UNIVERSITY OF CALIFORNIA, SANTA BARBARA
Department of Physics

Physics 100B — Summer 2004

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ASSIGNMENT 5

Part A due Wednesday, September 1. Part B due Wednesday, September 8. The final exam on Thursday, September 9 (in class). Part B of this assignment includes problems from last year's final exam (for review), as well as some problems from Chapter 15. This is really two assignments (due on two separate Wednesdays, because of the Labor Day holiday). I'm giving you all of the problems now so you can get a head start on the material.

This is the last assignment before the final exam, which covers Γ and β-functions, asymptotics, Fourier and Laplace Transforms, Dirac δ-functions, and Green's Functions. This is the material in Boas, Chapters 11 and 15 (but you will need to be able to do contour integrals to compute Fourier and Laplace transforms). This is a LOT of material. It would be a good idea to get a very early start on this homework (today!), and read ahead on the topics we will cover before coming to class.

PART A: DUE WEDNESDAY, SEPTEMBER 1: From Boas Chapter 15 (Do these right away!):

From Boas Chapter 15:

Section 2, page 641: problems 12, 27

Section 3, pages 646-647: problems 10, 30

Section 5, page 661: problem 14, 17

PART B: DUE WEDNESDAY, SEPTEMBER 8: From Boas Chapter 15 and last year's final exam:

Section 7, page 670: problem 6, 11

Section 8, page 675-676: problems 6, 18

Section 9, page 680: problem 5

Section 10, pages 681-683: problem 24

Also do the following problems from last year's final exam:

1). Using repeated integration by parts, find the two highest order terms (i.e. the leading order term, and the first correction to that term) in the asymptotic series of the *incomplete* gamma function $\Gamma(n, x)$ in the limit $x \to \infty$. Here

$$\Gamma(n, x) = \int_x^{\infty} t^{n-1} e^{-t} dt,$$

and you can assume n is a large, positive, real integer. Verify that magnitude of the first term is much bigger than that of the second term in the $x \to \infty$ limit.

2). For each of the following statements indicate whether it is true or false. In order for a statement to be true, it must always be true. Let $f(x)$ and $g(x)$ be two complex functions of a real variable x, and let $\mathcal{F}\{f(x)\} = \hat{f}(k) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)e^{-ikx}dx$ denote the Fourier transform of $f(x)$, and $\mathcal{F}\{g(x)\} = \hat{g}(k) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} g(x)e^{-ikx}dx$ denote the Fourier transform of $g(x)$.

a). $\mathcal{F}\{f(x)+g(x)\} = \hat{f}(k)+\hat{g}(k)$

b). $\mathcal{F}\{f(x)g(x)\} = \hat{f}(k)\hat{g}(k)$

c). If $f(x)$ is real for real values of x then $\hat{f}(k)$ is real for real values of k.

d). Let $g^*(x)$ denote the complex conjugate of the function $g(x)$ for x real, and $[\hat{g}(k)]^*$ denote the complex conjugate of $\hat{g}(k)$ for k real, then $\mathcal{F}\{g^*(x)\} = [\hat{g}(k)]^*$.

Now let $\mathcal{L}\{f(x)\} = \tilde{f}(s) = \int_0^{\infty} f(x)e^{-sx}dx$ denote the Laplace transform of $f(x)$, and $\mathcal{L}\{g(x)\} = \tilde{g}(s) = \int_0^{\infty} g(x)e^{-sx}dx$ denote the Laplace transform of $g(x)$. Again indicate whether each statement is true or false.

e). $\mathcal{L}\{f(x)+g(x)\} = \tilde{f}(s)+\tilde{g}(s)$

f). $\mathcal{L}\{f(x)g(x)\} = \tilde{f}(s)\tilde{g}(s)$

g). If $f(x)$ is real for real values of x then $\tilde{f}(s)$ is real for real values of s.

h). Let $g^*(x)$ denote the complex conjugate of $g(x)$ for x real, and $[\tilde{g}(s)]^*$ denote the complex conjugate of $\tilde{g}(s)$ for s real, then $\mathcal{L}\{g^*(x)\} = [\tilde{g}(s)]^*$.

3). Using Laplace transforms, solve the following equation

$$\phi(x) = x + \int_0^x (t-x)\phi(t)dt.$$

Derive all necessary transforms– you may not quote tables, though you may use the basic properties that were derived in class.

4). Using the method of obtaining an explicit solution to the homogeneous equation, find the Green's function $G(x,x')$ for the operator $\hat{\mathcal{L}} = d^2/dx^2$ on the interval $x \in [0,1]$ for the boundary conditions $y(0)=0$, and $y'(1)=0$. The Green's function satisfies

$$\frac{d^2G(x,x')}{dx^2} = \delta(x-x'),$$

with the same boundary conditions as $y(x)$.