UNIVERSITY OF CALIFORNIA, SANTA BARBARA
Department of Physics

Physics 100B — Summer 2004
Prof. Jean Carlson

ASSIGNMENT 4

Part A due Wednesday, August 25. Part B due Monday, August 30. The midterm exam on Thursday, August 26 (in class). The exam will be open Boas, and open notes, and covers material in chapters 2 and 14 of Boas. Part A of this assignment includes problems from last year's midterm exam (for review), as well as some problems from Chapter 11.

The rest of the quarter we will focus on material in Boas Chapters 11 and 15. There will be a lot of new material, especially in Chapter 15 (Fourier Transforms, Laplace Transforms, and Greens Functions). It would be a good idea to read ahead.

PART A: DUE WEDNESDAY, AUGUST 25: From Boas Chapter 11 and last year's midterm exam (Do these right away!):

(1). For the function $f(z)$ below: (i) locate all of the singularities, and (ii) determine the order of each singularity. Also, (iii) determine the residue at each singularity.

$$f(z) = \frac{1}{z(e^z - 1)}$$

(2). In this problem, assume a is some complex number.

(a). For the function $f(z)$ below, locate all of the singularities, determine the order of each singularity, and find the residue at each singularity.

$$f(z) = \frac{1}{(z-a)^2 + 1}$$

(b). Use your results from part (a) and contour integration to determine the possible values of the following integral $I(a)$, and the range of a for which the integral takes each of the possible values.

$$I(a) = \int_{-\infty}^{\infty} \frac{dx}{(x-a)^2 + 1}.$$

(Hint: the integral has three different possible values.)

(3). This problem makes use of the mapping $z = w^2$ to solve Laplace's equation $\nabla^2 T(x, y) = 0$ for the steady state temperature profile $T(x, y)$ in the upper half z-plane.

(a). For the transformation $z = w^2$, determine and label the specific points in $w-$plane which correspond to the points A, B, C, D, E which lie on the real axis in the $z-$plane. Here $A = -R$ and $E = R$ (where R is a large positive real number). The other points are $B = -1$, $C = 0$, and $D = 1$.

(b). Determine the set of points in the $z-$plane where the mapping $z = w^2$ is conformal. Determine the set of points where $z = w^2$ fails to be conformal. Justify your answer.

(c). Use the mapping $z = w^2$ to find the steady state temperature profile $T(x, y)$ on a semi-infinite, two-dimensional plate in the upper half z-plane ($y > 0$). The boundary conditions are constant temperature $T(x, 0) = 0$ on the positive x-axis, and insulating $\partial T/\partial n(x, 0) = T_y(x, 0) = 0$ on the negative x-axis. Sketch lines of constant temperature in both the w-plane and the z-plane.

From Boas Chapter 11:

Section 3, p. 459-460: problem 14

Section 7, page 464-465: problem 9

PART B: DUE MONDAY, AUGUST 30: From Boas Chapters 11 and 15:

From Boas Chapter 11:

Section 11, page 474: problem 3, 8

Section 13, page 481: problems 3, 4

From Boas Chapter 15:

Section 4, pages 652-654: problem 25

Section 10, page 681-683: problem 20