PHY101 HW #2
Due Friday, 1/18/13 @ 5pm
Functions of a Complex Variable

Reading

Read Sections 1-3 of Chapter 14 of Boas.

Problems

This problem set has a few more review problems from Chapter 2, but mostly involves problems on complex analysis from Chapter 14. The problems involve fundamental results which we will later apply to solve sums, integrals, and differential equations. By the end of this assignment you should be comfortable with complex functions and equations (including complicated examples involving multi-valued functions). You should understand how to determine if a function is analytic and some of the resulting consequences.

From Boas Chapter 2:

Section 17, page 81, Problems 19, 25. For 25(d), the problem should say: If $f(z)$ *can be* expanded in a power series with real coefficients about $z = 0$, show that $\overline{f}(z) = f(\bar{z})$.

Branch Cut Problems:

1) Consider a function $f(z)$ defined by $f(z) = z^\pi$, $f(1) = 1$, and having a single branch cut along the positive imaginary axis. What is $f(-1)$?

2) Consider a function $f(z)$ defined by $f(z) = z^\pi$, $f(1) = 1$, and having a single branch cut along the negative imaginary axis. What is $f(-1)$?

3) Consider two functions f_1, f_2 defined as follows: $f_1 = (z+1)^{1/2}$, $f_1(2) = +\sqrt{3}$ and f_1 has a single branch cut starting at $z = -1$ and running along the negative real axis. $f_2 = (z-1)^{1/2}$, $f_2(2) = 1$ and f_2 has a single branch cut starting at $z = +1$ and running to the left along the real axis to $-\infty$. The value of either function is not defined at $z = -5$. But one may discuss the limits of $f_{1,2}(z)$ as z approaches -5 from above and below the branch cuts. How do these values compare? Use your observation to argue that $f = \sqrt{(z+1)(z-1)}$ may be defined as a single-valued function with a branch cut only on the interval $[-1, 1]$ along the real axis. (I am not asking for a rigorous proof. A rough argument will suffice.)

From Boas Chapter 14:

Section 1, page 667, Problems 7, 15, 18.
Express your answers in terms of either x, y or $|z|, \theta$.

Section 2, page 672, Problems 23, 45, 58, 61. Also, is $u(x, y) = x^2 + y^2$ the real part of an analytic function of $z = x + iy$?

Section 3, page 676, Problems 10, 14, 15.
Notes:

i) Problem 10 does **not** *use the techniques of Chapter 14 Section 3. Instead, it uses the techniques of Chapter 6 Section 8. So, there is no obstacle to starting problem 10 immediately. Please choose some definite contour (any one you like) along which to evaluate the integral. On the other hand, problems 14, 15 use results that we will discuss on Wednesday.*

ii) It may be easier to work problem 15 before problem 14.

iii) You don't need to know anything about Fourier series for problem 14. That just provides context for why the result is interesting.