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Supplementary Information

1. VOODOO CAT STATE

In the main article we display the measured and calculated Wigner functions for the resonator states $|0\rangle + |N\rangle$ and for the states $|1\rangle + \exp(ik\pi/8)|3\rangle + |6\rangle$, $k = 0$ to 4. In Fig. S1 we display the "Voodoo cat" state, which involves Fock states as high as $|9\rangle$, fully demonstrating the range of states we can currently prepare.

2. WIGNER TOMOGRAPHY AND DENSITY MATRIX

The Wigner function $W(\alpha)$ and density matrix ρ are related via the trace

$$W(\alpha) = \frac{2}{\pi}\mathrm{Tr}\left(D(-\alpha)\rho D(\alpha)\,\Pi\right). \quad (4)$$

To measure the Wigner function, we first prepare the resonator state, as given by the density matrix ρ. During state analysis, microwaves drive the resonator and coherently displace the resonator state by $-\alpha = (1/2)\int \Omega_{\mathrm{r}}(t)\mathrm{d}t$, as described by the operator $D(-\alpha) = D^\dagger(\alpha) = \exp(\alpha^* a - \alpha a^\dagger)$. For the displaced resonator state $\rho' = D(-\alpha)\rho D(\alpha)$, we determine the diagonal elements ρ'_{nn} by measuring $P_{\mathrm{e}}(\tau)$ during a swap interaction[8] (see below). As the Fock states are eigenstates of the parity operator Π with eigenvalues 1 (-1) for even (odd) Fock states, the Wigner function can simply be calculated as

$$W(\alpha) = (2/\pi)\sum_n (-1)^n \rho'_{nn}(-\alpha). \quad (5)$$

We note that the Wigner function can also be calculated directly from the time trace $P_{\mathrm{e}}(\tau)$ via a Fresnel transform[30], requiring only a short time scan, but yielding slightly less precise results in our case. The parity can also be measured directly in the dispersive limit[24], obviating the time scan, but the dispersive regime is incompatible with the parameters we need for state preparation.

The amplitude scale and the phase of the microwave pulse α are calibrated by a best fit between the measured and calculated Wigner distributions. Small variations ($\sim 5\,\%$) in the scale calibration were found for the various states measured here, including the coherent state, and thus an average was used. The magnitude of the scale factor is in good agreement with the attenuation of the microwave line and its coupling capacitor.

The density matrix can be calculated from the Wigner function by inverting Eq. (4). However, to make full use of the measured data, we instead calculate the density matrix ρ directly from the full set of measured photon number probabilities[28] by solving the set of linear equations

$$\rho'_{nn}(\alpha_m) = \langle n|D(-\alpha_m)\rho D(\alpha_m)|n\rangle = \sum_{j,i} M_{nmji}\rho_{ji}, \quad (6)$$

one for each extracted photon number n and one for each measured displacement α_m. The matrix

$$M_{nmji} = \langle j|D(\alpha_m)|n\rangle^* \langle i|D(\alpha_m)|n\rangle\,, \quad (7)$$

is calculated by expanding the displacement operator $D(\alpha) = \exp(\alpha a^\dagger - \alpha^* a)$ in the Fock basis:

$$\langle p|D(\alpha)|q\rangle = e^{-|\alpha|^2/2}\sqrt{p!q!}\sum_{k=0}^{\min\{p,q\}} \frac{\alpha^{(p-k)}(-\alpha^*)^{(q-k)}}{k!(p-k)!(q-k)!}\,. \quad (8)$$

We solve the largely overdetermined linear system of Eq. (6) by least-squares while restricting ρ to be hermitian. Due to noise, ρ can have small negative eigenvalues. Therefore we diagonalise ρ, set the unphysical negative eigenvalues to zero, and then transform back to the Fock basis. Finally we normalise ρ.

3. PHOTON NUMBER READOUT

At the end of the state preparation sequence for the resonator, the qubit is ideally in its ground state. We verify this by performing state tomography of the qubit[15], yielding a qubit density matrix that is very close to the ground state. Typically, the off-diagonal elements of the density matrix are very small, but the excited state probability is not zero, corresponding to a Bloch vector pointing close to the $|\mathrm{g}\rangle$ state: For the state generation shown in Fig. 4, the angle θ between the Bloch vector and $|\mathrm{g}\rangle$ is always smaller than 5°. For the states described in Fig. 3, the angles are from left to right 15°, 3°, 13°, 4°, and 9°, due to less precise tune-up of the sequences for some of the states. The length of the Bloch vector is close to 0.8 in Fig. 4 and slightly larger in Fig. 3. This decrease in amplitude could be due to errors in the preparation sequence that leave the qubit and resonator somewhat entangled. However, we attribute the reduction in visibility mostly to decoherence: The preparation sequences for the states in Fig. 4 take approximately 200 ns, a time slightly longer than the Ramsey coherence time $T_2 = 150$ ns of the qubit. This implies that when the qubit is brought into an equal superposition of $|\mathrm{g}\rangle$ and $|\mathrm{e}\rangle$ and left there for a time of 200 ns (worst case), the length of the Bloch vector would be reduced to 0.25. The qubit decoherence is actually less than this because the state is typically not in an equal