UNIVERSITY OF CALIFORNIA, SANTA BARBARA
Department of Physics

Prof. S.B. Giddings **Physics 229A** Winter 2007

Gauge Theories

ASSIGNMENT #1

Due Thursday, January 18, 2007

Reading: Rothstein, TASI lectures, p. 1 - 22 (see "resources");
Background, as needed, from Srednicki's QFT.

1. One defines the "critical dimension" d_c for an operator to be the space-time dimension for which that operator is marginal. How will that operator behave in dimensions d when $d > d_c$ or $d < d_c$? In a theory of interacting relativistic scalars, Dirac fermions, and gauge bosons, determine the critical dimension for the following operators:
   1. A gauge coupling to either a fermion or a boson through the covariant derivative in the kinetic term;
   2. A Yukawa interaction, $\phi\bar{\psi}\psi$;
   3. An anomalous magnetic moment coupling $\bar{\psi}\sigma_{\mu\nu}F^{\mu\nu}\psi$ for a fermion;
   4. A four fermion interaction, $(\bar{\psi}\psi)^2$.

2. Rothstein, exercise 1.1

3. The one-loop β function for QCD is

$$\beta(g) = \mu \frac{d}{d\mu} g \simeq -\frac{g3}{16\pi^2} b_0$$

with

$$b_0 = \frac{11}{3}N_c - \frac{2}{3}N_F \ .$$

Integrate this equation to find

$$\alpha_s\left(\mu^2 = q^2\right) \ ,$$

the value of the strong coupling constant at energy scale q^2, in terms of the integration constant Λ_c.

4. Derive the conserved $SU(3)_L \times SU(3)_R$ Nöther currents for the QCD lagrangian with u, d, s quarks and $m_u = m_d = m_s = 0$. Find the divergences of these currents for non-zero masses.