UNIVERSITY OF CALIFORNIA, SANTA BARBARA
Department of Physics

Physics 100A — Summer 2004
Prof. Jean Carlson

ASSIGNMENT 3

Due Monday, July 12.

Reminder: Midterm exam in class on Thursday, July 15. The exam will be open book (Boas) and open notes, and will cover material through the end of the Spectr al Theorem for Normal Operators, i.e. through the end of the Linear Algebra (Se ction A of the syllabus).

Keep reading Chapter 3 of Boas, and the book chapters on the Physics 100A course web site (http://www.eres.library.edu under Physics 100A).

DUE MONDAY, JULY 12:

From Shankar Quanturm Mechanics:

(1). Exercise 1.7.2 (page 30): Show that the determinant of a matrix is unaffected by a unitary change of basis. [Equivalently, show detΩ=det($U^\dagger \Omega U$).]

Boas Chapter 10, Section 4 (page 418): Problem 7, 37

Boas Chapter 10, Section 5 (page 426): Problem 11

From Shankar Quantum Mechanics:

(2). Exercise 1.8.1 (page 41):

(a) . Find the eigenvalues and normalized eigenvectors of the matrix

$$\Omega = \begin{pmatrix} 1 & 3 & 1 \\ 0 & 2 & 0 \\ 0 & 1 & 4 \end{pmatrix}$$

(b). Is the matrix Hermitian? Are the eigenvectors orthogonal?

(3). Exercise 1.8.2 (page 41): Consider the matrix

$$\Omega = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

(a). Is the matrix Hermitian?

(b). Find its eigenvalues and eigenvectors.

(c). Verify that $U^\dagger \Omega U$ is diagonal, where U is the matrix of eigenvectors of Ω.

Also do the following problem from Shankar, Basic Training:

(4). Problem 9.3.2 (page 250): In the space of 3-tuples consisting of column vectors:

$$|a>= \begin{pmatrix} a_x \\ a_y \\ a_z \end{pmatrix}$$

Find the three 3×3 matricies corresponding to the projection operators P_x, P_y, and P_z which preserve the components of a vector in the directions x, y, or z respectively but kill the other two. In other words

$$P_x|a>= \begin{pmatrix} a_x \\ 0 \\ 0 \end{pmatrix}$$

What do you get when you form the sum of the projection operators? Give arguments for why the result has to be so.

(5). Discussion Problem** (**Note: you must turn in discussion problems. This is a tricky one, so definitely plan to attend one of the discussion sections.**): The matricies, U and H are related by $U = \exp(i\alpha H)$ where α is real. The exponential function is defined by its power series expansion,

$$\exp(A) = \sum_{n=0}^{\infty} A^n/n!, \qquad A^0 = I$$

and I is the identity operator. You can use without proof the fact that

$$\exp(i\alpha H)\exp(-i\alpha H) = \exp(i\alpha(H - H)) = 1.$$

a). If H is Hermitian, show that U is Unitary.

b). If U is Unitary, show that H is Hermitian.

Caution: Do not assume that $\exp(i\alpha H)\exp(-i\alpha H^\dagger) = \exp(i\alpha(H - H^\dagger))$

** This problem will be solved in detail as an example problem at the beginning of each discussion section (TA office hours), and *must* be handed in with the homework due on Monday, July 12.