+|R\rangle)\}/\sqrt{8}\\
\\
=&\{|+\rangle|-\rangle(|L\rangle-|R\rangle)\frac{(i-1)}{\sqrt{2}}\\
&+|-\rangle|+\rangle(|L\rangle+|R\rangle)\frac{(i+1)}{\sqrt{2}}\}/\sqrt{2}
\end{aligned}
\tag{5}
$$

Now it's quite clear exactly what is happening. Measuring the idler photon in the $\pm\pi/4$ basis collapses the spatial component of the signal photon's wave function to either $|L\rangle+|R\rangle$ or $|L\rangle-|R\rangle$. Both states create perfectly legitimate interference fringes, but because of the - sign, when one is a peak, the other is a trough. Since these two cases are equally likely, without certain polarization data, the peaks of one will fill in the troughs of the other to yield a composite image without interference fringes. Since the signal photon detection screen doesn't know about the idler photon, this must also be what happens when one measures the idler photon in the x-y basis, or not at all. The quantum eraser (if used) then gives us information to sift through the mess to reconstruct the respective interference fringes.

With regards to the delayed nature of the eraser, there's just a causally independent correlation between the spin states of the 2 photons and the position state of the signal photon. This is not unlike the experiment that broke Bell's Inequalities; the order in which things are measured has no impact on causality or measurement correlations.

One also notices that one can determine which fringe a signal photon belongs to by measuring its polarization, and discarding the idler photon. This is consistent with the earlier digression in §2. Quantum mechanically, this idler photon is completely superfluous. Classically though, this setup allows us to disturb object A by disturbing object B.

If you really think about it, you're not actually erasing anything that is already known, since you never knew the polarization (or which slit the photon went through) to begin with. The eraser does however, erase all possibility of *eventually* determining this data.

5 Classical Remarks

Though quantum mechanically not surprising, this experiment is classically shocking on many levels. Classically, when we make a measurement on one particle, we don't have to disturb other particles. However, due to entanglement, it is clear that making a measurement on one particle disturbs the entangled other. For instance, measuring the signal photon at the peak of a fringe tells us the polarization of the idler photon; this measurement collapses the states of both photons.

Furthermore, we classically expect the photon to come out of *one* slit, not both. The double slit experiment without the delayed eraser already told us that either outcome is possible, but we still semiclassically expected the photon to choose left slit, right slit, or both. This experiment, however, tells us that the photon doesn't even make the choice until after the system has been measured. The measurement, then, is nothing more than choosing a basis; looking back at those basis transformations earlier, it's not so surprising anymore.

Unfortunately, though classical ideas have been hammered into our heads since undergrad, the universe isn't classical. As Professor Bouwmeester said (paraphrased), "some quantum results seem astonishing at first, but when analyzed properly, is quite ordinary".