Physics 21 Final - 3 hours
3 pages - 10 problems

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Write your answers in a blue book. Calculators and two pages of notes allowed. No textbooks allowed. Please make your work neat, clear, and easy to follow. It is hard to grade sloppy work accurately. Generally, make a clear diagram, and label quantities. Make it clear what you think is known, and what is unknown and to be solved for. Except for extremely simple problems, derive symbolic answers, and then plug in numbers (if numbers provided) after a symbolic answer is available. You can take the acceleration of gravity near the earth as $g = 10\,\mathrm{m/s^2}$, to simplify calculations. **Put a box around your final answer... otherwise we may be confused about which answer you really mean, and you could lose credit.**

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1. You release a mass m_1 from rest at the top of a ramp shaped like a quarter-circle of radius R on to a horizontal table, where m_1 eventually has an elastic collision with mass m_2, as shown in Figure 1. You may choose any value for the mass m_1, but everything else in the situation stays the same, like R, m_2, and the acceleration of gravity g; everything is frictionless.

   (a) What is the velocity, v_1, of the mass m_1 just before it collides with m_2, in terms of the masses, R, and g?

   (b) What choice of m_1 will result in the largest velocity u_2 of m_2 after the collision, and what is the value of that velocity?

   (c) What choice of m_1 will result in m_1 to losing the largest *fraction* of its energy in the collision?

2. A line of rain drops, each with mass m and falling at terminal velocity v, falls on the pan of a weight scale. Each drop in the line is a distance ℓ from the next drop. When each rain drop falls on the pan, its water immediately comes to rest, accumulating in a puddle on the pan. At $t = 0$, the first rain drop hits the pan. What does weight is read on the scale, as a function of time t? Neglect any 'jerkiness' that arises because the drops reach the pan one at a time.

3. A particle of mass $m = 1\,\mathrm{kg}$ moves in one dimension from the origin to ∞ and is subject to the potential energy:

$$U(x) = \alpha(1 - \frac{x}{a}) + \frac{1}{2}\beta x^2 \tag{1}$$

   where $\alpha = 4\,\mathrm{Joules}$, $a = 1\,\mathrm{meter}$, and $\beta = 4\,\mathrm{Joules/meter^2}$.

   (a) Is there a stable equilibrium point for the particle, and if so, at what value of x does it occur (symbolically and numerically)?

   (b) Determine the (circular) frequency ω of small oscillations about any stable equilibrium point (both symbolically and numerically).

4. A bug of mass m_1 sits on a bar of mass m_2 and length L. The midpoint of the bar is connected by a massless strut of length d to a pivot point, as shown in Figure 2. The bar and strut sit atop a frictionless table. The bug starts at $t = 0$ from the point where the strut attaches to the bar, and walks along the bar with velocity **v with respect to the bar**. No net external force or torque acts on the system.

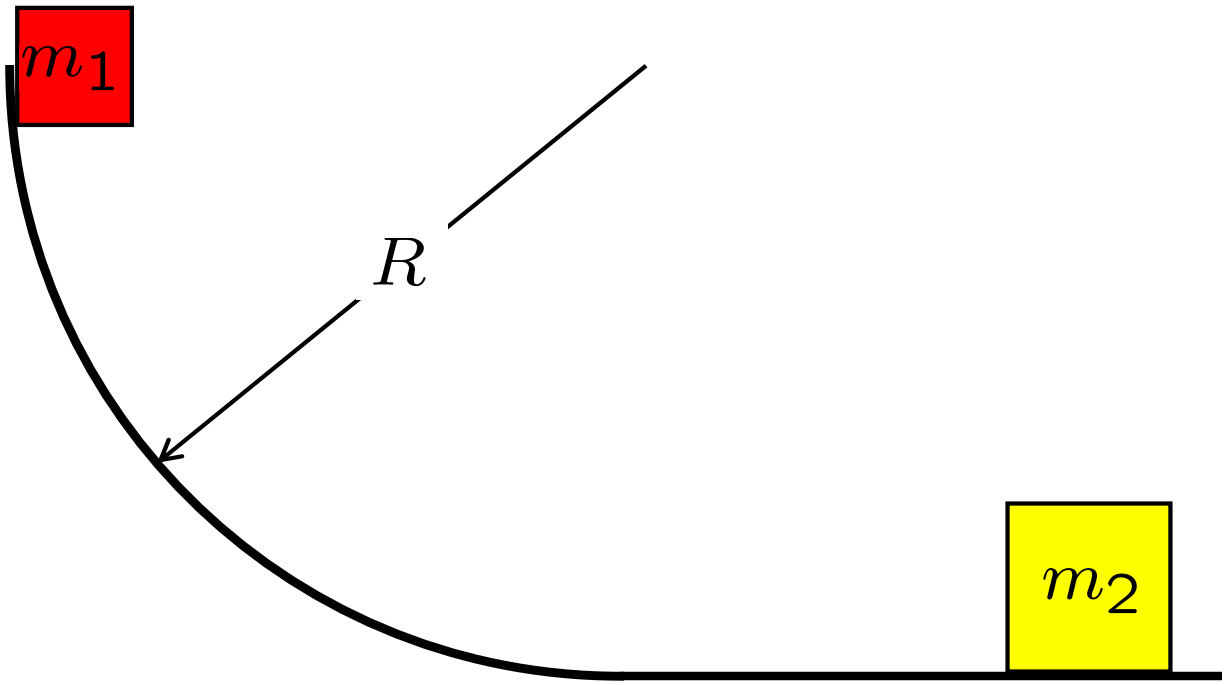

Figure 1: For use in Problem 1.

(a) Find the angular velocity of the bar and strut at $t = 0$, when the the bug has just started walking.

(b) Find the angular velocity of the bar and strut for general time t, for $0 \leq t \leq (L/2v)$, when the bug is still on the bar.

5. A mass m near the earth's surface is initally at rest, and is at coordinates (x, z), where the origin is at the level of the ground, z is the elevation above the ground, and x is the horizontal distance from the z axis. The mass m is dropped; find the torque vector on the mass with respect to the origin of the coordinates, as a function of time, from the time of release to the time it hits the ground.

6. A mass of 10 kg sits on a weight scale in an elevator on earth; the elevator accelerates upward with an acceleration of $5\,\mathrm{m/s^2}$. What is the reading on the weight scale?

7. A 10 kg mass is on an inclined plane that makes an angle of 45° with the horizontal, near the surface of the earth. The coefficient of friction between the mass and the incline is $\mu = 0.2$. What is the component of acceleration of the block parallel to the incline?

8. A mass m is attached to the end of a spring with spring constant k; the spring has negligible mass. The spring is used like a rope to swing the mass in a circle of radius R, at a constant angular velocity ω. What is the equilibrium length of the spring?

9. You throw a dense ball from ground level vertically upward with an initial velocity of 20 m/s (about 40 miles per hour). Ignore air resistance.

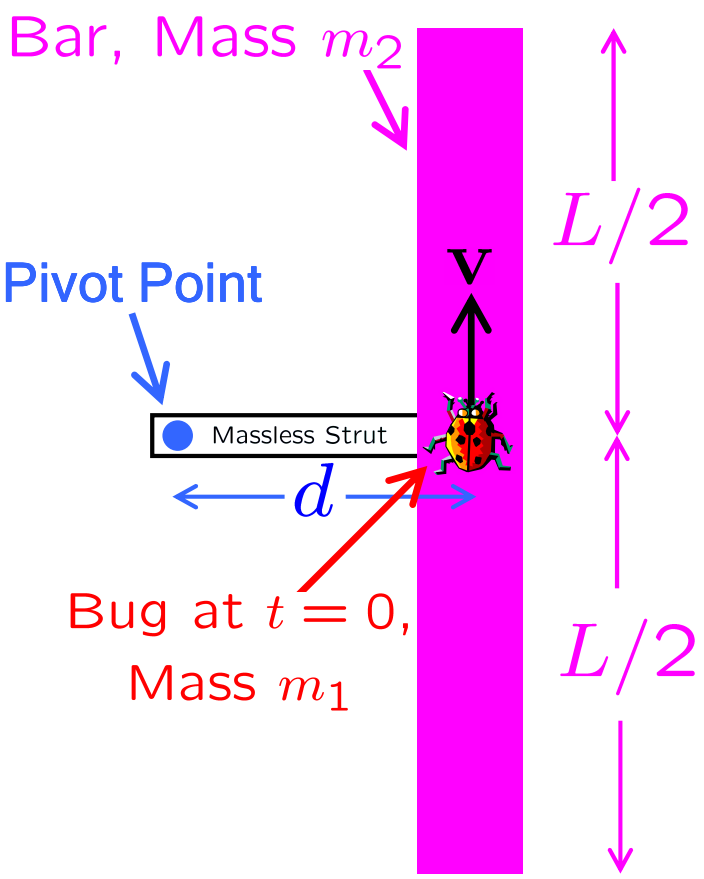

Figure 2: For use in Problem 4.

(a) How high does the ball go?

(b) For how much time is the ball in the air?

10. A mass M is split into uniform spheres of mass $m_1 = f \times M$, and $m_2 = (1 - f) \times M$, so note that $m_1 + m_2 = M$. The two masses are put into outer space, far from the gravitational influences of other objects, but they feel their each others' force of gravity. They are a distance R from one another, and both move in circular orbits.

(a) What are the respective radii of the circles that m_1 and m_2 move in, in terms of m_1, m_2, and R?

(b) Determine the angular velocity of the orbits about their center. Do the angular velocities depend on f?