Phys 110C: Problems for HW 2

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1 HW2 Problem 1

Wave Boost

Consider a plane, linearly-polarized electromagnetic wave traveling through vacuum in a frame S, toward $+\hat{\mathbf{z}}$. The wave is given by

$$\vec{E} = E_0 \quad \hat{\mathbf{x}} \cos(\phi) \tag{1}$$

$$\vec{B} = E_0/c \, \hat{\mathbf{y}} \cos(\phi) \tag{2}$$

where

$$\phi(x^\mu) = k_\mu x^\mu. \tag{3}$$

and the wave 4-vector is $k^\mu = (\omega/c, 0, 0, k_z)$. The Poynting vector is of course

$$\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B} = \frac{1}{\mu_0 c} E_0^2 \, \hat{\mathbf{z}} \cos^2(\phi) \tag{4}$$

a) Boost this wave to a frame S' traveling at velocity $\vec{u} = u\hat{\mathbf{z}}$. Using Lorentz transformations, express ϕ as a function of coordinates in the S' frame, $x^{\mu'}$. What are the wavelength and frequency in the boosted frame?

b) At a given point $x^{\mu'}$, use the Lorentz transformation to boost the electromagnetic field tensor. From the field tensor, find the fields in the boosted frame, $\vec{E}'$, $\vec{B}'$.

c) Find the Poynting vector in the boosted frame. Compare with that in the original frame.

2 HW2 Problem 2

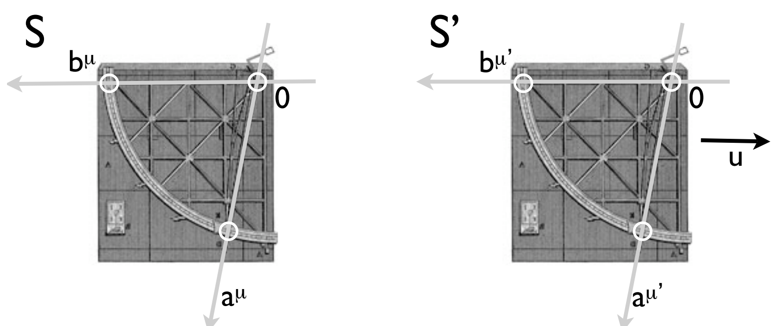
Relativistic Star Positions Medieval astronomers used quadrants to measure star positions. These involved two sights, mounted on the arc of an accurately calibrated circle. In one observing mode, two observers each viewed a star through one of the sights and through the center of the circle. The measured angle between the sights was then the angle between the stars. (Note: Sometimes one sight was replaced by a plumb bob, to define local vertical; in more modern sextants a mirror brings an image of the star into alignment with the horizon, a similar measure of local vertical.)

Two observers use a quadrant to measure the angle between two stars, in a frame S. Two photons, one from each star, travel along world lines

$$a^\mu(\tau) = (c\tau, (\sin\theta_a)\tau, 0, (\cos\theta_a)\tau) \quad (5)$$

$$b^\mu(\lambda) = (c\lambda, 0, 0, \lambda). \quad (6)$$

Here, of course, the parameters τ and λ give position along the curve; traditionally this parameterization is proper time, but photons have no proper time and so in this case the parameterization is arbitrary. Both photons pass through the center of the circle and then through the observers' sights, a distance ℓ from the center of the circle. The observers conclude that the stars are separated by an angle θ_a.



a) Assume that the spacetime origin is the event of the photons reaching the center of the circle. Find the coordinates of the events at which they pass through the observers' sights, each a distance ℓ away. At what *times* do the photons pass through the sights?

b) A starship travels past the quadrant at velocity $\vec{u} = u\hat{\mathbf{z}}$. The starship defines a frame S'. The quadrant and, of course, the stars remain at rest. Find the coordinates of the photons' passage through the sights. From the standpoint of a starship navigator, criticize the measurement of angle, made with the stationary quadrant. Can you correct the measurement to determine the actual angle between the stars?

c) Now suppose that the starship carries its own quadrant, and makes the same measurement, with the photons passing through the origin in S' in the boosted frame.

- Find the world lines of the same 2 photons in the boosted frame, $a^{\mu\prime}(\tau')$ and $b^{\mu\prime}(\lambda')$. (Note: You may use any convenient parameterizations τ' and λ'.)
- Find the coordinates of the events at which the photons pass through the observers' sights in the boosted frame. You may adjust the locations of the sights on the starship's quadrant, if you need to. What is the angle between the stars, as inferred by the navigator in the starship's frame?

d) Find wave 4-vectors k_a^μ and k_b^μ for the waves that carry the photons in the frame S. Boost these to the frame S'. Do these give the same direction as your previous result(s)?

3 HW2 Problem 3

Griffiths 9.5. Assume that the incident wave and the reflected and transmitted waves are sinusoidal: $g_I = A_I \exp(i(kz - \omega t))$, etc. This simplifies the math quite a bit. (Answers: $A_T/A_I = 2v_2/(v_1 + v_2)$, $A_R/A_I = (v_2 - v_1)/(v_1 + v_2)$).

Are the coefficients derived using the assumption of sinusoidal waves less general, than those using waves of arbitrary shape? If the incident wave is sinusoidal, can you assume that reflected and transmitted waves will also be sinusoidal? Explain your answers.

4 HW2 Problem 4

Griffiths 9.7. For part b), you may assume that the wave in molasses takes the form $\tilde{A}e^{i(\tilde{k}z - \omega t)}$, where $\tilde{k}$ is complex, and $\tilde{A}$ is an arbitrary complex constant. Answers:

$$a) \qquad \frac{\partial^2 f}{\partial z^2} = \frac{\mu}{T}\frac{\partial^2 f}{\partial t^2} + \frac{\gamma}{T}\frac{\partial f}{\partial t} \qquad (7)$$

$$b) \qquad \mathrm{Re}[k] = \omega\sqrt{\frac{\mu}{2T}}\sqrt{1+\sqrt{1+(\gamma/\mu\omega)^2}} \tag{8}$$

$$\mathrm{Im}[k] = \frac{\gamma}{\sqrt{2T\mu}}\frac{1}{\sqrt{1+\sqrt{1+(\gamma/\mu\omega)^2}}} \tag{9}$$

$$d) \qquad \text{Hint : Use } 9-5 \text{ with complex } k. \tag{10}$$

5 HW2 Problem 5

Griffiths 9.10. An active human in the industrialized world metabolizes about 2000 Cal per day (1 Cal = 10^3 cal). Convert this to Joules, and then to the sunlight-capturing area required to support one human's personal energy needs. Could the needs be supplied if humans were genetically altered to have chlorophyll in their palms? How about over their entire bodies?

6 Additional Problems from Griffiths

9.2, 9.4, 9.11