UNIVERSITY OF CALIFORNIA, SANTA BARBARA
Department of Physics

Physics 100A — Summer 2004

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ASSIGNMENT 2

Part A due Wednesday, June 30. Part B due Monday, July 5.

Keep reading Chapter 3 of Boas, and the book chapters on the Physics 100A course web site (http://www.eres.library.edu under Physics 100A).

PART A: DUE WEDNESDAY, JUNE 30 (Do these right away!):

Boas Section 8, page 137: 29, 35

(1). Consider the space of functions defined on the whole $x-$axis. Starting from the basis $(1, x, x^2, x^3, ...)$, construct the first four polynomials which are orthonormal, with the following definition of the scalar product:

$$< f|g >= \int_{-\infty}^{\infty} e^{-x^2} f^*(x)g(x)dx$$

(These are Hermite polynomials.) You can use the following facts without proof:

$$\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi},$$

$$\int_{-\infty}^{\infty} xe^{-x^2} dx = 0,$$

$$\int_{-\infty}^{\infty} x^2e^{-x^2} dx = \sqrt{\pi}/2,$$

$$\int_{-\infty}^{\infty} x^3e^{-x^2} dx = 0,$$

$$\int_{-\infty}^{\infty} x^4e^{-x^2} dx = \frac{3}{4}\sqrt{\pi},$$

$$\int_{-\infty}^{\infty} x^5e^{-x^2} dx = 0,$$

$$\int_{-\infty}^{\infty} x^6e^{-x^2} dx = \frac{15}{8}\sqrt{\pi}.$$

In addition, do the following problem from Shankar QM:

(2). Exercise 1.3.1 (page 15): Form an orthogonal basis in two dimensions starting with $|A >= (3, 4)$ and $|B >= (2, -6)$. Can you generate another orthonormal basis starting with these two vectors? If so, produce another.

(3). Exercise 1.3.4 (page 17): Prove the triangle inequality starting with $||V+W||^2$ (i.e. the norm of $V+W$ squared). You must use Re$< V|W > \leq | < V|W > |$ and the Schwarz inequality. Show that the final inequality becomes an equality only if $|V >= a|W >$, where a is a positive real scalar.

PART B: DUE MONDAY, JULY 5:

Boas Section 9, p. 141: 15

Boas Miscellaneous Problems p. 144: 18

In addition, do the following problem from Shankar QM:

(4). Exercise 1.6.2 (page 27): Given Ω and Λ are Hermitian operators, what can you say about (a) the product $\Omega\Lambda$, (b) $\Omega\Lambda + \lambda\Omega$ where λ is a complex number, (c) the commutator $[\Omega, \Lambda]$, and (d) $i[\Omega, \Lambda]$?

(5). Exercise 1.6.3 (page 28): Show that the product of two unitary operators is unitary.

(6). Exercise 1.6.6 (page 29): Verify that the following matricies are unitary:

$$\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}$$

$$\frac{1}{2}\begin{pmatrix} (1+i) & (1-i) \\ (1-i) & (1+i) \end{pmatrix}$$

Verify that the determinant is of the form $\exp i\theta$ in each case. Are either of the matricies Hermitian?

(7). Discussion Problem** (**Note change: you must turn in discussion problems**): Show that $(AB)^\dagger = B^\dagger A^\dagger$.

** This problem will be solved in detail as an example problem at the beginning of each discussion section (TA office hours), and *must* be handed in with the homework due on Monday, July 5.