Physics 25 Final - 3 hours
2 Pages

Harry Nelson

Wednesday, June 14

Write your answers in a blue book. Calculators and one page of notes allowed. No textbooks allowed. Please make your work neat, clear, and easy to follow. It is hard to grade sloppy work accurately. Generally, make a clear diagram, and label quantities. Make it clear what you think is known, and what is unknown and to be solved for. Except for extremely simple problems, derive symbolic answers, and then plug in numbers (if necessary) after a symbolic answer is available. **Put a box around your final answer... otherwise we may be confused about which answer you really mean, and you could lose credit.**

---

1. Give numerical answers to at least two significant figures for:
   (a) The Bohr radius.
   (b) The radius of a negative tau lepton (which has a mass that is 3500 times that of the electron, and the same electric charge as an electron) in the lowest Bohr orbit about a Calcium nucleus ($Z = 20$). Assume that the Calcium nucleus has infinite mass.
   (c) The speed of an electron bound in the lowest Bohr orbit about a Fermium nucleus ($Z = 100$), relative to the speed of light. Neglect relativity, and assume the Fermium nucleus has infinite mass.
   (d) The value of the fine structure constant, $\alpha = e^2/(\hbar c)$
   (e) The value of $\hbar c$ in eV-nm.
   (f) The wavelength of a photon with energy of 1×10^9 eV, also called 1 GeV.
   (g) The wavelength of a uranium atom, for which $mc^2 = 2.21 \times 10^5$ MeV. with kinetic energy of 10 MeV.

2. You shoot a photon of energy E at a stationary electron, and the photon comes right back at you, with energy $E/2$. Find E, and express E as a fraction of $m_e c^2$, where m_e is the electron mass and c is the speed of light. You might find the following relationship for Compton scattering to be useful: $\lambda' = \lambda + 2\pi(\hbar/mc)(1 - \cos\theta)$.

3. A particle of mass m moves in a radially symmetric potential $V(r)$ where:

$$V(r) = -\frac{\xi}{r^{3/2}}$$

   where ξ is a real, positive number.
   (a) Find both the radius and the energy of the lowest Bohr orbit.
   (b) Find the *ratio* of the radius of the second Bohr orbit (with $n = 2$) and the first.

(c) Find the *ratio* of the energy of the second Bohr orbit (with $n = 2$) and the first.

4. An electron with kinetic energy $E = 2.73\,\text{eV}$ moves in one dimension from $x = -\infty$ to $x = 0$, where the potential energy is constant and equal to zero. At $x = 0$, the electron encounters a *step down* in potential energy by $V_0 = -90\,\text{eV}$. What is the probability that the electron is reflected back toward $x = -\infty$, both symbolically and numerically?

5. A particle of mass m is in a potential of the following form:

$$\begin{array}{rcllr} V(x) & = & +\infty & x < 0 & \\ V(x) & = & 0 & 0 < x < a/2 & \text{(Region I)} \\ V(x) & = & V_0 & a/2 < x < a & \text{(Region II)} \\ V(x) & = & +\infty & x > a & \end{array}$$

(a) Make a clear plot of the potential $V(x)$.

(b) When in Region I, the particle has a total energy E. Find the expression for the wave vector or wave vectors k_1 for the particle when it is in Region I.

(c) Find the form of the wave functions in Region I that satisfy the boundary condition at $x = 0$; don't consider (yet) any other boundary condition.

(d) For $E < V_0$, find the expression for the wave vector or wave vectors k_2 for the particle when it is in Region II. If (the) k_2(s) is(are) imaginary, explicitly factor your expression into i and a real part.

(e) For $E < V_0$, find the form of the wave functions in Region II that satisfy the boundary condition at $x = a$; don't consider (yet) any other boundary condition.

(f) Find a condition on m, V_0, and a that must be satisfied for a bound state to exist with $E < V_0$.
