22 MAY

TRANSFORM TO FRAME: $V \neq 0$

RELATIVISTIC EFFECTS

GEOMETRICAL

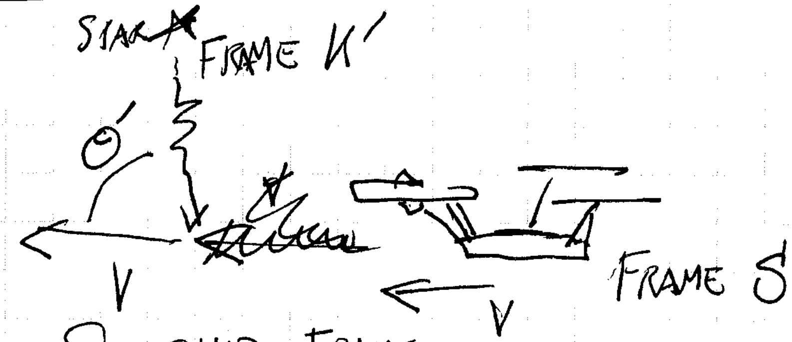
QUADRANT ABOARD SS ENTERPRISE

ENERGY TRANSFORM

ACCELERATION (HW8)

BREMSSTRAHLUNG

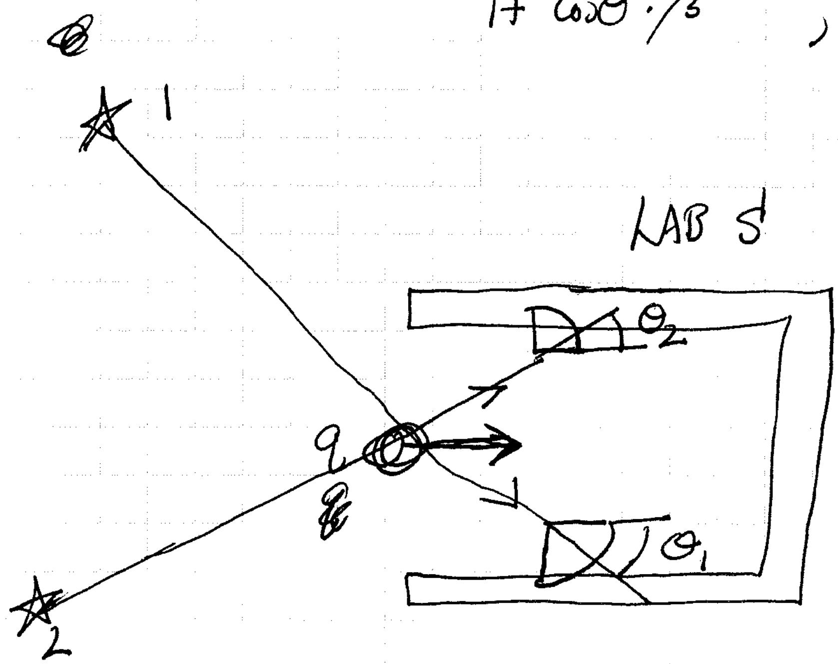
STARSHIP ENTERPRISE! $V = c - \varepsilon$



BOOST TO STARSHIP FRAME

The apparent angle in the frame of the starship is:

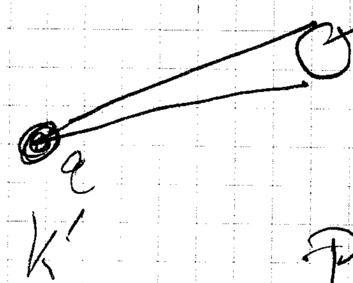
$$\cos\theta = \frac{\cos\theta' + \beta}{1 + \cos\theta' \cdot \beta}, \qquad \beta = V/c$$



~~RAD~~ ~~ISOTR~~

ISOTROPIC RADIATION IN PARTICLE FRAME K' IS BEAMED FORWARD IN LAB FRAME S

TRANSFORM TO STARSHIP LAB:



$$\frac{dW'}{dt' d\Omega'} = \frac{\text{ENERGY EMITTED}}{\text{TIME} \cdot \text{SOLID ANGLE}}$$

$$p^{\mu'} = \left(\frac{\Delta W'}{c\Delta t'}, \frac{\Delta W'}{c\Delta t'}\hat{r}\right)$$ a good 4-vector

✓ $$\frac{\Delta W}{c} = P^0 = \frac{\Delta W'}{c} \cdot \gamma \cdot (1+\beta)$$

$$d\Omega = d\varphi \sin\theta \, d\theta = d\varphi \cdot d(\cos\theta)$$

FIND:

✓ $$d\varphi \, d\theta = d\varphi' \frac{\sin\theta' d\theta'}{\gamma^2 (1+\beta\cos\theta')^2}$$

~~Δt_{ABS}~~

$$\Delta t_{emit} = \gamma \Delta t'_{emit}$$

✓ $$\Delta t_{abs} = \gamma (1-\beta\mu) \Delta t'_{emit}$$

FIND:

$$\cancel{\frac{dW_{abs}}{d\Omega}} = \frac{dP_{abs}}{d\Omega} = \frac{dW_{abs}}{d\Omega \, dt_{ABS}} = \underbrace{\frac{1}{\gamma^4 (1-\beta\cos\theta)^4}}_{\text{Sharply peaked near } \theta = 0, \text{ for } \beta \to 1} \frac{dW_{emit}}{d\Omega \, dt_{emit}}$$

γ Big!

RECALL

IN FRAME K':

$\vec{V}_q = 0$ AT t_R, (IMPORTANT: DEFINES K')

$\vec{x}_q(t_R) = \vec{0}$, $\vec{a} \parallel \hat{z}$ (CONVENIENT & NOTATION)

SHIP
$$\begin{cases} \vec{E}_{rad} = \frac{q}{4\pi\epsilon_0} \frac{1}{r} \frac{|\vec{a}|}{c^2} \sin\theta\, \hat{\theta} \\ \vec{B}_{rad} = \frac{\hat{r}}{c} \times \vec{E}_{rad} \end{cases}$$

RADIATED ENERGY

$$\cancel{\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B}} \quad \vec{S}' = \frac{1}{\mu_0} \vec{E}_{rad} \times \vec{B}_{rad} = \frac{\mu_0}{c} \frac{q^2}{(4\pi)^2} |a|^2 \sin^2\theta\, \hat{r}$$

$$\Gamma' = \frac{dW'}{dt'} = \int_{\text{SPHERE}} \vec{S}' \cdot d\vec{a}' = \frac{\mu_0}{c} \frac{q^2}{(4\pi)^2} a^2 \frac{8\pi}{3}$$

~~How does $\frac{dW'}{dt'}$ compare to $\frac{dW}{dt}$?~~ ③ ④

How does the total power radiated in the lab frame compare to that in K' — the instantaneous frame of the particle?

LAB FRAME: $\frac{dW}{dt}$

Note that $\frac{dW'}{dt'}$ is the 0 component of a 4-vector:

$$P^\mu = \left(\frac{1}{c}\frac{dW'}{dt}, \frac{d\vec{P}_{rad}}{d\tau}\right)$$

The other 3 components are the 3-momentum of the radiation

You can think of this as the energy ~~lost by the partl~~ and momentum lost by the particle, per unit time,

or the energy and momentum within an expanding sphere about the particle, with thickness corresponding to the proper time interval $d\tau$.

Because the radiation pattern is <u>symmetric</u> about the charge,

$$S^{\mu}_{TOT} = \left(\frac{dW'}{d\tau}, 0, 0, 0\right)$$

$$\cancel{\frac{dW}{dt}} \quad \frac{dW}{d\tau} = \Lambda^{0}{}_{\mu'} S^{\mu'}$$

$$= \gamma \frac{dW'}{d\tau}$$

NOTE: $d\tau = dt'$

$dt = \gamma dt'$ "Moving clocks run slow"

So that

$$\frac{dW}{dt} = \frac{dW'}{dt'}$$

HOLDS FOR SYMMETRIC RADIATION PATTERN.

IN MOVING FRAME,

$$a_\parallel' = \gamma^3 a_\parallel$$

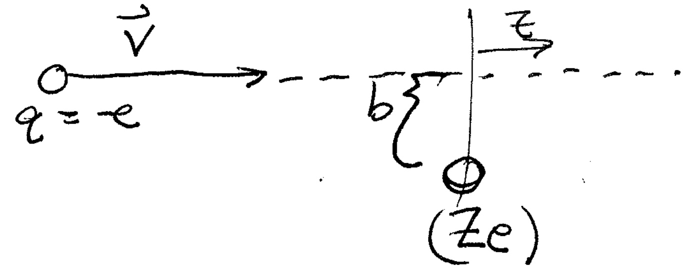
$$a_\perp' = \gamma^2 a_\perp$$

(SEE PREV LECT, HW8)

SO: IN LAB FRAME:

$$\boxed{\frac{dW}{dt} = \frac{\mu_0}{c}\,\frac{q^2}{\cancel{(4\pi)^2}}\cdot\cancel{\frac{8\pi}{3c}}\,\frac{1}{6\pi}\left(\gamma^6 a_\parallel^2 + \gamma^4 a_\perp^2\right)}$$

EXAMPLE: BREMSTRAHLUNG



ASSUME: e^- NEARLY UNDEFLECTED

~~V BIG OR b B~~

(V, b BIG)

~~COLLISION LAST~~

~~$q(a)t = (Ze)\dot{v}$~~

Change in ~~$\vec{a}$~~:

$$a(t) = \frac{Ze^2}{4\pi\epsilon_0 m (b^2+z^2)}$$

$$= \frac{Ze^2}{4\pi\epsilon_0 m (b^2+v^2t^2)}$$

For $v \ll c$,

$$a_\perp = \frac{Ze^2}{4\pi\epsilon_0 m}\frac{b}{(b^2+z^2)^{3/2}}$$

$$a_\parallel \simeq \frac{Ze^2}{4\pi\epsilon_0 m}\frac{z}{(b^2+z^2)^{3/2}}$$

Net ~~Power radiated~~

~~$\int_{-\infty}^{\infty} \frac{dt}{(b^2+v^2t^2)^3}$~~

$$\int_{-\infty}^{\infty} \frac{dt}{(1+t^2)^3} = \frac{3\pi}{8}$$

$$\int_{-\infty}^{\infty} \frac{t^2\,dt}{(1+t^2)^3} = \frac{\pi}{8}$$

Net power radiated

$$W_{rad} = \frac{\mu_0}{c}\frac{e^2}{(4\pi)^2}\frac{8\pi}{3}\int_{-\infty}^{\infty} a_\perp^2 \, dt$$

$$= \frac{\mu_0}{c}\frac{e^2}{(4\pi)^2}\frac{8\pi}{3}\int_{-\infty}^{\infty}\left(\frac{Ze^2}{4\pi\epsilon_0 m}\right)^2 \frac{b^2}{(b^2+(vt)^2)^3}\, dt$$

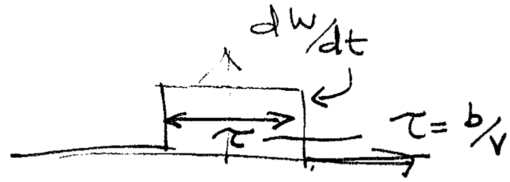
$$\cancel{= \frac{\mu_0}{c}\frac{e^2}{(4\pi)^2}\frac{8\pi}{3} Z^2}$$

$$= \frac{\mu_0}{c}\frac{Z^2 e^6}{(4\pi\epsilon_0 m)^2}\frac{1}{(4\pi)^2}\frac{8\pi}{3}\cdot\left\{\left(\frac{b}{v}\right)\cdot\frac{1}{b^4}\cdot\frac{3\pi}{8} + \left(\frac{b^3}{v}\right)\frac{1}{b^6}\cdot\frac{\pi}{8}\right\}$$

$$= \frac{\mu_0}{c}\frac{Z^2 e^6}{(4\pi\epsilon_0 m)^2}\frac{1}{vb^3}\underbrace{\cancel{\frac{1}{(4\pi)^2}}\; 2\cdot\cancel{\frac{4\pi}{3}}\cdot\frac{4\pi}{8}}_{\frac{1}{12}}$$

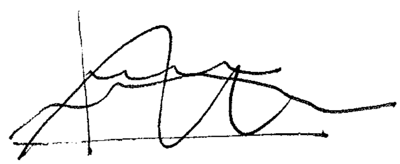
Assume $a_\perp \sim$ const $\sim \begin{cases} \frac{Ze^2}{4\pi\epsilon_0 m}\frac{1}{b}, & z \le b \\ 0 & \text{otherwise} \end{cases}$

$a_\parallel \sim 0$



We seek to find the frequency structure of W

$$F(\omega) = \int_{-\infty}^{\infty} e^{i\omega t}\cdot\left(\frac{dW}{dt}\right)dt$$



$$\omega \ll \frac{1}{\tau} = v/b$$

$$\cancel{\frac{1}{2\pi}\left(\frac{Ze^2}{4\pi\epsilon_0 m}\right)^2\frac{t}{b}}$$

Final: $F(\omega) \sim \frac{\mu_0}{c}\frac{e^2}{6\pi}\left(\frac{Ze^2}{4\pi\epsilon_0 m}\frac{1}{b}\right)^2\cdot\frac{b}{v}$

$$\sim \frac{\mu_0}{c}\frac{1}{6\pi}\frac{Z^2e^6}{(4\pi\epsilon_0 m)^2}\frac{1}{bv}$$

Flat spectrum.

From populations $F_{net} = \int_{b_{min}}^{b_{max}} F(\omega)\cdot 2\pi b\,db \propto \ln\left(\frac{b_{max}}{b_{min}}\right)$