Physics 101 Homework 6 Solutions

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1 Ch. 14, §9, 2

$$w = \frac{z+1}{2i} = \frac{x+iy+1}{2i} = \frac{y}{2} - i\frac{x+1}{2} \tag{1.1}$$

Lines of constant u and v are plotted on the next page.

2 Ch. 14, §9, 8

$$w = \cosh(z) = \frac{e^x e^{iy} + e^{-x} e^{-iy}}{2} = \frac{e^x \cos(y) + e^{-x}\cos(y) + ie^x \sin(y) - ie^{-x}\sin(y)}{2} = \cosh(x)\cos(y) + i\sinh(x)\sin(y) \tag{2.1}$$

Lines of constant u and v are plotted on the next page.

3 Ch. 14, §9, 11

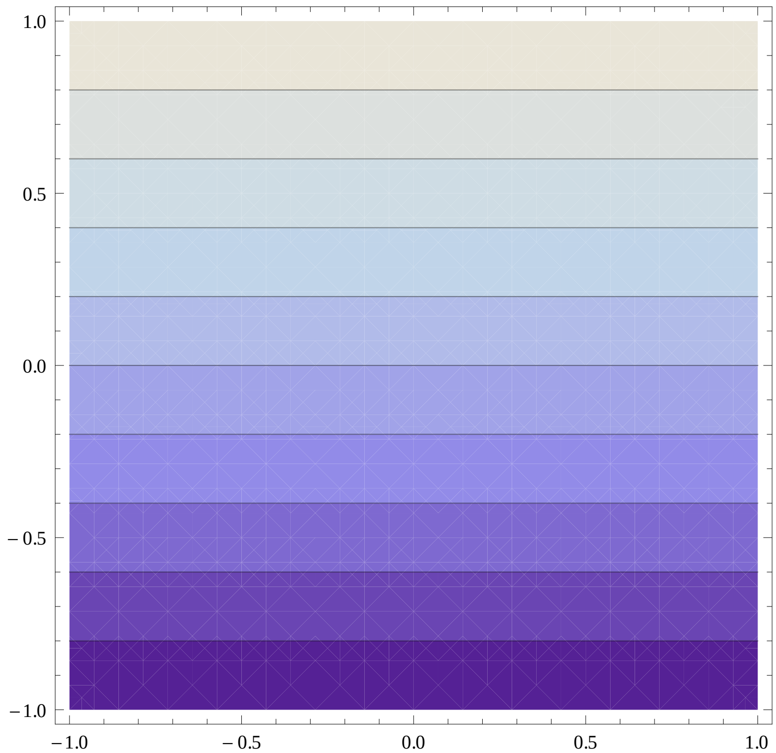
The Riemann surface for $\log z$ is an infinite sheeted spiral, like an infinite rotelli noodle. Going from one sheet to the one above (or below) it corresponds to increasing (or decreasing) $\Im(\log z)$ by 2π.

4 Ch. 14, §10, 4

Under the map $w = \log z$, the quarter disk with $r \in (0,1), \theta \in (0, \pi/2)$ is mapped to the rectangular region $u \in (-\infty, 0), v \in (0, \pi/2)$ in the $u + iv$ complex plane. Thus, we have to solve the heat equation with insulating boundary conditions at $u = -\infty, 0$, $T = 0$ at $v = 0$ and $T = 100$ at $v = \pi/2$. The solution is given by $T = 200v/\pi$. Thus, $T = 200\theta/\pi = 200 \arctan(y/x)/\pi$. The isotherms are given by $y = \tan(\pi T/200)x$, which are lines going radially outward from the origin, as you might have expected.

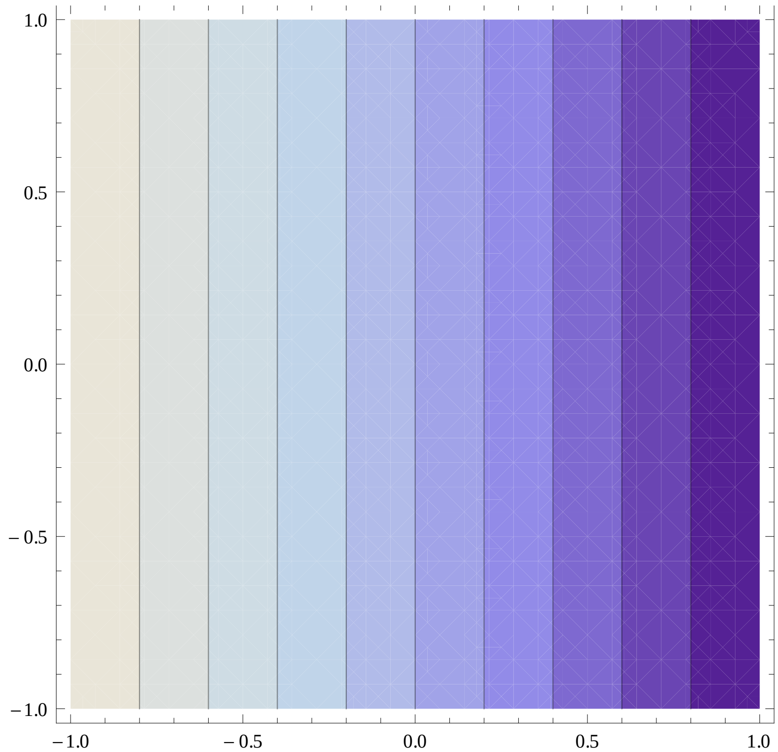
```
(*Section 9, Problem 2*)
(*Lines of constant u*)
ContourPlot[{y / 2}, {x, -1, 1}, {y, -1, 1}]
```

Out[2]=

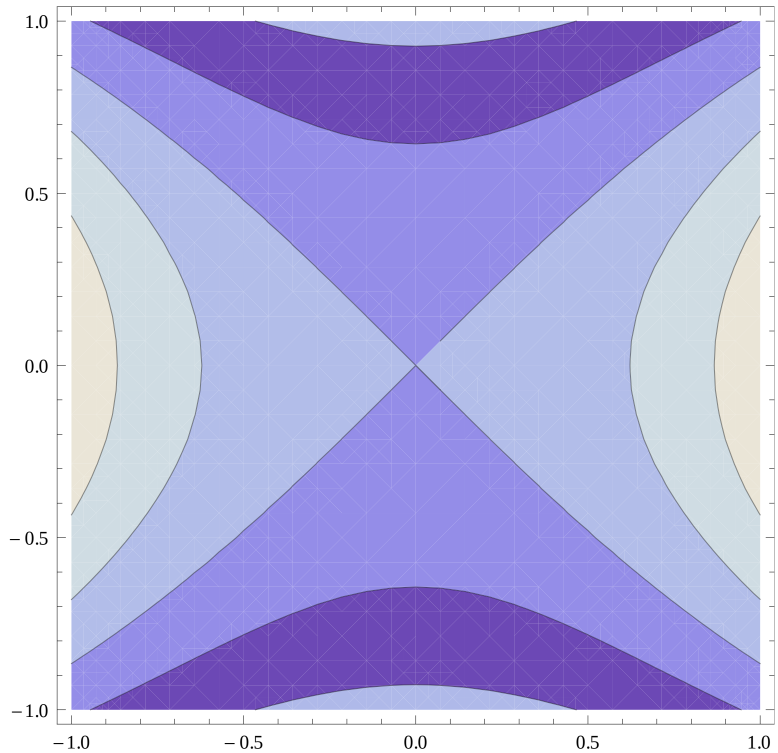


```
ContourPlot[{-(x+1)/2}, {x, -1, 1}, {y, -1, 1}]
(*Lines of constant v*)
```

Out[3]=



```
(*Section 9, Problem 8*)
(*Lines of constant u*)
ContourPlot[{Cos[y] Cosh[x]}, {x, -1, 1}, {y, -1, 1}]
```



```
(*Lines of constant v*)
ContourPlot[Sin[y] Sinh[x], {x, -1, 1}, {y, -1, 1}]
```