PHY101 HW #5
Due Friday, 2/8/13 @ 5pm
Principal Values and Branch Cuts

Reading

Read the rest of Section 7 as well as sections 8,9, and 10 of Chapter 14 of Boas. (I won't lecture on section 8 or assign HW, but it is good for you to see it. The material from sections 9 and 10 will not be on the upcoming exam, but we will begin to discuss it in class on Friday, so it would be good to at least skim this quickly.)

A note about the exam on 2/11:

Most of the exam will be similar to certain HW problems you have done. The best way to study for the exam is to study the HW solutions and to work similar problems until you are comfortable doing so.

Problems

The new material in this week's problems concerns yet more techniques for evaluating tricky integrals. This week's techniques typically involve integrating through singularities, either through poles (using the Cauchy Principal Value prescription) or through branch points. Like last week's assignment, this is great material for the exam.

From Boas Chapter 14:

Section 7, page 699, Problems 25, 26, 34, 36, 40 *Hint: You may want to do the new problem below before doing problem 40. The problems are similar in style, but I have given detailed instructions below for each step of the new problem.*

Section 11, page 718, Problems 27, 28

New Problem:
For $0 < a < 1$, use the steps below to show that

$$I = \int_{-\infty}^{\infty} \frac{e^{ax}}{1+e^x} dx = \frac{\pi}{\sin a\pi}$$

A) Find the singularities of $f(z) = \frac{e^{az}}{1+e^z}$ and show that they are all simple poles.

B) For real $R > 0$, consider the "rectangular" contour Γ_R in the complex plane that i) runs from $-R$ to R along the real axis, ii) then runs from R to

$R + 2i\pi$ parallel to the imaginary axis, iii) and runs from $R + 2i\pi$ back to $-R + 2i\pi$ parallel to the real axis and iv) finally runs from $-R + 2i\pi$ back down to $-R$ parallel to the imaginary axis. Draw this "rectangular" contour in the complex plane.

C) Evaluate the integral $I' = \lim_{R\to\infty} \int_{\Gamma_R} f(z)dz$ using the residue theorem.

D) Show that $I' = (1 - e^{i2\pi a})I$.

E) Combine the results from (A-D) to show that I has the value stated above.