UNIVERSITY OF CALIFORNIA, SANTA BARBARA

Department of Physics

Physics 23 Fall 2006

Prof. Gary Horowitz TA Ted Tao

SECOND MIDTERM EXAM

1. An electric dipole consists of charges $+2e$ and $-2e$ separated by 5nm in an electric field of strength $10^6 N/C$. What is the magnitude of the torque when the dipole moment is (a) parallel, and (b) at right angles to the direction of the field?

Solution: (a) (4 points) $\vec{\tau} = \vec{r} \times \vec{F}$. When the dipole moment is parallel to the field, there is no torque: $\tau = 0$.

(b) (6 points) At right angles: We compute the torque (about the center of the dipole) from each charge and add them together

$$\tau = \frac{d}{2}qE + \frac{d}{2}qE = dqE = (5 \times 10^{-9})(2e)(10^6) = 1.6 \times 10^{-21} Nm$$

2. A thin, infinitely long cylinder has charge $\lambda = 10^{-8} C/m$. The cylinder is $.2m$ in radius.
   a. What is the electric field at a radius r? Consider both $r > .2m$ and $r < .2m$.
   b. What is the electric potential at radius r? Assume $V = 0$ at the center of the cylinder.

Solution: (a) (8 points) Use Gauss' Law with cylindrical surfaces of radius r and length L. There is no contribution to the flux from the caps. For $r < .2m$, there is no enclosed charge, so $\vec{E} = 0$. For $r > .2m$, we have

$$2\pi r L E(r) = \frac{\lambda L}{\epsilon_0}$$

so

$$E(r) = \frac{\lambda}{2\pi\epsilon_0 r} = \frac{(10^{-8} C/m)(18 \times 10^9 Nm^2/C^2)}{r} = \frac{180}{r} N/C$$

(b) (7 points) Integrating the electric field out from $r = 0$, we find $V = 0$ for $r < .2m$, and

$$V(r) = -\frac{\lambda}{2\pi\epsilon_0} \int_{.2}^{r} \frac{dr}{r} = -180 \ln(r/.2m) \text{ Volts}$$

for $r > .2m$.

3. A charge $10^{-14}C$ is added to a conducting ball of radius 2cm.
   a. How much of the charge lies on the surface and how much lies inside?
   b. Compute the escape velocity for an electron from the surface of the ball. Neglect gravitational forces.

Solution: (a) (3 points) Inside a conductor the electric field is zero, so the charge must be zero by Gauss' Law.
(b) (7 points) The potential at the surface of the sphere is

$$V = \frac{q}{4\pi\epsilon_0 r}$$

The potential energy of an electron on the surface is $U = -eV$ and the escape velocity is defined so that the kinetic energy plus this potential energy is zero:

$$\frac{1}{2}m_e v^2 = \frac{eq}{4\pi\epsilon_0 r}$$

So

$$v^2 = \frac{eq}{2\pi\epsilon_0 m_e r} = \frac{(1.6\times 10^{-19}C)(10^{-14}C)(18\times 10^9 Nm^2/C^2)}{(9\times 10^{-31}kg)(2\times 10^{-2}m)}$$

$$v^2 = 16\times 10^8 m^2/s^2 \qquad \text{or} \qquad v = 4\times 10^4 m/s$$