Supermassive Black Holes
As Gravitational Wave Sources for LISA

Omer Blaes

Department of Physics, University of California at Santa Barbara,
Santa Barbara, CA 93106

Abstract. I briefly review the astrophysics of supermassive black holes that is relevant to their role as sources of gravitational waves for the proposed *Laser Interferometer Space Antenna* mission.

INTRODUCTION

Probably the most exciting of the realistic gravitational wave sources for LISA are supermassive black holes (SMBH's) [1]. By astrophysical standards, the case for the existence of SMBH's is very strong. Stellar and gas dynamics can be used to infer the presence of massive dark objects at the centers of most bright galaxies. The most spectacular extragalactic case is the galaxy NGC 4258, where water masers have been observed in an edge-on, warped disk of gas in the nucleus. The masing gas has a Keplerian velocity profile, consistent with a central compact mass of 3.6×10^7 $M_\odot$ inside a radius less than 0.13 pc, implying a density greater than 4×10^9 $M_\odot$ pc^{-3} [2]. Stellar proper motions in our own Galactic center imply the existence of a compact mass of 2.6×10^6 $M_\odot$ inside a radius of $\sim 10^{-2}$ pc, giving a density greater than 10^{12} $M_\odot$ [3,4]. Such high densities can be used to rule out alternatives such as dense clusters of stars or compact objects [5], leaving SMBH's or perhaps SMBH binaries as the only plausible explanation. Massive dark objects appear to be ubiquitous in the nuclei of bright galaxies, and there is even a correlation between the dark mass and the galaxy bulge mass: $M_\bullet \simeq 0.006 M_{\rm bulge}$ [9]. This is roughly consistent with the integrated quasar light in the universe, albeit with an average radiative efficiency of around a percent [10], somewhat less than the ten percent usually assumed in accretion.

It is the existence of active galactic nuclei and quasars that is perhaps the most compelling argument for the existence of SMBH's. These sources can exhibit compact relativistic outflows as well as relativistically broadened X-ray lines [6]. Rapid variability in the X-rays and gravitational microlensing in the optical [7] can constrain the size of the central source to be very small, while variability in the broad optical emission lines can be used to infer high central masses [8].

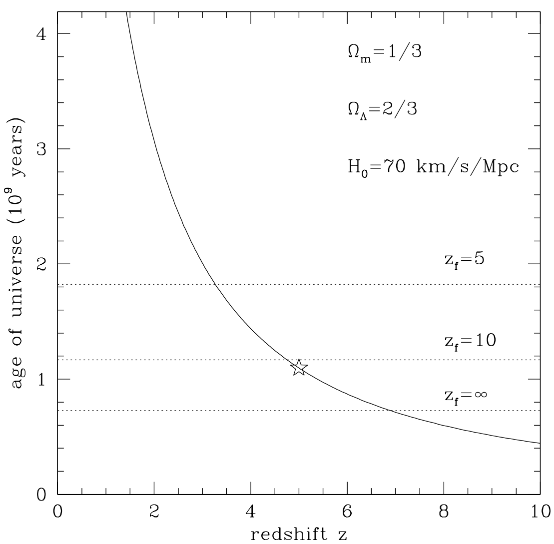

FIGURE 1. Age of the universe as a function of redshift, for the currently popular flat universe model with nonzero cosmological constant (adapted from [14]). Dashed horizontal lines indicate the time when a 10^8 $M_\odot$ SMBH could have formed by Eddington limited accretion with ten percent radiative efficiency, assuming an initial mass of 10 $M_\odot$ at the indicated redshift z_f. The star indicates the redshift of the most distant quasars known.

SMBH FORMATION

The SMBH mass/bulge mass correlation strongly suggests that SMBH formation is an integral part of the overall process of galaxy formation. The peak of quasar activity occurred at redshifts between two and three, comparable to the epoch of peak star formation in the universe (e.g. [10]). Quasars have been detected out to higher redshifts, the current record holders being at $z = 5.00$ and 5.03 [11,12]. These high redshift objects indicate that structure formation was well on its way early on in the universe, and that SMBH's formed fast (Figure 1, [13]).

There are a variety of formation pathways for SMBH's which are summarized in a famous flowchart by Rees [15], some of which might be copious sources of gravitational waves for LISA. An oversimplified viewpoint is that there are two classes of mechanisms to form SMBH's: stellar dynamical and gas dynamical.

The most detailed work has been done with stellar dynamics. An exciting scenario from LISA's point of view is the dynamical collapse and merger of a dense cluster of neutron stars and/or black holes. Quinlan & Shapiro [16] explored this using Fokker-Planck calculations and found that mass segregation in the merging remnants of the cluster resulted in several 100 $M_\odot$ black holes at the center. They could not follow the subsequent evolution, but if ~ 500 $M_\odot$ seeds could form and merge, this would make a very strong gravitational wave source for LISA. Lee [17] followed up on this work by performing a full N-body calculation of 1000 compact objects, and found runaway growth of *one* object. This would therefore not be as spectacular a source of gravitational waves in the LISA bandpass.

Both of these calculations suffer from the problem of artificial initial conditions. It is hard to see how such a cluster of compact objects could form in the first place without also having more ordinary stars which have much larger collision cross-sections. Fokker-Planck calculations of the evolution of a dense cluster of main sequence stars have been performed [18]. Bearing in mind that the physics of the collisions is very uncertain, it appears that a $> 10^3$ $M_\odot$ massive star forms rapidly within the cluster relaxation time.

It therefore seems reasonable to assume that gas dynamics dominates the SMBH formation process, involving either the collapse of a gas cloud, or the formation of supermassive stars or disks. Unfortunately, this is a much harder problem (cf. our poor theoretical understanding of star formation!). Rees has argued for a long time that the gas dynamical formation of a SMBH would be a very weak source of gravitational waves (e.g. [19]). In the collapse of a supermassive star by the post-Newtonian instability, for example, the gravitational wave energy could be spread out over a long time because not all the matter would reach the center at once (i.e. within a time interval comparable to GM/c^3, where M is the mass of the star or resulting black hole). This is because the free-fall time $\sim (G\rho)^{-1/2}$ depends only on the average density ρ inside a given radius, and supermassive stars do not have a uniform density distribution. However, early numerical spherical collapse calculations (e.g. [20]) found that, in spite of the initial density nonuniformity, the collapse proceeds homologously, so that it could be a strong source of gravitational waves provided the necessary asphericities do not affect this conclusion. Supermassive stars are essentially $n = 3$ polytropes, and such models are known to have (Newtonian) homologous collapse solutions [21]. (The inner parts of the collapsing stars have substantial pressure support, and are therefore *not* in complete free fall and need not run away from the outer parts.) Even so, given that the energy released in gravitational waves is very sensitive to the collapse time scale, this scenario is asking for a lot of optimism. Even small departures from a homologous collapse would greatly reduce the luminosity. More recent post-Newtonian spherical collapse calculations by Fuller et al. [22] find in fact that electron-positron pair production and neutrino cooling destroy the homologous nature of the collapse over all but the inner ten percent of the mass of the star.

Loeb and Rasio [23] have simulated the isothermal collapse of rotating, optically thin gas clouds. Assuming the gas can continue to cool, much of it fragments into clumps, with a small fraction forming a rotating supermassive disk in the center, leading to the possibility that a newborn SMBH grows by accretion from such a disk. These disks may themselves be subject to instabilities, and this begs the obvious question: can galaxies be born with binary or multiple SMBH's in their nuclei? After all, we know this is common in star formation, although making quantitative theoretical predictions is currently not possible.

SMBH BINARIES

Various pieces of observational evidence for the actual existence of SMBH binaries have been presented over the years, but none are as yet compelling. Tantalizing evidence that time variations in the double peaked Hβ emission line profile of the quasar 3C 390.3 could be interpreted as being due to a binary SMBH [24] were ruled out by later observations when the line did not behave as predicted by the model [25]. Periodic "wiggles" in VLBI radio jets have been interpreted as being due to the orbital motion in a SMBH binary (e.g. the quasar 1928+738, [26]), but this is not the only explanation for this phenomenon. The optical light curve of the BL Lac OJ 287 exhibits flares which regularly recur every twelve years, and a binary SMBH has been proposed to explain this periodicity [27,28]. SMBH binary evolution can scour out the density profile of a surrounding star cluster [29–31], which provides a plausible explanation for the formation of observed inner optical surface brightness "core" profiles of bright elliptical galaxies. However, adiabatic growth of a single SMBH can also produce the same profiles, and can even explain some of the observed correlations in those profiles [32].

Even if SMBH's are not born as binaries, binaries are likely to form when galaxies merge, a common process in the currently popular hierarchical scenarios for structure formation. We even see such mergers in the local universe - the most interesting example being the radio source 3C 75, which actually consists of two radio galaxies in the process of merging, with radio jets that appear to interact and merge together on one side [33]. At least in this case we *know* that the two merging galaxies each contain a SMBH.

The evolution of the two SMBH's in the galaxy merger is thought to proceed in four stages [34]: (1) Dynamical friction brings the SMBH's to the center of the merged stellar core on a characteristic time scale $t_{fric} \sim 6 \times 10^7$ yr$(M_\bullet/10^6 \mathrm{M}_\odot)^{-1}$, assuming the stellar core has a density $\sim 10^3$ $\mathrm{M}_\odot$ pc^{-3}, velocity dispersion ~ 300 km s^{-1}, core radius ~ 100 pc, and core mass $\sim 8 \times 10^9$ $\mathrm{M}_\odot$. (2) The SMBH's form a bound binary when they approach within ~ 5 pc$(M_\bullet/10^6 \mathrm{M}_\odot)^{1/3}$ of each other. Dynamical friction causes the semimajor axis of the binary to continue to shrink. (3) The binary becomes "hard", meaning that its orbital velocity exceeds the velocity dispersion of the surrounding stars. Stellar encounters continue to shrink the binary, causing it to harden further. (4) Eventually, gravitational radiation takes over the job of driving the binary coalescence when the semimajor axis shrinks to $\sim 2 \times 10^{-3}$ pc$(M_\bullet/10^6 \mathrm{M}_\odot)^{3/5}$, the time scale for shrinking being a rapidly decreasing function of semimajor axis. The longest and most uncertain stages of this process are stages (2) and (3). Ejection of stars on radial orbits ("loss cone depletion"), which are the ones which have close encounters with the binary and therefore dominate the hardening, is a major uncertainty. The eccentricity of the merging binary can evolve, either increasing (reducing the time when gravitational radiation starts to take over) or decreasing.

Many authors have explored aspects of this problem, but the most detailed work so far is by Quinlan [35] and Quinlan & Hernquist [30]. The first paper explored the evolution of SMBH binaries in a fixed stellar background. Among the interesting results was the fact that the eccentricity of the binary could increase while it was hardening, but if the SMBH's first became bound on low eccentricity (i.e. circular) orbits, then the eccentricity stayed low. The initial eccentricity of the bound binary is therefore an important quantity to determine. More worrying for LISA, Quinlan could calculate the rate of mass ejection of stars by the binary and use it to estimate when the loss cone would be entirely depleted, even if he was not calculating the evolution of the star cluster itself. He found that *all* low eccentricity binaries stalled before they were able to get close enough for gravitational radiation to take over. Quinlan & Hernquist addressed this problem in their second paper by calculating the back reaction of the binary on the stars and found a new, important effect. Because the star cluster is approximately spherical, the net force on the SMBH binary is very small, and so the center of mass of the binary *wanders* within the star cluster. This allows the loss cone to be sufficiently replenished to continue to shrink the binary. They were unable to make large explorations of parameter space, but they estimated that wandering implies that the overall hardening rate would occur 10 to 50 times slower than if the binary was in an absolutely fixed stellar background. This means that most high mass SMBH binaries will merge within a Hubble time. Unfortunately, SMBH binaries with masses less than 10^7 $M_\odot$ may still take too long to merge, and it is these which are of primary interest for the LISA bandpass. There are still considerable uncertainties, e.g. the effects of gas dynamics which would presumably accelerate mergers. Surprisingly, there are even uncertainties as to whether a SMBH binary can form at all in galaxy mergers, because some simulations find that the SMBH's end up on wide orbits where dynamical friction is slow in bringing them into the nucleus [36], although these results may have been due to unrealistic initial conditions [29]. Clearly much more work needs to be done on the dynamical evolution of SMBH's in galaxy mergers.

All the SMBH binary merger rate estimates for LISA ignore these details and assume that every galaxy merger produces a merged SMBH binary. Currently popular hierarchical structure formation scenarios (e.g. ΛCDM) lead to many mergers over time, and can successfully model both the historical star formation rate and quasar luminosity function, albeit with assumptions! (See e.g. [37] for recent work along these lines.) A very crude estimate based on multiplying the local number density of bright galaxies by a Hubble volume and assuming that every one underwent one merger in its history leads to a merger rate of roughly 1 per year [38], i.e. it only takes every galaxy to do one exciting thing in its lifetime in order to get interesting event rates for LISA!

The observed black hole mass/galaxy bulge mass relationship combined with the quasar luminosity function can be used to argue that black hole mergers do in fact occur [10,38]. Quasar activity peaked in the universe at redshifts between two and three, and the brightest quasars we observe then have luminosities corresponding to black hole masses of at least 4×10^8 $M_\odot$, assuming that their luminosity is at most the Eddington luminosity. The comoving number density at that time was 10^{-6} Mpc^{-3}, but if the black hole masses remained unchanged since these quasars faded, then the corresponding galaxy bulges today have number densities $\sim 10^{-3}$ Mpc^{-3}! One way to explain this discrepancy is to assume that the time over which quasars are bright is only $\sim 10^6$ yr, i.e. 10^{-3} times shorter than the duration of the bright quasar epoch ($\sim 10^9$ yr between redshifts two and three, cf. Figure 1), giving a much higher quasar number density. But then why is the black hole mass/galaxy bulge mass relationship roughly consistent with the total integrated quasar light, implying that black hole mass growth was around 1 percent radiatively efficient (mass e-folding time of 5×10^6 yr)? An alternative possibility is that these black holes later merged - a factor 4 increase in black hole mass (two equal mass mergers) would mean that the corresponding bulges would be several hundred times rarer. On the other hand, perhaps the bright quasars are not quite at the Eddington limit, and black holes are already four times larger at those epochs. Such (ad hoc) sub-Eddington growth to 2×10^9 M$_\odot$ from an initial mass of 10 M$_\odot$ would require $\sim 3 \times 10^9$ yr at ten percent radiative efficiency, which is way too long (cf. Figure 1). However, the 1 percent radiative efficiency inferred from the black hole mass/galaxy bulge mass relationship would allow this to be possible.

Finally, I should briefly mention that SMBH's could be lit up today in gravitational waves by orbiting stellar mass compact objects. Such sources would be extremely interesting from the physics point of view, as they make ideal high precision tests of strong field general relativity. This is mainly for the simple reason that orbits in a black hole spacetime are relatively easy to compute compared to, say, the gravitational wave signal from two merging equal mass black holes. Once again, the event rate is extremely uncertain: Sigurdsson [39] has recently estimated ~ 1 yr^{-1} to ~ 1 day^{-1}!

CONCLUSIONS

The predicted event rates for LISA are terribly uncertain when it comes to SMBH's. It is perhaps sobering to compare this situation with the predictions of neutron star binary mergers for ground-based gravitational wave observatories. The predicted event rates are based on only *three* known binaries which will merge within a Hubble time, of which only one (PSR 1534+12) dominates the merger rate extrapolation [40,41]. So while the event rates are clearly highly uncertain, at least one can take some consolation in knowing that these sources really exist! In contrast, we do not know, but strongly suspect, that SMBH binaries exist and merge. On the other hand, there is astronomical (never mind physics!) justification enough for LISA, if only because it is guaranteed to see Galactic binary stars. The fact that we do not know the SMBH merger rate, or how SMBH's formed, actually implies that LISA will be a unique astronomical tool in probing the origin and evolution of SMBH's, which, as is becoming increasingly clear, are intimately tied to the growth of structure in the universe.

REFERENCES

1. Thorne, K. S., & Braginsky, V. B., *ApJ*, **204**, L1-L6 (1976).
2. Miyoshi, M., et al., *Nature*, **373**, 127-129 (1995).
3. Genzel, R., Eckart, A., Ott, T., & Eisenhauer, F., *MNRAS*, **291**, 219-234 (1997).
4. Ghez, A. M., Klein, B. L., Morris, M., & Becklin, E. E., *ApJ*, **509**, 678-686 (1998).
5. Maoz, E., *ApJ*, **494**, L181-L184 (1998).
6. Tanaka, Y., et al., *Nature*, **375**, 659-661 (1995).
7. Wambsganss, J., Paczyński, B., & Schneider, P., *ApJ*, **358**, L33-L36 (1990).
8. Peterson, B. M., & Wandel, A., *ApJ*, **521**, L95-L98 (1999).
9. Magorrian, J., et al., *AJ*, **115**, 2285-2305 (1998).
10. Richstone, D., et al., *Nature*, **395**, A14-A19 (1998).
11. Fan, X., et al., *AJ*, **118**, 1-13 (1999).
12. Fan, X., et al., *AJ*, submitted (1999); astro-ph/9909169.
13. Turner, E. L., *AJ*, **101**, 5-17 (1991).
14. Krolik, J. H., *Active Galactic Nuclei*, Princeton: Princeton University Press, 1999.
15. Rees, M. J., *ARA&A*, **22**, 471-506 (1984).
16. Quinlan, G. D., & Shapiro, S. L., *ApJ*, **343**, 725-749 (1989).
17. Lee, M. H., *ApJ*, **418**, 147-162 (1993).
18. Quinlan, G. D., & Shapiro, S. L., *ApJ*, **356**, 483-500 (1990).
19. Rees, M. J., *Class. Quantum Grav.*, **14**, 1411-1415 (1997).
20. Shapiro, S. L., & Teukolsky, S. A., *ApJ*, **234**, L177-L181 (1979).
21. Goldreich, P., & Weber, S. V., *ApJ*, **238**, 991-997 (1980).
22. Fuller, G. M., Woosley, S. E., & Weaver, T. A., *ApJ*, **307**, 675-686 (1986).
23. Loeb, A., & Rasio, F. A., *ApJ*, **432**, 52-61 (1994).
24. Gaskell, C. M., *ApJ*, **464**, L107-L110 (1996).
25. Eracleous, M., et al., *ApJ*, **490**, 216-226 (1997).
26. Roos, N., Kaastra, J. S., & Hummel, C. A., *ApJ*, **409**, 130-133 (1993).
27. Sillanpää, A., et al., *ApJ*, **325**, 628-634 (1988).
28. Lehto, H. J., & Valtonen, M. J., *ApJ*, **460**, 207-213 (1996).
29. Makino, J., & Ebisuzaki, T., *ApJ*, **465**, 527-533 (1996).
30. Quinlan, G. D., & Hernquist, L., *New Astronomy*, **2**, 533-554 (1997).
31. Faber, S. M., et al., *AJ*, **114**, 1771-1796 (1997).
32. van der Marel, R. P., *ApJ*, **117**, 744-763 (1999).
33. Owen, F. N., O'Dea, C. P., Inoue, M., & Eilek, J. A., *ApJ*, **294**, L85-L88 (1985).
34. Begelman, M. C., Blandford, R. D., & Rees, M. J., *Nature*, **287**, 307-309 (1980).
35. Quinlan, G. D., *New Astronomy*, **1**, 35-56 (1996).
36. Governato, F., Colpi, M., & Maraschi, L., *MNRAS*, **271**, 317-322 (1994).
37. Kauffmann, G., & Haehnelt, M., *MNRAS*, submitted (1999); astro-ph/9906493.
38. Richstone, D., *Laser Interferometer Space Antenna*, ed. W. M. Folkner, Woodbury: American Institute of Physics, 41-44 (1998).
39. Sigurdsson, S., *Laser Interferometer Space Antenna*, ed. W. M. Folkner, Woodbury: American Institute of Physics, 53-56 (1998).
40. Narayan, R., Piran, T., & Shemi, A., *ApJ*, **379**, L17-L20 (1991).
41. Phinney, E. S., *ApJ*, **380**, L17-L21 (1991).