Statement of Research Interests

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I am currently a graduate student in the Physics Department at the University of California, Santa Barbara. My primary research interests focus on the subject of thermalization of quantum systems, specifically the question of whether an isolated quantum system can be appropriately described by the postulates of conventional equilibrium statistical mechanics. The eigenstate thermalization hypothesis (ETH), as originally postulated by Mark Srednicki and Josh Deutsch, suggests that the thermal behavior of individual energy eigenstates in a quantum system is responsible for the presence (or lack) of thermalization in isolated quantum systems. A variety of theoretical and numerical results suggest that individual energy eigenstates in “most” non-integrable quantum systems do indeed exhibit properties resembling that of a system in thermal equilibrium. In broad terms, my aim is to understand how general this behavior is.

Because thermalization is typically expected to occur in systems sufficiently far from integrability and in states sufficiently far from zero temperature, most analytic techniques available to condensed matter theorists do not succeed in identifying thermalization. For this reason, most of my work has focused on numerical exact diagonalization (with some recent analytical exceptions). Since most quantum mechanical systems possess a Hilbert space dimension which grows exponentially with system size, the understanding and extensive use of symmetries is typically required, in order to reduce computation times. Also, since attaining the full spectrum is not feasible for many systems due to their size, and since it is often useful to compare the behaviour of small systems to the behaviour of large, thermodynamic systems, it is sometimes necessary to use quantum Monte Carlo techniques, for example Stochastic Series Expansion. Most of my calculations are implemented in C++ (with some occasional use of Fortran, Python, and C), with some use of Mathematica (and at times other computational tools, for example the FEAST eigensolver).

Previous Work

Recently I have been interested in the compatibility of eigenstate thermalization with other paradigms of thermodynamics, in particular spontaneous symmetry breaking. In a recent paper (Phys. Rev. E (Rapid Communication) 92, 040103, 2015), Mark Srednicki and I examined the extent to which eigenstate thermalization could be adapted to the case of the 2D transverse-field Ising model (TFIM), a system which is non-integrable and features a finite temperature phase transition. Our results suggest that, with appropriate modifications of the notion of eigenstate thermalization, spontaneous symmetry breaking can indeed be accommodated into the framework of eigenstate thermalization.

In a follow-up to this work, Mark Srednicki, Marcos Rigol, Rubem Mondaini and I attempted to understand in more detail the interplay between eigenstate thermalization and spontaneous symmetry breaking. This resulted in a publication (Phys. Rev. E 93, 032104, 2016) in which we more carefully analyzed various signatures of eigenstate thermalization in the TFIM across several different parameter regimes and lattice geometries. While detecting signatures of ETH in this model, one of the shortcomings of this work was the inability to identify enough eigenstates in the broken symmetry phase to determine whether or not quantum chaos was present in this phase. Mark Srednicki and I have remedied this in a new paper (arXiv:1611.03992), in which we conclusively identified the presence of quantum chaos in the broken symmetry phase of the one-dimensional transverse-field Ising model with power-law interactions.

Current Work

The possibility that individual energy eigenstates reflect information about a finite-temperature phase transition, possibly including correlation lengths and critical exponents, is an intriguing one, and is motivated by recent work by James Garrison and Tarun Grover (arXiv:1503.00729). This work has presented an interesting corollary of eigenstate thermalization, namely that in a system satisfying ETH, it may be possible to explicitly calculate the thermal behavior of observables across a wide range of temperatures, using only one many-body eigenstate. Evidence for this was provided by studying a one-dimensional model without any symmetry breaking. However, a natural question is whether or not this behavior persists in a system with symmetry breaking, and whether individual energy eigenstates with an energy corresponding to the symmetric phase of the model contain any information about the symmetry broken phase. Tentative, unpublished numerical results which I have found suggest that in the model we have studied in our previous work (arXiv:1611.03992), individual energy eigenstates may possess some signatures of the finite-temperature phase transition in that model. Mark and I hope to elucidate this phenomenon in at least one upcoming publication, and James and I are interested in collaborating on the matter further in the future.

In addition to our ongoing study of the interplay between ETH and SSB, Mark Srednicki and I have also been interested in the onset of quantum chaos in non-integrable quantum systems, extending the work of previous authors (Silvestrov, Phys. Rev. E 58, 5629, 1998 and Jacquod and Shepelyansky, Phys. Rev. Lett. 79, 1837, 1997). Tentative results we have found suggest that the criterion for chaos in non-disordered systems, formulated in terms of the critical coupling, obeys a simple scaling law. We are attempting to motivate this claim with a more rigorous analytic argument. Additionally, we are interested in understanding the well-known problem in quantum chaos regarding whether the presence of a small perturbation will be enough to induce quantum chaos in an integrable model in the thermodynamic limit. We believe we have a numerical scaling argument involving the delocalization of eigenstates in the highly degenerate energy subspaces of clean, non-disordered, integrable Fermi gases which we can use to further elucidate this question. We hope to present these results regarding the onset of quantum chaos in a series of papers to be published in the coming months.

Future Work

There are several research avenues which I would like to pursue, some of them directly related to the question of isolated quantum thermalization, and some of them involving other areas of condensed matter research.

First, Mark Srednicki and I hope to continue our work with Marcos Rigol and Rubem Mondaini in order to fully understand the interplay between ETH and SSB. Our questions include, for example, how the statement regarding off-diagonal matrix elements in ETH must be modified in the presence of SSB, and also the time evolution in the symmetry broken regime, in particular the precise manner in which the system's isolated quantum dynamics realizes ergodicity breaking, expanding upon some recent work by other authors (see, for example, Blaß and Rieger, arXiv:1605.06258). One specific question, for example, is whether the eigenstates of the Hamiltonian encode any specific information about the magnetization plateau one would expect to appear on short-time scales when quenching from a polarized state in the ferromagnetic phase, and whether this value agrees with the one predicted by a “restricted” canonical ensemble in equilibrium. Additionally, I would like to understand whether the recently developed “emergent eigenstate” method pioneered by Vidmar et. al. (arXiv:1512.05373) can be used to describe the evolution of the order parameter on short time scales.

Second, I would like to explore the extent to which machine learning and (quantum) Monte Carlo methods can be combined to improve the study of (quantum) statistical models. I have a particular interest in this subject due to my previous work involving the interplay of physics and machine learning as an undergraduate student, and have been intrigued by several recent publications on the subject (Broecker et. al., arXiv:1608.07848; Ch'ng et. al., arXiv:1609.02552; Liu et. al., arXiv:1610.03137). One specific question I would like to investigate in detail is which classical systems can be effectively understood using a combination of cluster update techniques and machine learning algorithms, thus expanding upon the work outlined in Liu et. al. I would also like to investigate the extent to which similar methods could be used to improve quantum Monte Carlo techniques. In broader terms, I'd like to further understand which (Q)MC techniques and which machine learning techniques result together in the most efficient mechanism for understanding the behavior of (quantum) statistical models. I would also like to understand the extent to which these machine learning methods may be applicable to the study of dynamic, non-equilibrium (quantum) problems. Over the last few months, I have been mentoring an undergraduate student, Syrian Truong, in the subject of quantum Monte Carlo (we have a brief report regarding a Stochastic Series Expansion study of the long-range Ising model to be posted to the arXiv soon). Right now the two of us are in the process of deciding which of these questions regarding machine learning and (quantum) Monte Carlo we would like to investigate together in the immediate future.

Aside from these two specific projects, I have a particular interest in a variety of other long-standing, more broad questions in isolated quantum thermalization. These include, but are not limited to,

- The connection between random matrix models and quantum thermalization, and the sense in which physical Hamiltonians behave “randomly”
- The extent to which numerical linked cluster expansions (NLCEs) can be used to understand which quantum systems are capable of coming to thermal equilibrium under their own dynamics, expanding upon previous work by Marcos Rigol (Phys. Rev. Lett. 112, 170601, 2014)
- Whether the previously mentioned results of Garrison and Grover, in which reduced density matrices constructed from exact eigenstates of non-integrable models behave like thermal density matrices, applies to models with long-range interactions, and whether it applies even when the sub-system is not simply connected (we have unpublished results which suggest this is the case).
- The extent to which Shnirelman's Theorem, which guarantees eigenstate thermalization in the semi-classical limit, can be extended beyond semi-classical considerations
- Whether the conclusions of the renormalization group, that the physics of long length scales is independent of microscopic physics, has an appropriate statement in the context of eigenstate thermalization. Additionally, whether the scale-invariant behaviour of a system near a phase transition is reflected in individual eigenstates of a quantum Hamiltonian, and if so, how to study this phenomenon analytically or numerically without the need for full exact diagonalization.
- Under what circumstances ETH fails, and the extent to which disorder may result in a lack of thermalization (due to many-body localization)
- The extent to which machine learning methods may be an effective tool for studying isolated quantum thermalization
- Whether signatures of off-diagonal long range order, and Bose-Einstein condensation, can be detected in individual energy eigenstates, similar to how our previous work has detected signatures of SSB in individual energy eigenstates in the transverse field Ising model
- The extent to which eigenstate thermalization is compatible with (lattice) gauge theories