UNIVERSITY OF CALIFORNIA, SANTA BARBARA
Department of Physics

Physics 221C — Spring 2009

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FINAL EXAM

Due 3 PM Wednesday, June 10, my office in Broida 3019

You may use the following materials during this exam: the course textook; anything from the web sites for 221A, B, and C, including homework solutions; any personal notes and homework assignments. You may make maximal use of anything derived in the book, in homework solutions, or in class without re-deriving it. You may not consult any other materials, or any person other than me. I will be in my office Tuesday 11-12 AM and 4-5 PM, and will check email (mark@physics.ucsb.edu) as often as possible at other times.

1) Consider scattering of an electron neutrino ν_e off a proton p. Write down an effective lagrangian for this process in the form

$$\mathcal{L}_{\rm eff} = \tfrac{1}{\sqrt{2}} G_{\rm F}\, \overline{\mathcal{N}}\gamma^\mu(1-\gamma_5)\mathcal{N}\, \overline{p}\gamma_\mu(C_{\rm V}-C_{\rm A}\gamma_5)p\ ,$$

where $\mathcal{N}$ is the electron neutrino field and p is the proton field, and determine the values of $C_{\rm V}$ and $C_{\rm A}$.

2) Consider an alternate model of electroweak interactions based on the gauge group SU(2), in which the left-handed Weyl fields e, $\overline{e}$, and ν are grouped into a $\mathbf{3}$ of SU(2),

$$\psi = \begin{pmatrix} \overline{e} \\ \nu \\ e \end{pmatrix} \sim \mathbf{3}\ .$$

A Higgs field in the $\mathbf{3}$ breaks SU(2) down to the U(1) of electromagnetism; the unbroken generator is taken to be T^3, where

$$T^1_{\mathbf{3}} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \qquad T^2_{\mathbf{3}} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \qquad T^3_{\mathbf{3}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}.$$

a) In this model, what is the relation of the SU(2) gauge coupling g_2 to the electromagnetic coupling e?

b) Find the currents $J^{\pm\mu}$ that couple to the $W^{\pm\mu}$ fields in this model, and express them in terms of the four-component fields $\mathcal{E}$ and $\mathcal{N}$ and/or their charge conjugates $\mathcal{E}^{\rm C}$ and $\mathcal{N}^{\rm C}$. How does your result compare with the Standard Model?

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3) Consider a theory of N Dirac fermions in $d = 2$ spacetime dimensions with

$$\mathcal{L} = i\overline{\Psi}_i \partial\!\!\!/ \Psi_i - m\overline{\Psi}_i\Psi_i + \tfrac{1}{2}g^2\big(\overline{\Psi}_i\Psi_i\big)^2 \ ,$$

where $i = 1, \ldots, N$, and a repeated index is summed. The usual Dirac γ matrix algebra applies (with $d = 2$ in the formulae for general d), but with $\mathrm{Tr}\, 1 = 2$ instead of 4.

a) In d spacetime dimensions, what is the mass dimension of g? Do you expect this theory to be renormalizable or nonrenormalizable in two spacetime dimensions?

b) Show that the functional integral for the theory can be written as

$$\int \mathcal{D}\Psi\mathcal{D}\overline{\Psi}\, e^{i\int d^2x\, \mathcal{L}} = \int \mathcal{D}\Psi\mathcal{D}\overline{\Psi}\mathcal{D}\varphi\, \exp\left[i\int d^2x\, \Big(i\overline{\Psi}_i\partial\!\!\!/\Psi_i - m\overline{\Psi}_i\Psi_i + g\varphi\overline{\Psi}_i\Psi_i - \tfrac{1}{2}\varphi^2\Big)\right],$$

where $\varphi(x)$ is a new scalar field with no kinetic term and unit mass. Thus, its momentum-space propagator is $\tilde{\Delta}(k) = 1$, rather than $1/(k^2 + m^2)$.

c) Introduce renormalizing factors Z_Ψ, Z_φ, Z_g, and Z_m, and compute the first three of these using the $\overline{\mathrm{MS}}$ renormalization scheme in $d = 2 - \varepsilon$ dimensions.

d) Compute the one-loop beta function for g.

e) Under what circumstances, if any, is the theory asymptotically free?

Hints: Remember that there are N fermion fields! Remember that the tree-level scalar propagator is one!

That's it!