UNIVERSITY OF CALIFORNIA, SANTA BARBARA

Department of Physics

Physics 221A — Fall 2008

Prof. Mark Srednicki — TA Idse Heemskerk

FINAL EXAM

Due 11 AM, Wednesday, Dec 10, Chair's office in Broida 3019

You may use the following materials during this exam: the course textook; anything from the course web site, http://www.physics.ucsb.edu/∼phys221A, including homework solutions; any personal notes and homework assignments. You may make maximal use of anything derived in the book, in homework solutions, or in class without re-deriving it (e.g, the values of various Feynman diagrams). You may not consult any other materials, or any person other than me. I will be in my office Tuesday 2:00–3:30, and will check email (mark@physics) as often as possible at other times.

1) Consider a theory of a real scalar field φ in *two* spacetime dimensions, with lagrangian

$$\mathcal{L} = -\tfrac{1}{2}\partial^\mu\varphi\partial_\mu\varphi - \tfrac{1}{2}m^2\varphi^2 + \tfrac{1}{6}g\varphi^3 + Y\varphi \ ,$$

with Y adjusted so that $\langle 0|\varphi(x)|0\rangle = 0$. Note that g has dimensions of mass-squared. Because all Z factors have been set equal to one, the physical particle mass $m_{\rm ph}$ is not necessarily equal to the lagrangian parameter m. Show that

$$m^2_{\rm ph} = m^2 + c\,g^2/m^2 + O(g^4) \ ,$$

and evaluate the numerical constant c.

2) Consider a theory of a *complex* scalar field φ in *four* spacetime dimensions, with lagrangian

$$\mathcal{L} = -Z_\varphi\partial^\mu\varphi^\dagger\partial_\mu\varphi - Z_m m^2\varphi^\dagger\varphi - \tfrac{1}{4}Z_\lambda\lambda(\varphi^\dagger\varphi)^2 \ .$$

We define m to be the physical particle mass, and the coupling λ such that the exact two-particle scattering amplitude for two positively-charged a particles at threshold (zero kinetic energy: Mandelstam variables $s = 4m^2$ and $t = u = 0$) is $\mathcal{T}_{aa\to aa} = -\lambda$, with no higher-order (in λ) corrections.

a) Compute the $O(\lambda^2)$ correction to $\mathcal{T}_{aa\to aa}$ for general values of s, t, and u. Write your answer in a form that is explicitly independent of μ.

b) Compute the $O(\lambda^2)$ correction to the scattering amplitude $\mathcal{T}_{ab\to ab}$ for one positively-charged a particle and one negatively-charged b particle, for general values of s, t, and u. Write your answer in a form that is explicitly independent of μ. What is the value of $\mathcal{T}_{ab\to ab}$ at threshold?

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3) Consider a theory of three real scalar fields A, B, and C in *six* spacetime dimensions, with lagrangian

$$\begin{aligned}\mathcal{L} = &-\tfrac{1}{2}Z_A\partial^\mu A\partial_\mu A - \tfrac{1}{2}Z_{mA}m_A^2A^2 \\ &-\tfrac{1}{2}Z_B\partial^\mu B\partial_\mu B - \tfrac{1}{2}Z_{mB}m_B^2B^2 \\ &-\tfrac{1}{2}Z_C\partial^\mu C\partial_\mu C - \tfrac{1}{2}Z_{mC}m_C^2C^2 \\ &+\tfrac{1}{6}Z_g\, gABC\ .\end{aligned}$$

a) Draw all *tree-level* diagrams that contribute to the process $AA \to ABC$.

b) Consider a process with E_i external particles (incoming or outgoing) of type i ($i = A, B, C$). Show that not all such processes are allowed (even when loop corrections are included), and find the restrictions on the allowed values of E_A, E_B, and E_C.

c) Do any terms need to be added to $\mathcal{L}$ for renormalizability?

d) Compute the $O(\alpha^2)$ term in the beta function for $\alpha = g^2/(4\pi)^3$.

Possibly useful integrals:

$$\int_0^1 \frac{dx}{1 - x(1-x)} = \frac{2\pi}{3\sqrt{3}}$$

$$\int_0^1 dx\ \ln[1 - x(1-x)] = -2 + \frac{\pi}{\sqrt{3}}$$

$$\int_0^1 dx\ \ln[1 - 4x(1-x)] = -2$$