Color from Geometry: Some Gauge Theory Applications of AdS/CFT

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Main Message

Through the **gauge/gravity (or gauge/string, or holographic) correspondence**, string theory provides a useful tool to study *some* aspects of *certain* **strongly-coupled non-Abelian gauge theories**

Why do we care?

- Results are useful to develop some intuition on (at least some) strongly-coupled non-Abelian theories
- Overlap with other field(s) gives good opportunity to learn some interesting real-world physics!
- Simple extraction of plausible gauge theory physics from string counterparts provides additional support for gauge/gravity correspondence

Why should *you* care?

- Equivalence of gravitational and non-gravitational theories is stunning!! New theoretical paradigm
- Novel perspective on some difficult gravity problems
- Tools familiar to a relativist suddenly find new applications in high energy / nuclear (and condensed matter) physics

Disclaimers/Clarifications

- We are NOT claiming to have solved QCD!! The gauge theories under present control, while interesting, are at best toy models of real-world QCD
- This string theory **application** is orthogonal to the search for a unified theory. We're NOT looking for the Standard Model here.

  Then again, this IS string theory, arguably being **useful**.

Some Useful Reviews

- Gubser, Karch, arXiv:0901.0935
- Mateos, arXiv:0709.1523
- Aharony, Gubser, Maldacena, Ooguri,Oz, hep-th/9905111
- Myers, Vázquez, arXiv:0804.2423
- Hubeny, Rangamani, arXiv:1006.3675
- Son, Starinets, arXiv:0704.0240
- Erdmenger, Evans, Kirsch, Threlfall, arXiv:0711.4467
- Edelstein, Shock, Zoakos, arXiv:0901.2534
- Peeters, Zamaklar, arXiv:0708.1502
- Edelstein, Portugues, hep-th/0602021
- Klebanov, hep-ph/0509087
- Aharony, hep-th/0212193
- Gubser, Pufu, Rocha, Yarom, arXiv:0902.4041

Some Useful Reviews

Condensed Matter / Atomic Physics Applications

- Horowitz, arXiv:1002.1722
- McGreevy, arXiv:0909.0518
- Herzog, arXiv:0904.1975
- Hartnoll, arXiv:0903.3246
- Sachdev, arXiv:1002.2947
- Schäfer, Teaney, arXiv:0904.3107

Overall Plan

- L1: Motivation, String Theory & AdS/CFT Basics
- L2: AdS/CFT Dictionary, Entropy, Viscosity,
  Mesons, Screening, Energy Loss
- L3: Brownian, Damping, Unruh
  Generalizations of AdS/CFT (Gauge/Gravity)

Plan for Lecture 1

- Motivation: QCD and QGP
- String Theory Basics
- AdS/CFT `derivation’

QCD

- Quarks $q_C^{(F)}(x)$ $C=1,2,3$ $F=1,\ldots,6$

  \+ **3 colors** (*kinds* of strong charge) $SU(3)_c$

- Gluons $A^{\mu}_{CC'}(x) \equiv A_I^{\mu}(x) t^I_{CC'}$ $C,C'=1,2,3$ $I=1,\ldots,8$

$$S_{QCD} = \int d^4x \left[-\frac{1}{2g_{YM}^2} \mathrm{Tr}(F_{\mu\nu}F^{\mu\nu}) + \bar{q}^{(F)}(i\gamma^{\mu}D_{\mu} + m^{(F)})q^{(F)} \right]$$

$$D_{\mu} \equiv \partial_{\mu} - iA_{\mu} \qquad F_{\mu\nu} \equiv i\left[D_{\mu}, D_{\nu}\right]$$

(Note that $\left(A_{\mu}\right)_{\mathrm{here}} = g_{YM}\left(A_{\mu}\right)_{\mathrm{usual}}$)

QCD

- Quarks $\quad q_C^{(F)}(x) \quad C=1,2,3 \quad F=1,\ldots,6$

  $+$

- Gluons $\quad A^{\mu}_{CC'}(x) \equiv A^{\mu}_{I}(x) t^{I}_{CC'} \quad C,C'=1,2,3 \quad I=1,\ldots,8$

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CLASSICALLY: invariant under local symmetry $SU(3)_c$, and, if $m^{(F)}=0$, rescalings and global internal symmetry

$$SU(N_f)_V \times SU(N_f)_A \times U(1)_V \times U(1)_A$$

QCD

- Quarks $\quad q_C^{(F)}(x) \quad C=1,2,3 \quad F=1,\ldots,6$

  $+$

- Gluones $\quad A^\mu_{CC'}(x) \equiv A^\mu_I(x) t^I_{CC'} \quad C, C' = 1,2,3 \quad I = 1,\ldots,8$

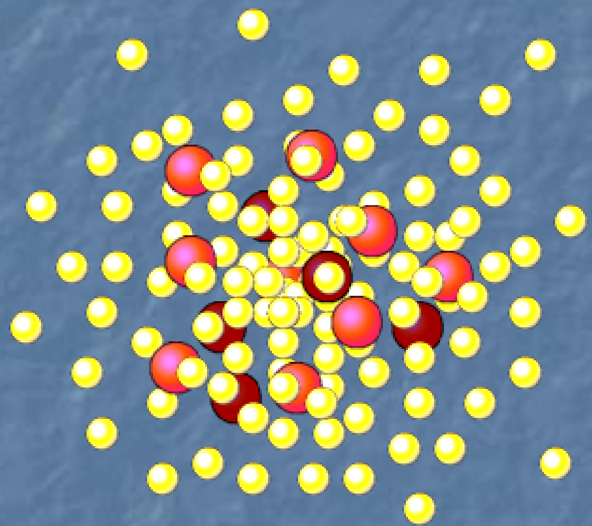
$$S_{QCD} = \int d^4x \left[-\frac{1}{2g_{YM}^2} \mathrm{Tr}(F_{\mu\nu}F^{\mu\nu}) + \bar{q}^{(F)}(i\gamma^\mu D_\mu + m^{(F)})q^{(F)} \right]$$

QUANTUMLY: invariant under local symmetry $SU(3)_c$, and, even if $m^{(F)}=0$, internal symmetry is only $SU(N_f)_V \times U(1)_V$ (chiral and rescaling symm. broken)

QCD: Asymptotic Freedom

The vacuum in a QFT is a polarizable medium…

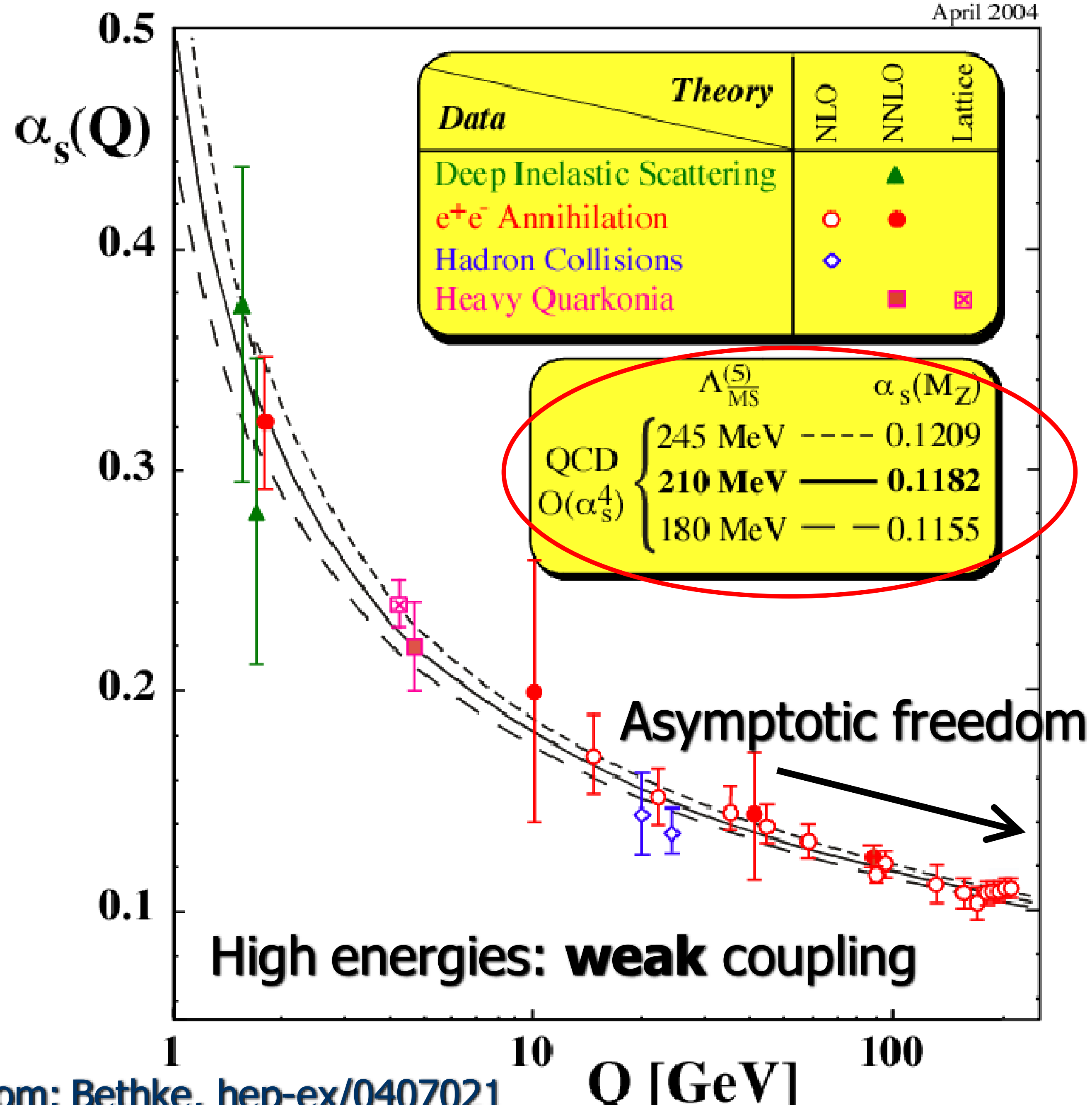
QCD: Asymptotic Freedom



Virtual quarks screen, virtual gluons antiscreen

$$\Longrightarrow \qquad \alpha_{YM}(Q) \equiv \frac{g_{YM}^2(Q)}{4\pi} \approx \frac{6\pi}{(11N_c - 2N_f)\log(Q/\Lambda_{\mathrm{QCD}})}$$

[Gross, Wilczek; Politzer]

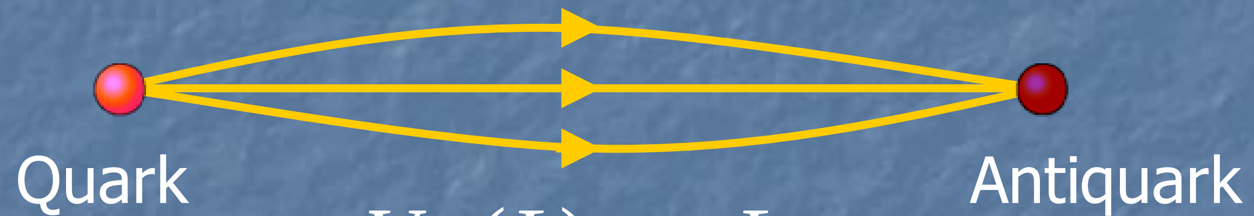


From: Bethke, hep-ex/0407021

QCD: Confinement

At low energies, we do NOT observe directly quarks and gluons, but hadrons (mesons, baryons, glueballs, etc.)

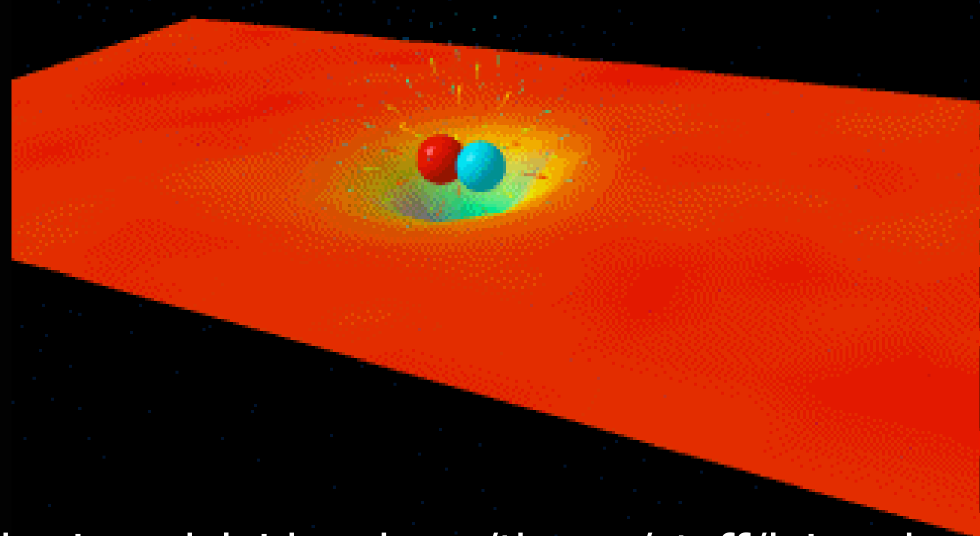
Only particles that are NEUTRAL under $SU(3)_c$



$$V_{q\bar{q}}(L) \simeq \sigma L$$

QCD: Confinement

These ‘flux tubes’ are visible in numerical calculations on discretized spacetime– lattice QCD



http://www.physics.adelaide.edu.au/theory/staff/leinweber/VisualQCD/Nobel/

Suggests connection between QCD and ‘fat’ strings (phenomenological model: `QCD string’ can reproduce “Regge behavior” $J= \alpha' m^2 + \alpha(0)$)

QCD: Deconfinement

- Note that strong coupling is necessary (although not sufficient!) to have confinement
- As we heat up a gas of hadrons, the coupling decreases…
- We therefore expect phase transition to

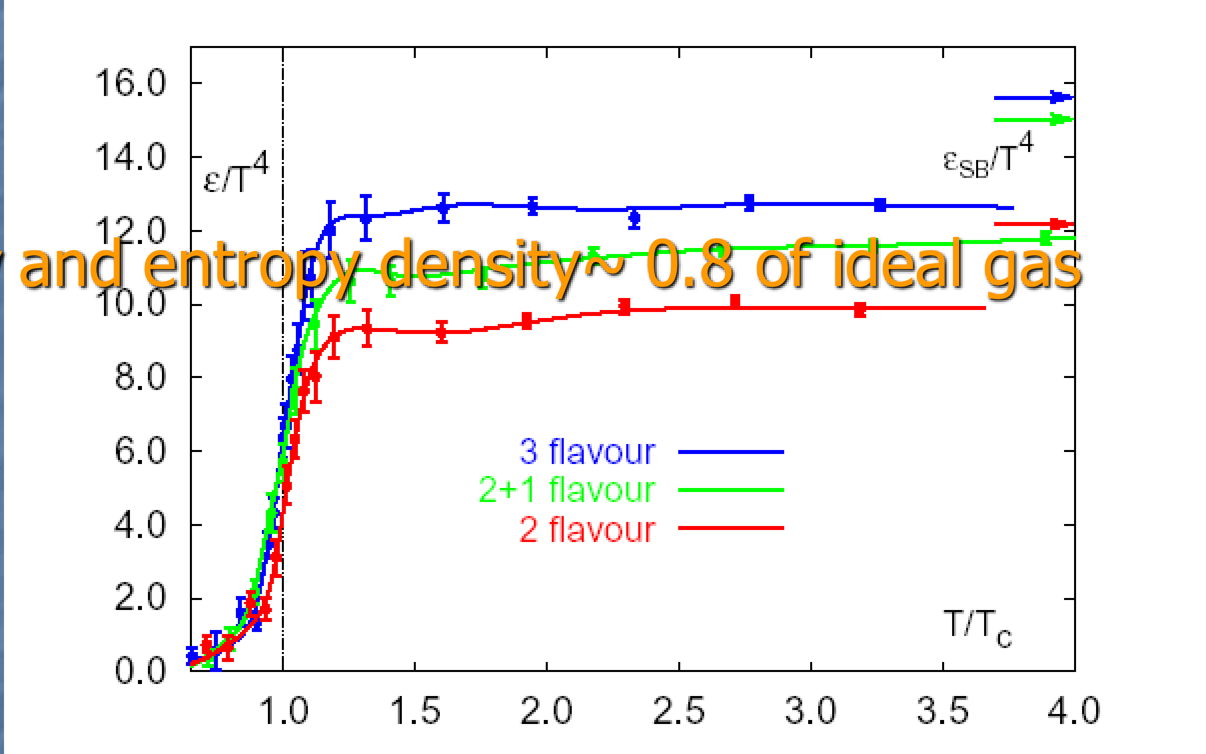
  Quark-Gluon Plasma (QGP)

  at a certain **deconfinement temperature**

$$T_c \approx \Lambda_{\mathrm{QCD}} \simeq 200\ \mathrm{MeV} \approx 2\times 10^{12}\mathrm{K}$$

QCD: Deconfinement

Lattice calculations confirm this, with $T_c \simeq 170-190\ \mathrm{MeV}$



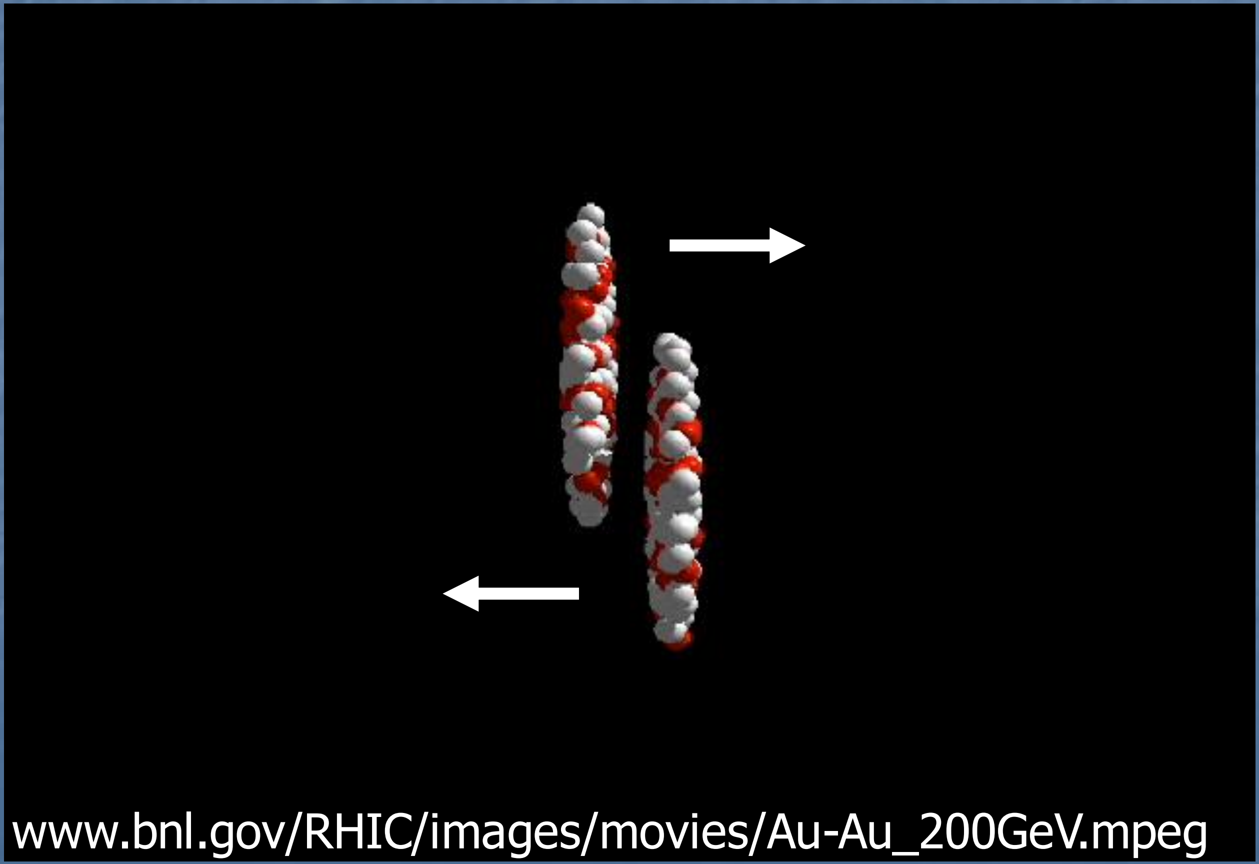
Energy and entropy density~ 0.8 of ideal gas

From: F. Karsch, hep-lat/0106019

QGP at RHIC (Brookhaven)

Au+Au (400 nucleons)

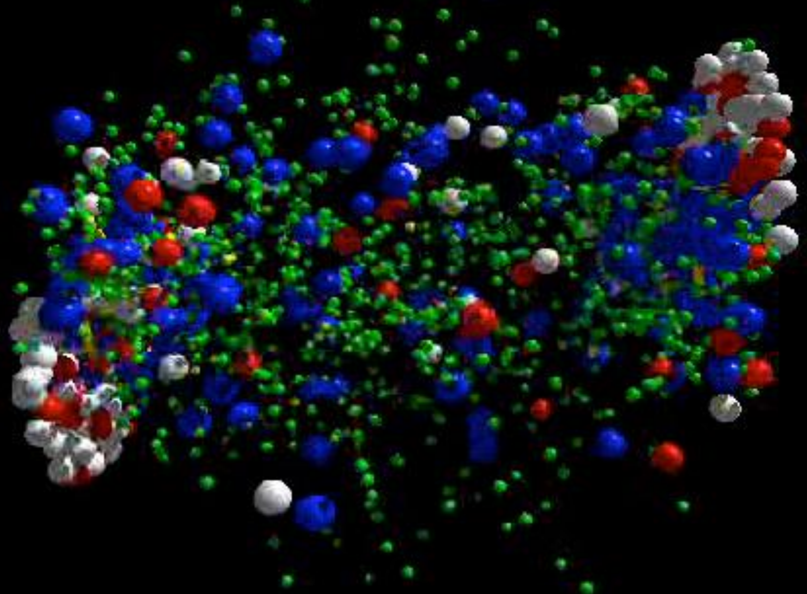
100 GeV/nucleon



QGP at RHIC (Brookhaven)

Au+Au (400 nucleons) → QGP → 5000 hadrons, etc.
100 GeV/nucleon ~10 GeV/hadron

Size~ 10^{-14} m
Duration ~ 10^{-22} s



www.bnl.gov/RHIC/images/movies/Au-Au_200GeV.mpeg

QGP at RHIC (Brookhaven)

- Temperature not far from $\Lambda_{\mathrm{QCD}} \simeq 200\ \mathrm{MeV}$:

$$T \sim (1-2)T_c \sim 170-300\ \mathrm{MeV}$$

- Low viscosity/entropy [Teaney;Romatschke,Romatschke] :

$$\frac{\eta}{s} \sim (0.1-0.2)\frac{\hbar}{k_B} \ll \left(\frac{\eta}{s}\right)_{\mathrm{pQCD}} \sim \frac{1}{g_{YM}^4 \log\left(1/g_{YM}^2\right)}\frac{\hbar}{k_B}$$

[Arnold,Moore,Yaffe]

- Short thermalization time, etc.

QGP at RHIC (Brookhaven)

$\Rightarrow$ **Strongly-Coupled Quark-Gluon Plasma (sQGP)**

$$g^2_{QCD} \sim 3-10 \quad \alpha_{QCD} \equiv g^2_{QCD}/4\pi \sim 0.3-1$$

- **Perturbative** expansion unreliable
- (Euclidean) **Lattice** calculations useful to determine static properties, but NOT dynamical ones
- Can construct effective **phenomenological models**…
- Or can try to do first-principles calculations in a **different (but hopefully similar) theory**

Cousins of QCD

- 'QCD' w/o quarks ($N_f = 0$) = $SU(3)_c$ Yang-Mills

  Preserves asymptotic freedom & confinement

- $SU(N_c)$ Yang-Mills (w/ or w/o quarks) with $N_c > 3$

  Preserves asymptotic freedom & confinement

Take $N_c \to \infty$ with $\lambda \equiv g_{YM}^2 N_c$ fixed ['t Hooft]

Try $SU(3)_c \simeq \underbrace{SU(\infty)_c} + O(1/N_c^2)$

Planar diagrams only; organized by 2D topologies {P. Kraus}

$\lambda \ll 1$ Perturbative expansion is valid

$\lambda \gg 1$ Surfaces filled: strings with coupling $1/N_c$?!

Lattice: $N_c \to \infty$ is decent approximation to $N_c = 3$!

[Teper et al.]

A (Distant) Cousin of QCD…

- $SU(N_c)$ Yang-Mills (w/o quarks): $A^{\mu}_{CC'}(x) \quad C, C' = 1, \ldots, N_c$

  + 6 real massless scalars: $\Phi^{I}_{CC'}(x) \quad I = 1, \ldots, 6$

  + 4 massless Weyl fermions: $\lambda^{A}_{CC'}(x) \quad A = 1, \ldots, 4$

  + carefully synchronized 3-pt and 4-pt interactions

= $SU(N_c)$ Super-Yang-Mills with $\mathcal{N} = 4$ supersymmetry

$$\mathscr{L} = \mathrm{Tr}\left\{ -\frac{1}{2g_{YM}^2} F_{\mu\nu} F^{\mu\nu} - i\bar{\lambda}^{A} \bar{\sigma}^{\mu} D_{\mu} \lambda^{A} - D_{\mu} \Phi^{I} D^{\mu} \Phi^{I} \right.$$

$$\left. + g_{YM} \mathrm{C}^{AB}_{I} \lambda_{A} \left[\Phi^{I}, \lambda_{B} \right] + g_{YM} \bar{\mathrm{C}}^{AB}_{I} \bar{\lambda}_{A} \left[\Phi^{I}, \bar{\lambda}_{B} \right] + \frac{g_{YM}^2}{2} \left[\Phi^{I}, \Phi^{J} \right] \left[\Phi^{I}, \Phi^{J} \right] \right\}$$

A (Distant) Cousin of QCD…

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Theory invariant under **rescalings** even at the quantum level… g_{YM} does NOT run with energy!! [Sohnius,West]

Spacetime symmetry: $SO(4,2) \supset \text{Poincaré}(3,1) \supset SO(3,1)$

Conformal group (+ fermionic part)

(dilatations + special conformal transf. + Poincaré)

Internal (R-)symmetry: $SU(4) \simeq SO(6)$

A (Distant) Cousin of QCD…

- $SU(N_c)$ Yang-Mills (w/o quarks): $A^{\mu}_{CC'}(x) \quad C, C' = 1, \ldots, N_c$

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= $SU(N_c)$ Super-Yang-Mills with $\mathcal{N} = 4$ supersymmetry

Is this theory at least qualitatively similar to QCD??

QCD vs. $\mathcal{N}=4$ SYM

- $T=0$:

Asympt. free $dg_{YM}^2/dE<0$		Conformal $dg_{YM}^2/dE=0$
Confined in IR		Deconfined
Only massive particles	$\neq$	No mass scale
Linear Potential		Coulomb Potential
Non-Supersymmetric		Supersymmetric

- $T>T_c$:

Approx. conformal $\varepsilon \sim T^4$		Temp. is only scale $\varepsilon \propto T^4$
Deconfined		Deconfined
Non-abelian plasma of gluons and matter in **fundamental** rep.	$\approx$	Non-abelian plasma of gluons and matter in **adjoint** rep.
Screened Potential		Screened Potential
No Supersymmetry		Supersymmetry broken

The AdS/CFT correspondence states that

$SU(N_c)$ $\mathcal{N}=4$ SYM $=$ [Maldacena] A particular **String Theory** on a certain curved spacetime

On RHS, easy to compute in the **strong-coupling** region

$$N_c \gg 1 \qquad g_{YM}^2 N_c \gg 1$$

(Remember: for sQGP we're ultimately interested in **QCD** at $N_c = 3 \qquad g_{YM}^2 N_c \sim 10-30$)

N.B.: We can similarly study other (closer) cousins of QCD, but even this crudest setup gets us some mileage

Now, where does the correspondence come from?

String Theory Basics

Perturbative definition about some (not necessarily flat) spacetime background {C. Johnson}

$$m = \sqrt{8} / l_s$$

$$m = \sqrt{4} / l_s$$

$$m = 0$$

For models of unification, $10^{3}\,\mathrm{GeV} < m_s < 10^{19}\,\mathrm{GeV}$ / $10^{-35}\,\mathrm{m} < l_s < 10^{-19}\,\mathrm{m}$

For application to QCD, $m_s \sim 1\,\mathrm{GeV}$

String Theory Basics

$m = 0$

φ , $h_{\mu\nu}$, $B_{\mu\nu}$, A_μ , $C_{\mu_1 \ldots \mu_{p+1}}$ + Fermions

Dilaton

Graviton

Gauge Bosons

Field content of $\mathcal{N} = 1, 2$ SUGRA in $D = 9 + 1$

String Theory Basics

Single basic interaction

Gravitational

Yang-Mills

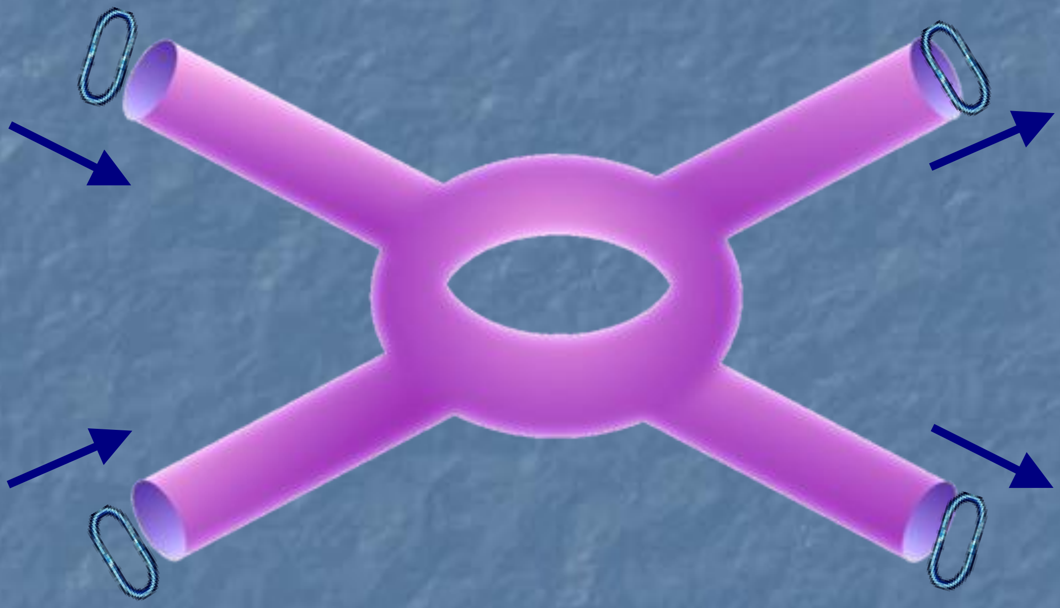
...

$$g_s = e^{\varphi}$$

$$G_N \sim g_s^{\,2} l_s^8, \quad g_{YM}^2 \sim g_s l_s^{p-3}$$

String Theory Basics

Can compute scattering amplitudes:



UV finite!

String Theory Basics

Results summarized in effective action, e.g.,

$$S_{\mathrm{IIB}} = \frac{1}{2\kappa^2}\int d^{10}x\sqrt{-g}\left[e^{-2\varphi}\left(R + 4(\partial_\mu \varphi)^2 - \frac{1}{12}\left(\partial_{[\mu} B_{\nu\lambda]}\right)^2 \right) \right.$$

$$\left. - \sum_{p=-1,1,3} \frac{1}{(p+2)!}\left(\partial_{[\mu_1} C_{\mu_2\cdots\mu_{p+2}]}\right)^2 + \cdots + \text{fermions} \right]$$

$$+ \text{higher derivative terms suppressed by } l_s^n$$

It is important to bear in mind that string theory is NOT really a theory of strings…

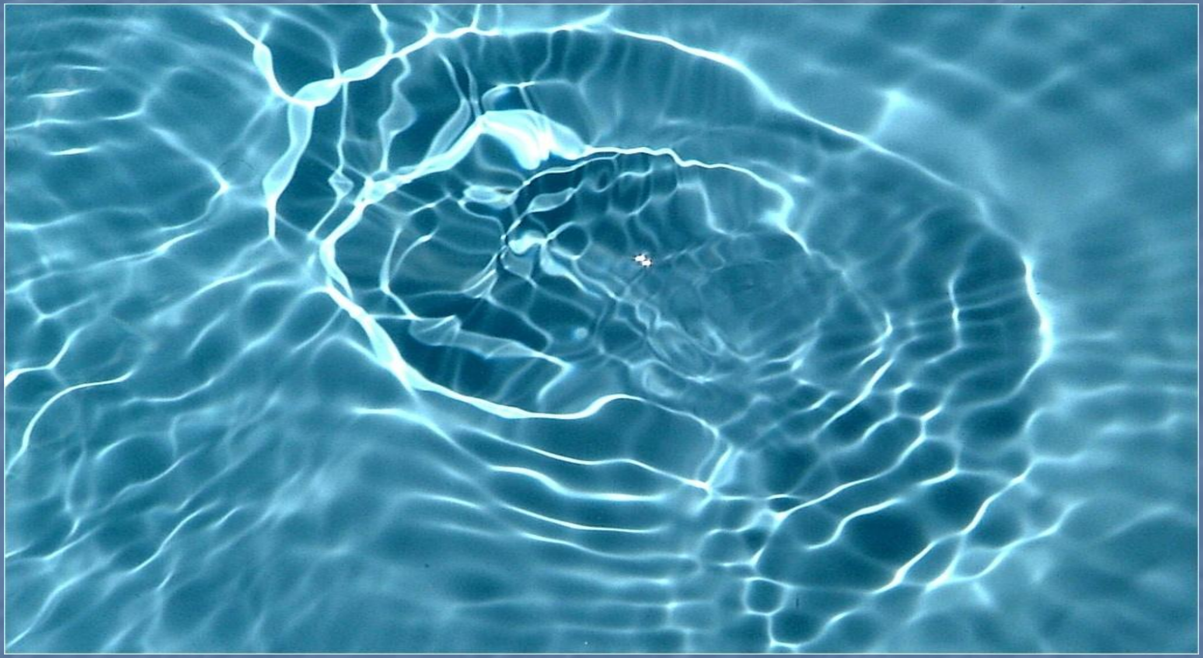
String Theory Basics

Recall that a ‘particle’ physicist is really a “**field** physicist”:

Particles= small excitations of a quantum field

String Theory Basics

Recall that a ‘particle’ physicist is really a “**field** physicist”:

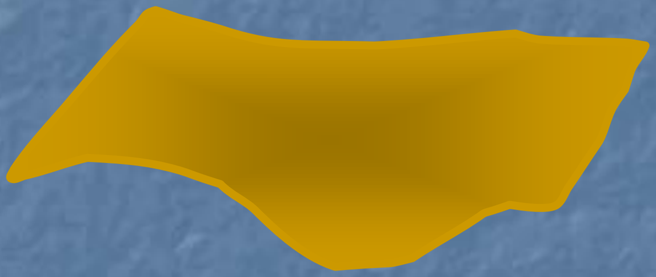


Soliton= large (finite energy) excitation of a quantum field

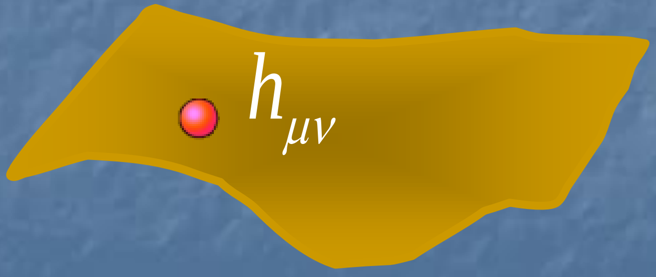
String Theory Basics

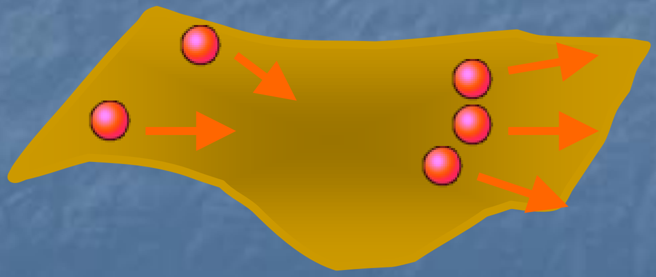
In particular, when confronted with

Gravity $\longleftrightarrow$ Spacetime ,



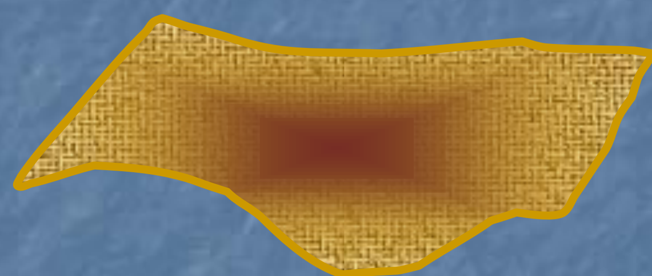
the particle/field physicist would start studying it as follows:





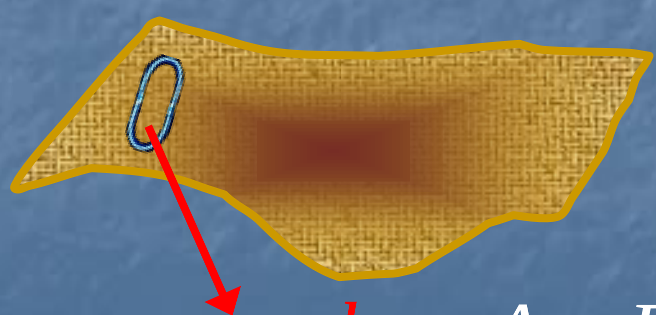
String Theory Basics

Within **string theory**, spacetime is only part of a much more complex structure (~ a "string field")



whose small excitations are described by **strings**:

size l_s , coupling g_s



$\varphi\,,\, h_{\mu\nu}\,,\, A_\mu\,,\, B_{\mu\nu}\,,\, C_{\mu_1\ldots\mu_{p+1}}$ + fermions + etc.

String Theory Basics

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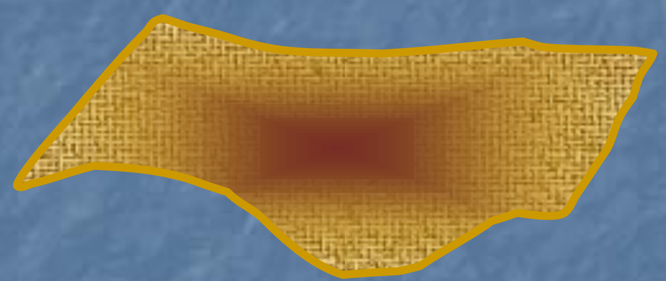


and whose large, solitonic, excitations include various **branes**:

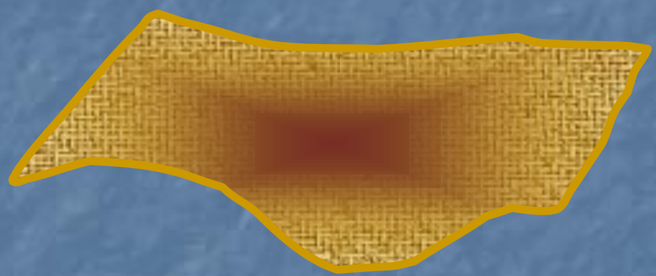
0-brane 1-brane 2-brane 3-brane

…

with masses $m \propto 1/g_s^{\,2}$ or $m \propto 1/g_s$ → D-branes

String Theory Basics

Within **string theory**, spacetime is only part of a much more complex structure (~ a "string field")



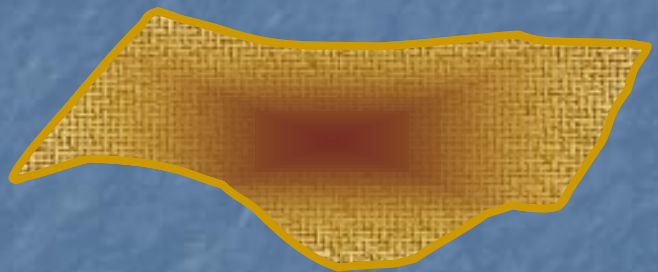
and whose large, solitonic, excitations include various **branes**:

...

(Interestingly, each of these objects can be reinterpreted as the fundamental string of a *different* string theory! E.g., D1-brane of Type I = string of Heterotic SO(32))

String Theory Basics

Within **string theory**, spacetime is only part of a much more complex structure (~ a "string field")



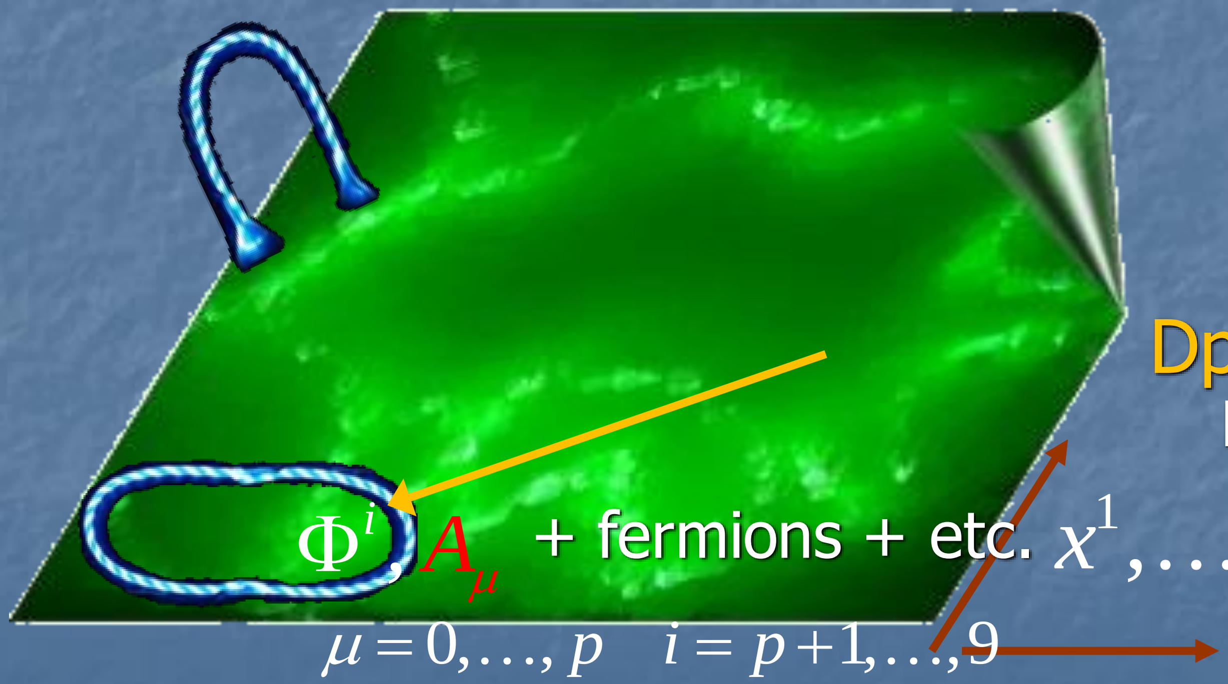
and whose large, solitonic, excitations include various **branes**:

...

(And, upon closer inspection, all of them have inner structure. Strings and branes are NOT fundamental.)

D-branes

D-branes are dynamical objects whose excitations are described by **open** strings



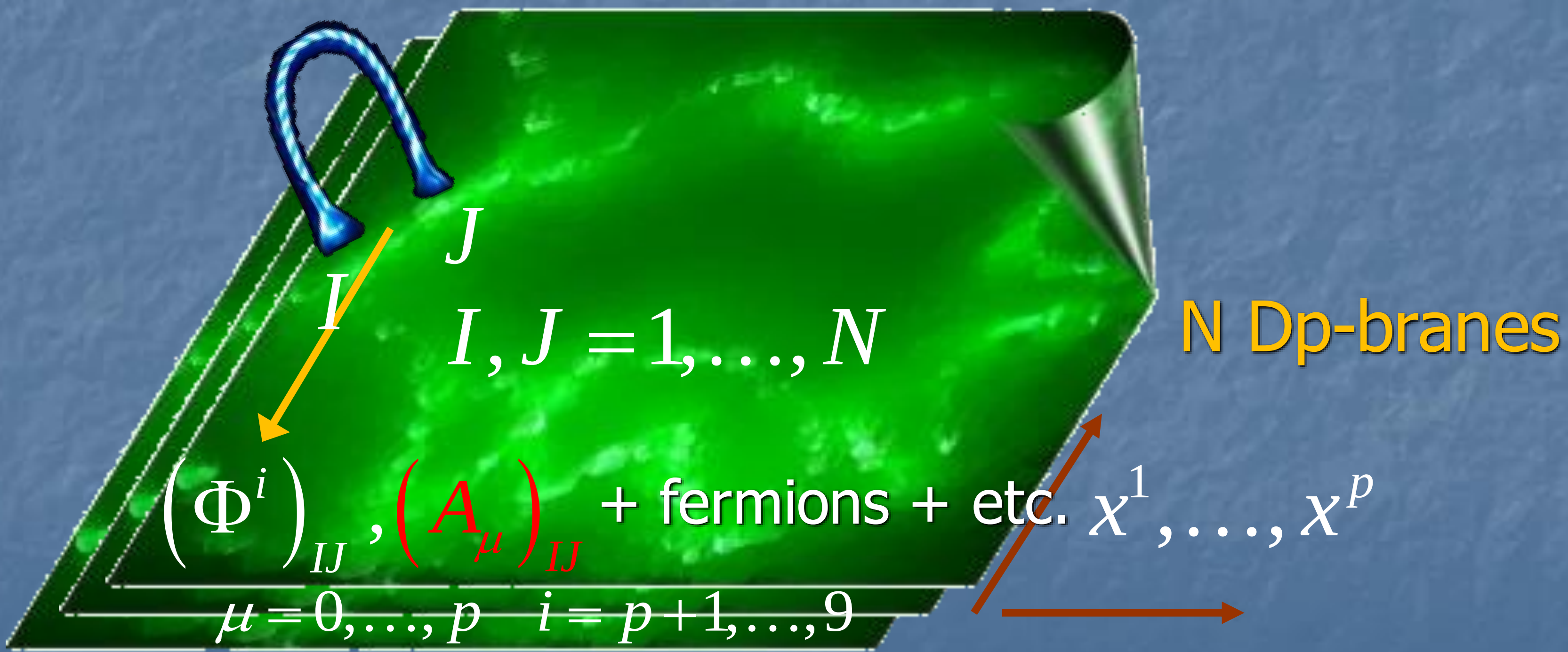
Multiple D-branes

$E \ll 1/l_s$

$\Rightarrow \quad \mathcal{N} = 2^{(7-p)/2}$ Super-Yang-Mills in $D = p+1$

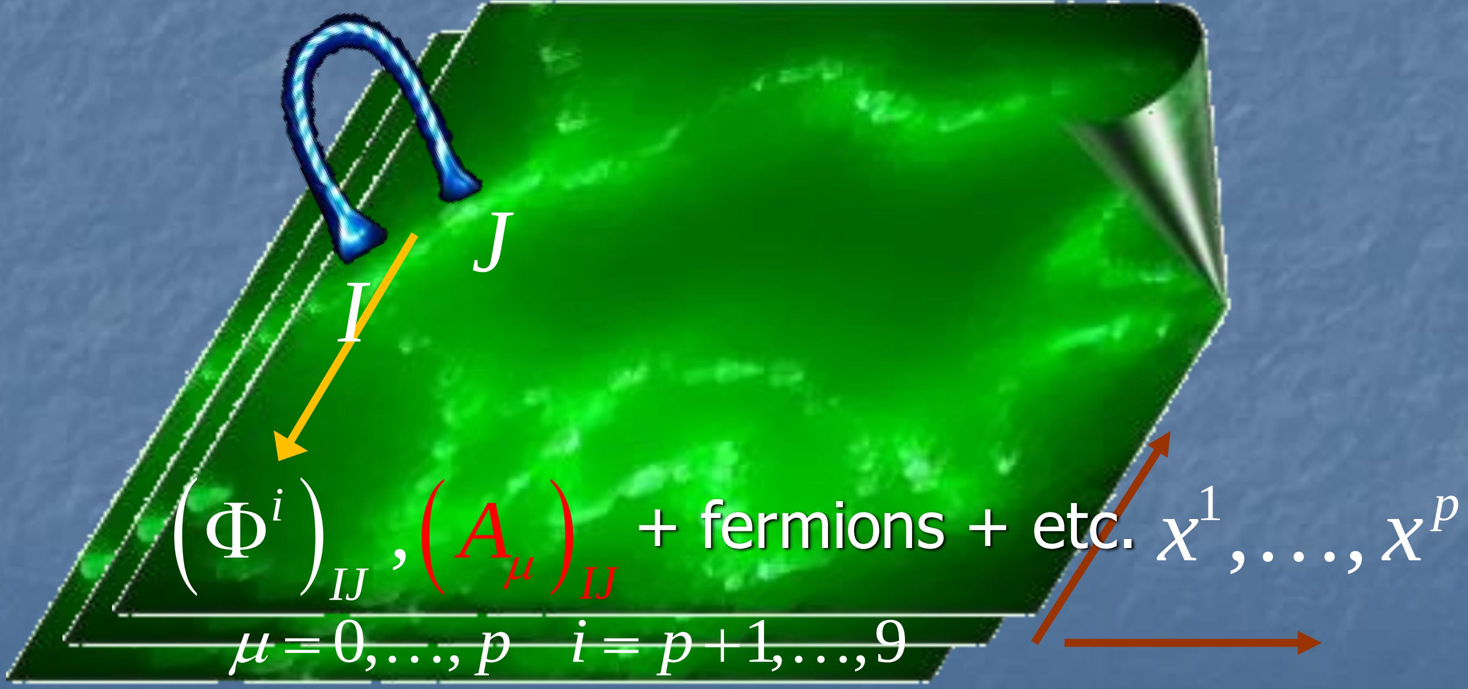
with $U(N)$ gauge group and $g_{YM}^2 = (2\pi)^{p-2} g_s l_s^{p-3}$

[Witten]



In SYM: $\left\langle \left(\Phi^9\right)_{11} \right\rangle \neq 0 \implies \left(A_\mu\right)_{1J}$ acquires mass

[Higgs]

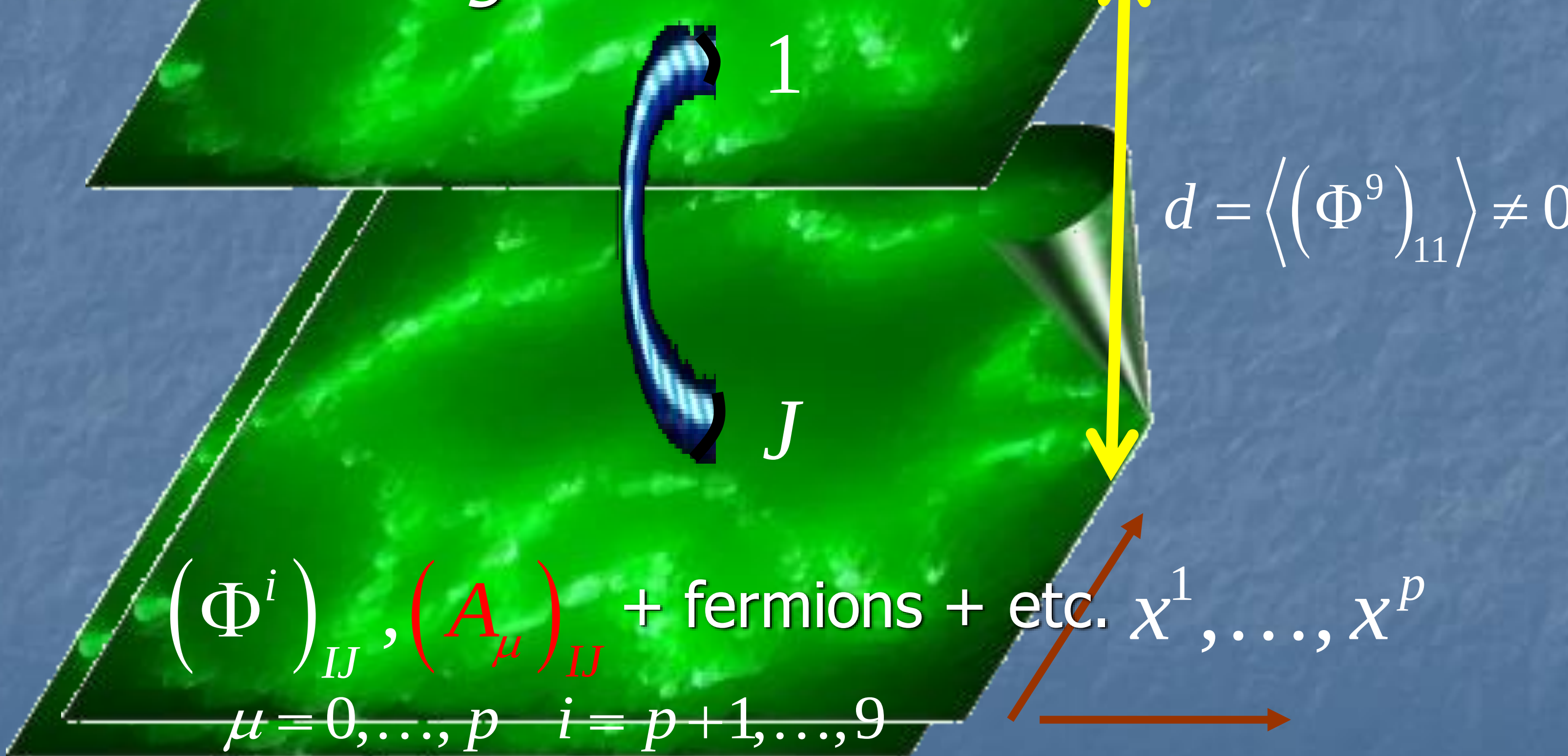


In SYM: $\left\langle \left(\Phi^9\right)_{11} \right\rangle \neq 0 \Rightarrow \left(A_\mu\right)_{1J}$ acquires mass

[Higgs]

In string theory: corresponding string acquires minimal *length*

[Witten]



Multiple D-branes

$$M_{\mathrm{D}p} = \frac{NV_p}{(2\pi)^p g_s l_s^{p+1}}, Q_{\mathrm{D}p} = N$$

Do these branes deform spacetime?

Black Branes

Supergravity e.o.m. have **solitonic** solutions

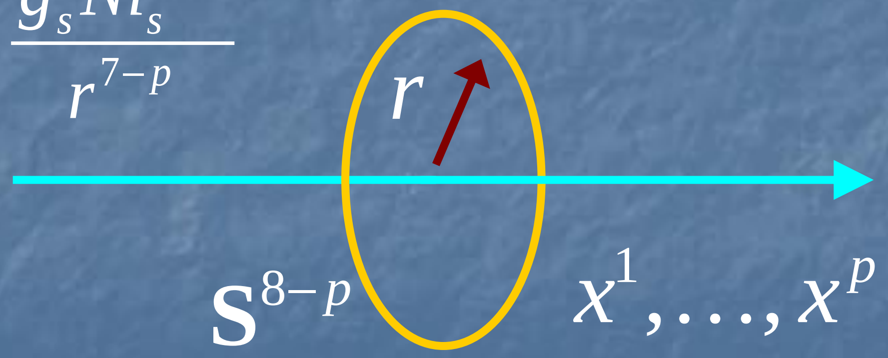
[Horowitz,Strominger]

Explicitly, for extremal (mass=charge) case:

$$g_{\mu\nu}(r)dx^\mu dx^\nu = H^{-1/2}\left(-dx_0^2+\cdots+dx_p^2\right)+H^{1/2}\left(dr^2+r^2 d\Omega_{8-p}^2\right),$$

$$\exp\left(\varphi(r)\right)=g_s H^{(3-p)/4}, \qquad C_{01\cdots p}(r)=1-H^{-1},$$

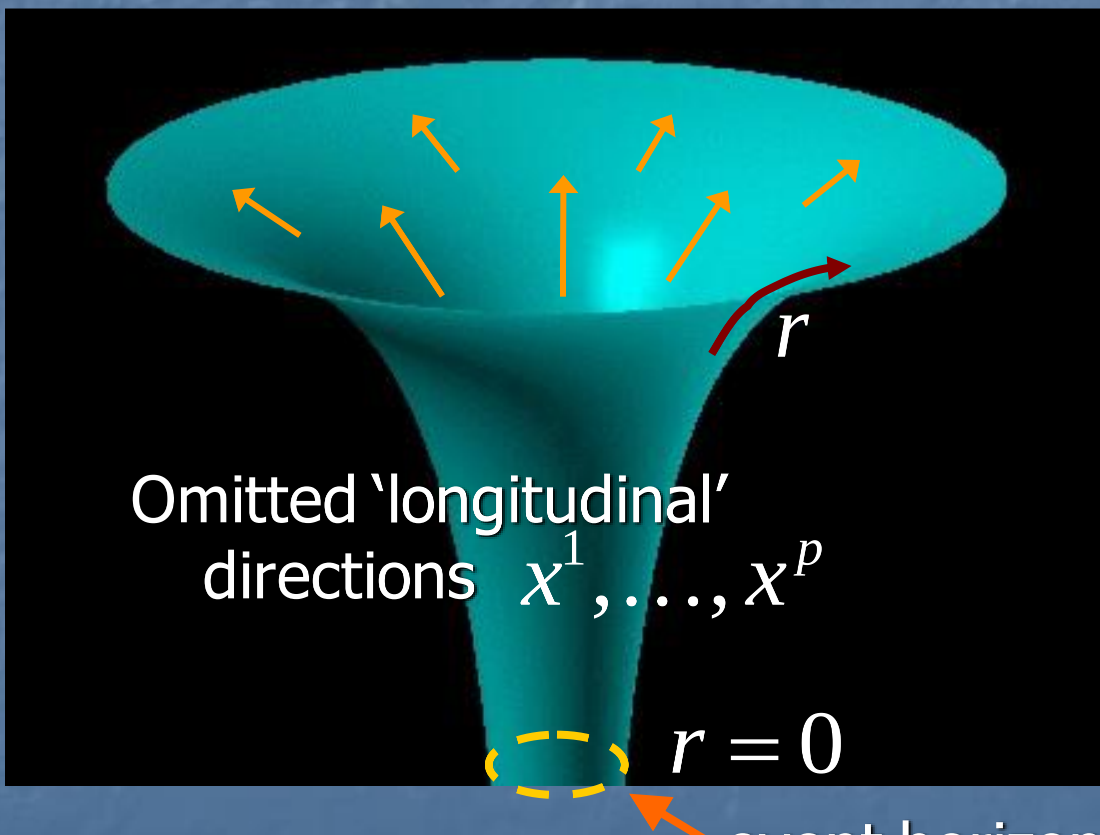
$$H(r)\equiv 1+\left(2\sqrt{\pi}\right)^{5-p}\Gamma\left(\tfrac{7-p}{2}\right)\frac{g_s N l_s^{7-p}}{r^{7-p}}$$



Black Branes

Extended (p-dim) versions of charged black holes:

[Horowitz,Strominger]

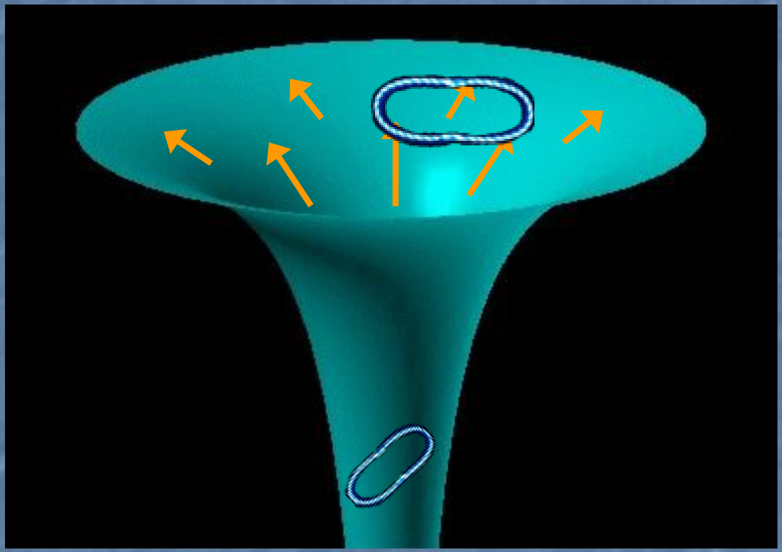


$$M = \frac{NV_p}{(2\pi)^p g_s l_s^{p+1}}$$

$$Q = N$$

$$\mathbf{S}^{8-p} \times \mathbf{R}^p$$

Black Branes vs. D-branes



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[Polchinski]

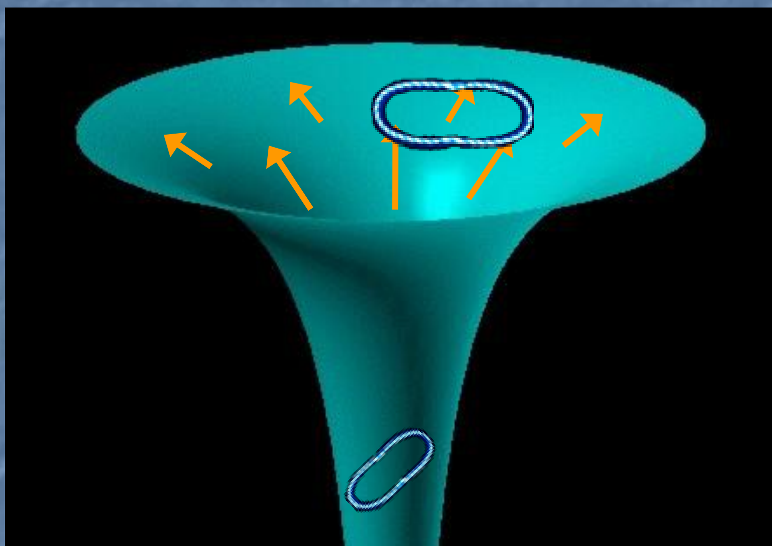
Black p-brane

N Dp-branes
+ Flat Spacetime

D-branes/Open strings?

Curved Spacetime?

Black Branes vs. D-branes



Black p-brane

=

[Polchinski]

N Dp-branes
+ Flat Spacetime

D-branes/Open strings? ⟷ Curved Spacetime?

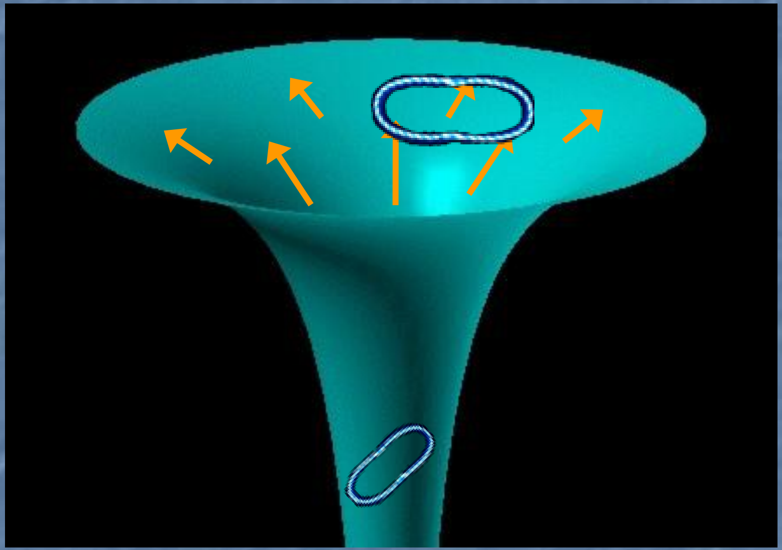
Overall Plan

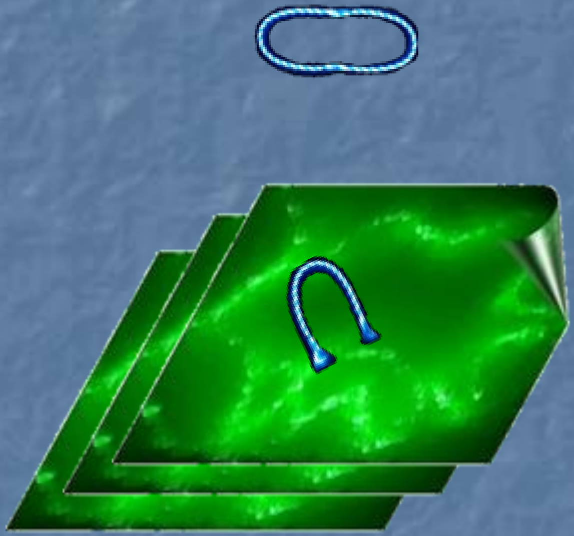
- L1: Motivation, String Theory Basics
- L2: AdS/CFT `Derivation' & Dictionary,
  Entropy, Viscosity, Screening, Energy Loss
- L3: Mesons, Brownian, Damping, Unruh,
  Generalizations of AdS/CFT (Gauge/Gravity)

Plan for Lecture 2

- `Derivation' of AdS/CFT
- AdS/CFT Dictionary
- A Few Applications

‘Derivation’ of AdS/CFT



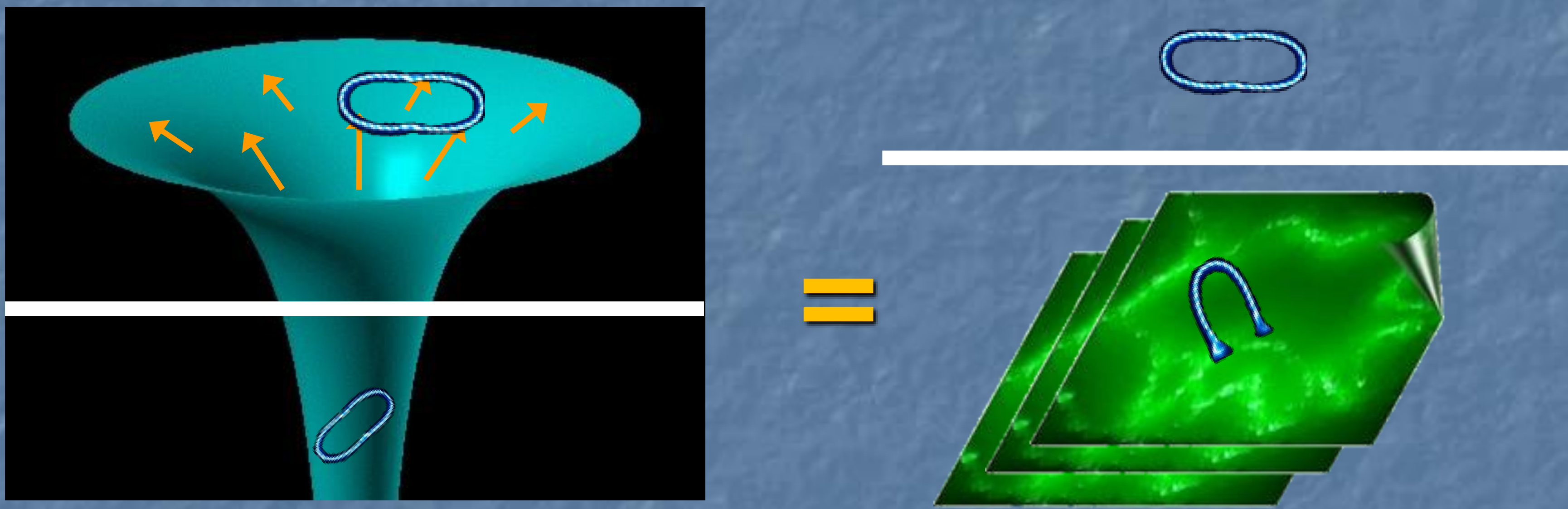


Let’s take $p = 3$. At **low energies**,

$E \ll 1/(g_s N)^{1/4} l_s \ll 1/l_s$,

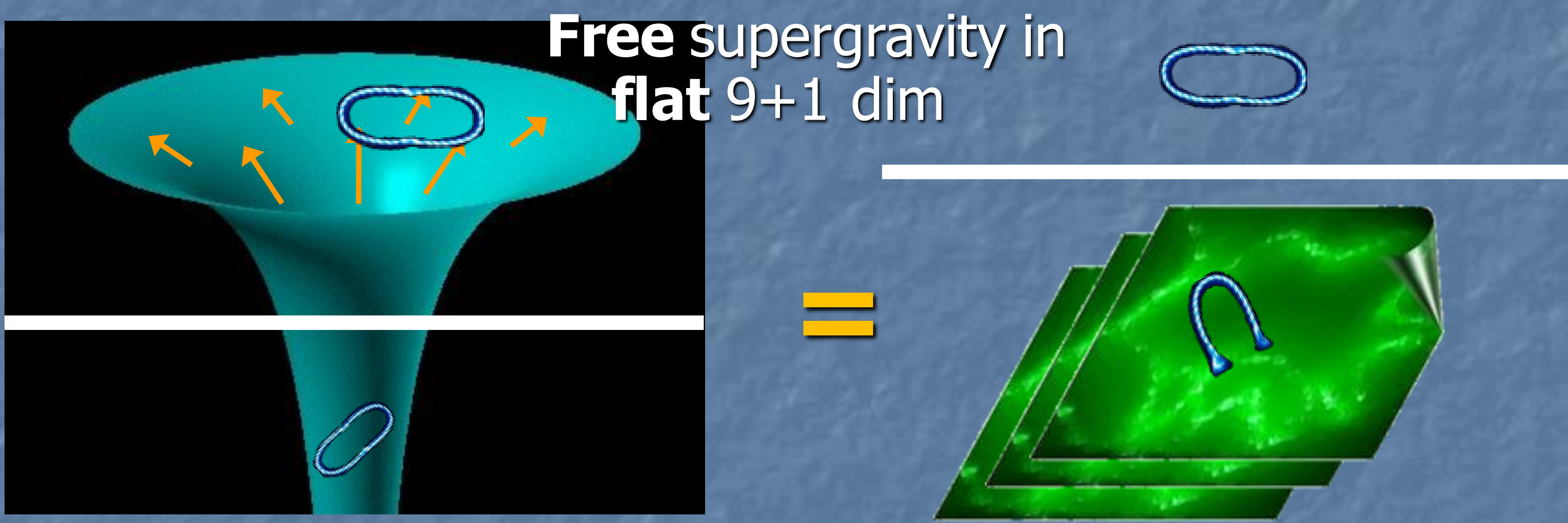
2 important things happen…

‘Derivation’ of AdS/CFT



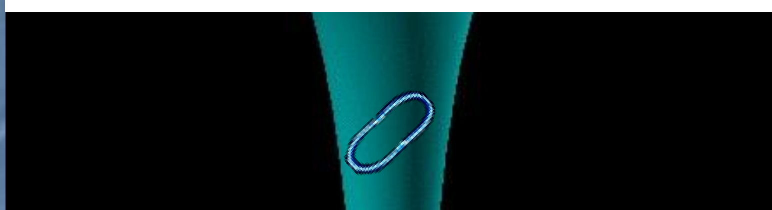
Each system splits into 2 decoupled components…

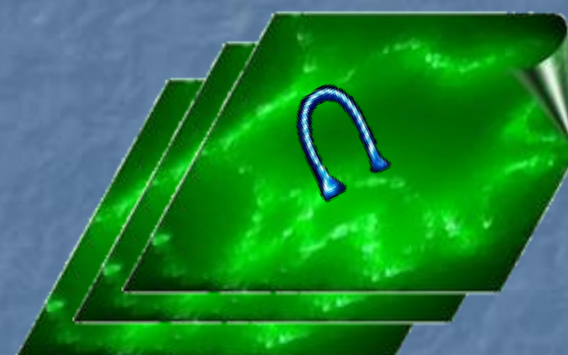
‘Derivation’ of AdS/CFT



and these components simplify drastically…

‘Derivation’ of AdS/CFT



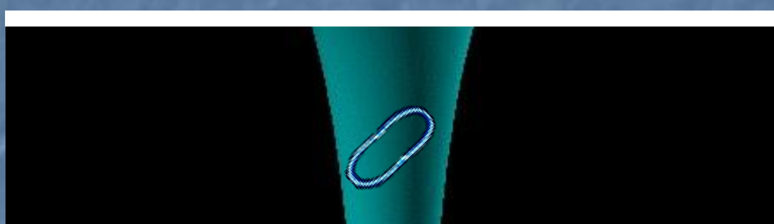


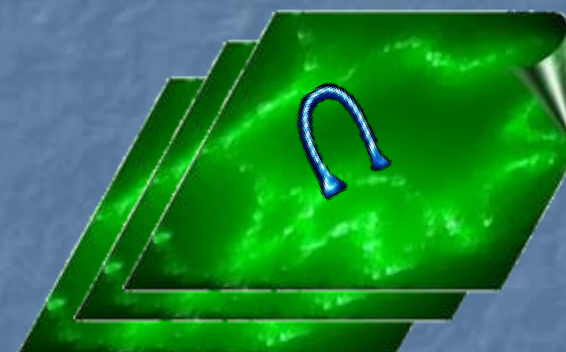
$$g_{\mu\nu}(r)dx^{\mu}dx^{\nu} = H^{-1/2}\left(-dx_0^2 + \cdots + dx_3^2\right) + H^{1/2}\left(dr^2 + r^2 d\Omega_5^2\right),$$

$$e^{\varphi(r)} = g_s, \quad C_{0123}(r) = 1 - H^{-1},$$

$$H(r) \equiv \cancel{1} + \frac{4\pi g_s N l_s^4}{r^4} \equiv \frac{L^4}{r^4}$$

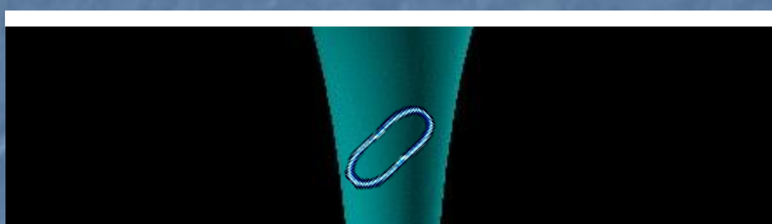
‘Derivation’ of AdS/CFT

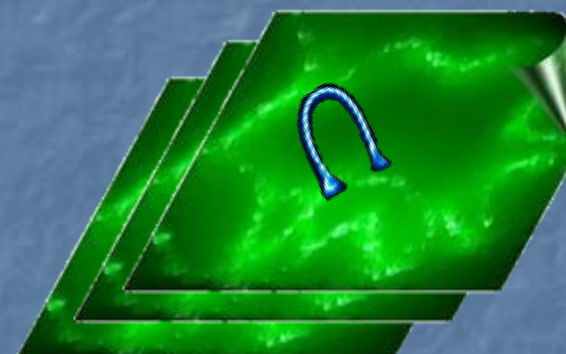




$$g_{\mu\nu}(r)dx^{\mu}dx^{\nu} = \underbrace{\left[\frac{r^2}{L^2}\left(-dt^2 + d\vec{x}^2\right) + \frac{L^2}{r^2}dr^2\right]}_{\text{5-dim anti-de Sitter (Poincaré patch)}} + L^2 d\Omega_5^2,$$

‘Derivation’ of AdS/CFT





Type IIB ST on AdS_5xS^5 = SU(N) SYM in 3+1 dim !!

[Maldacena; Gubser,Klebanov,Polyakov; Witten]

Anti-de Sitter Space

Maximally symmetric space with negative curvature, defined as (universal covering of) hyperboloid in $R^{4,2}$:

$$-X_0^2 + X_1^2 + X_2^2 + X_3^2 + X_4^2 - X_5^2 = -L^2$$

Global coords: $0 \le \rho, \quad -\infty < \tau < \infty, \quad 0 \le \Omega_A \le 1$

$(A = 1, \ldots, 4, \quad \Omega_A \Omega_A = 1)$

$$X_0 = L \cosh \rho \cos \tau$$

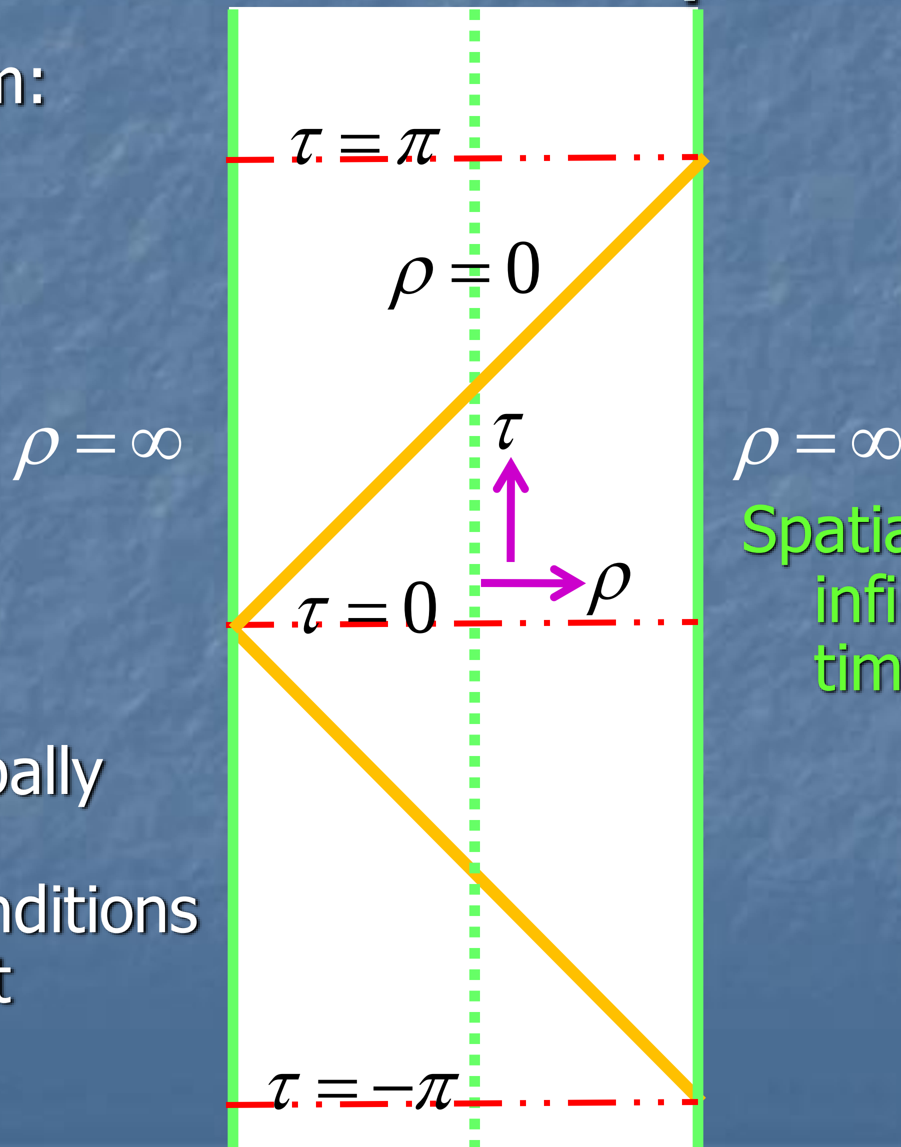
$$X_A = L \sinh \rho \, \Omega_A$$

$$X_5 = L \cosh \rho \sin \tau$$

$$\Rightarrow \quad ds^2 = L^2 \left(-\cosh^2 \rho \, d\tau^2 + d\rho^2 + \sinh^2 \rho \, d\Omega_3^2 \right)$$

Anti-de Sitter Space

Penrose diagram:



Spatial (and null) infinity is timelike surface

AdS is NOT globally hyperbolic. Boundary conditions are important

Anti-de Sitter Space

Maximally symmetric space with negative curvature, defined as (universal covering of) hyperboloid in $R^{4,2}$:

$$-X_0^2 + X_1^2 + X_2^2 + X_3^2 + X_4^2 - X_5^2 = -L^2$$

Poincaré (or horospheric) coords: $0 \le r, \quad -\infty < t, \vec{x} < \infty$

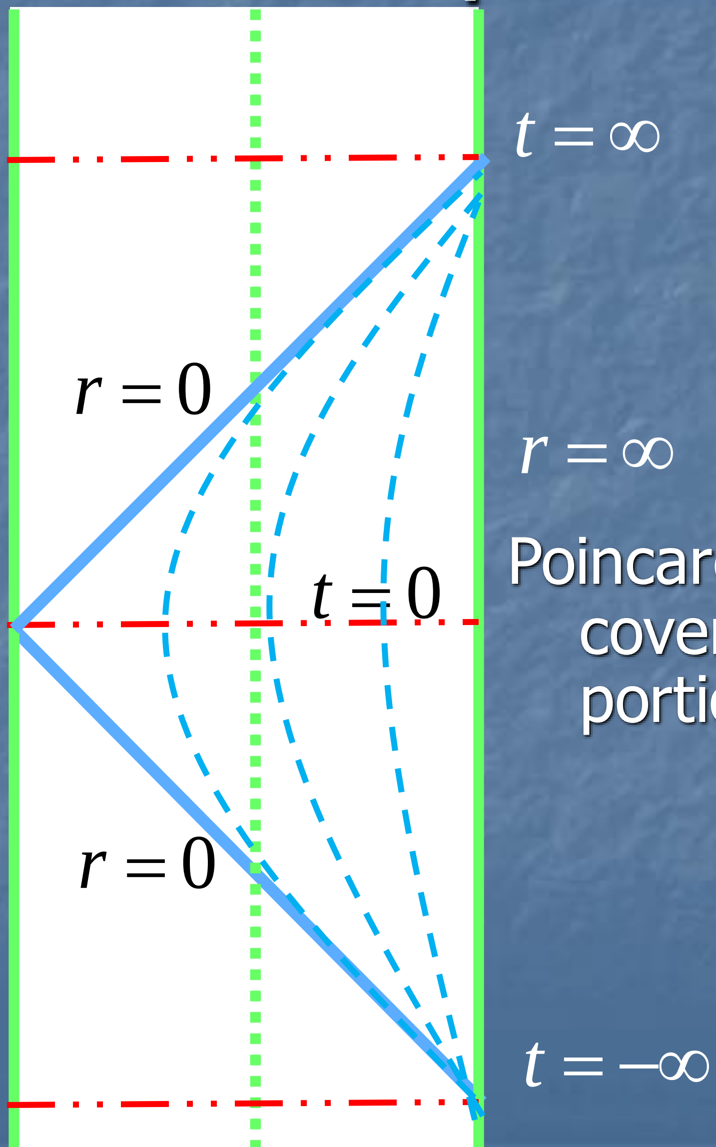
$$X_0 = \frac{L^2}{2r}\left(1 + \frac{r^2}{L^4}\left(L^2 + \vec{x}^2 - t^2\right)\right), \qquad \vec{X} = \frac{r}{L}\vec{x}$$

$$X_4 = \frac{L^2}{2r}\left(1 - \frac{r^2}{L^4}\left(L^2 - \vec{x}^2 + t^2\right)\right), \qquad X_5 = \frac{r}{L}t$$

$$\Rightarrow \qquad ds^2 = \frac{r^2}{L^2}\left(-dt^2 + d\vec{x}^2\right) + \frac{L^2}{r^2}dr^2$$

Anti-de Sitter Space

Penrose diagram:



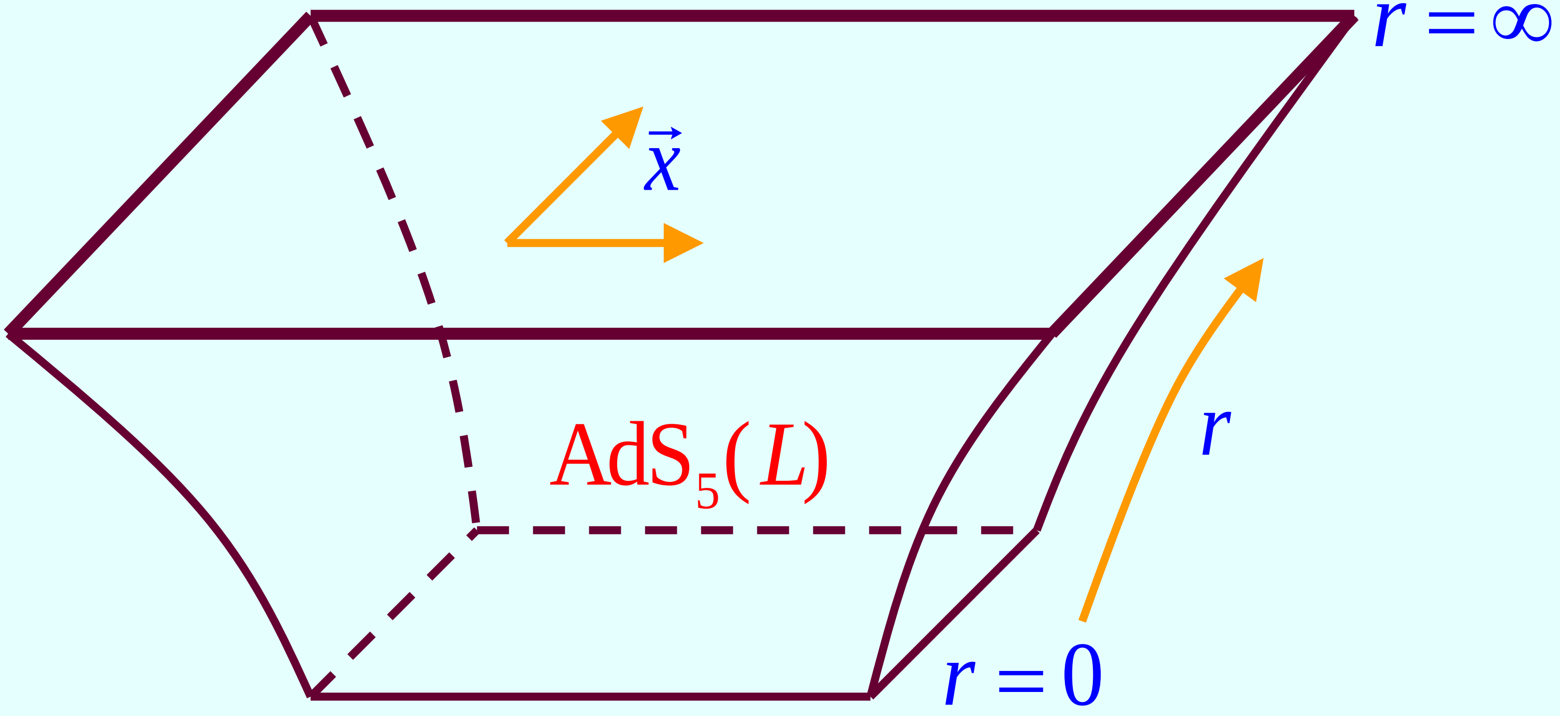
Poincaré coords. cover only a portion of AdS

Constant r observers share *acceleration horizon* at r=0 (with T=0)

Anti-de Sitter Space

From now on, will depict Poincaré patch like this:

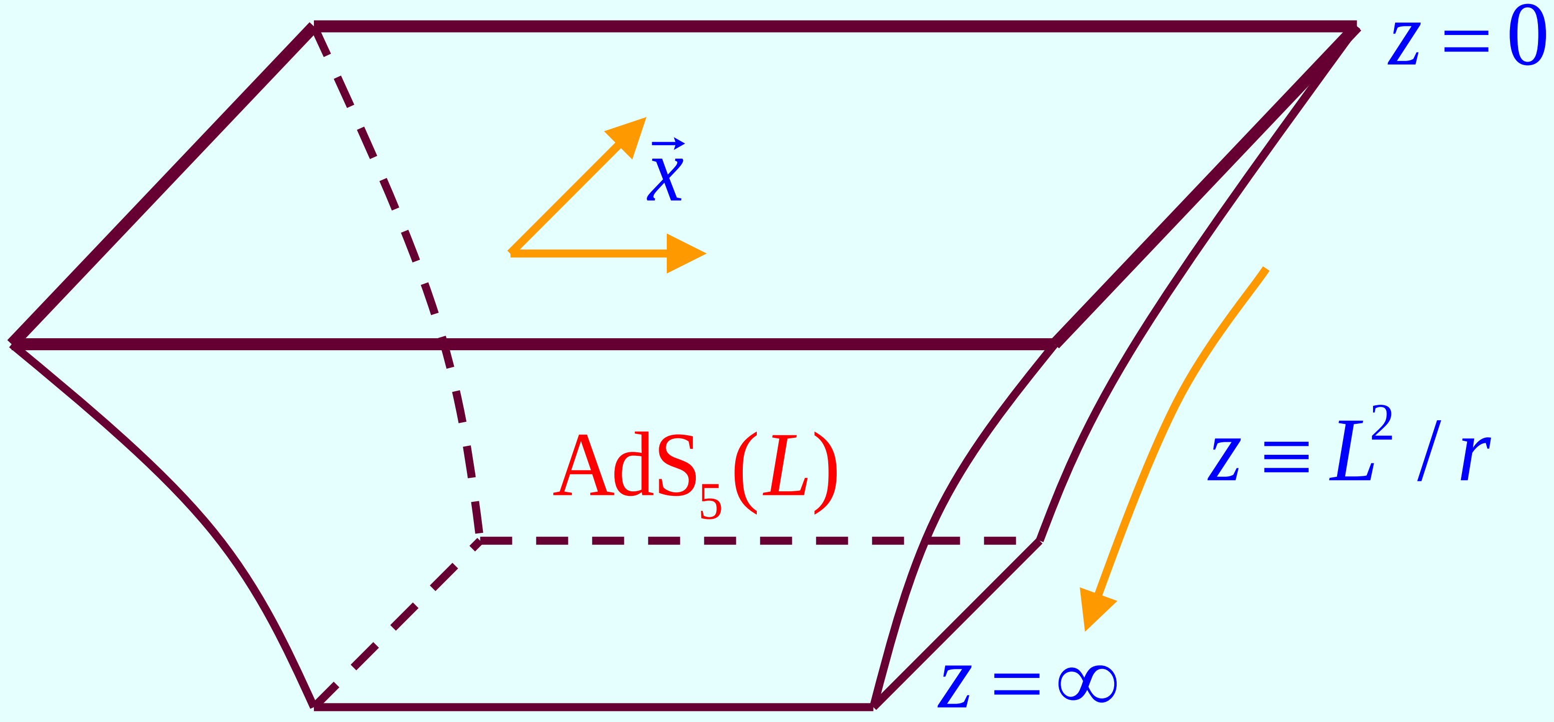
$$ds^2 = (r/L)^2(-dt^2 + d\vec{x}^2) + (L/r)^2 dr^2$$



Anti-de Sitter Space

Or, equivalently:

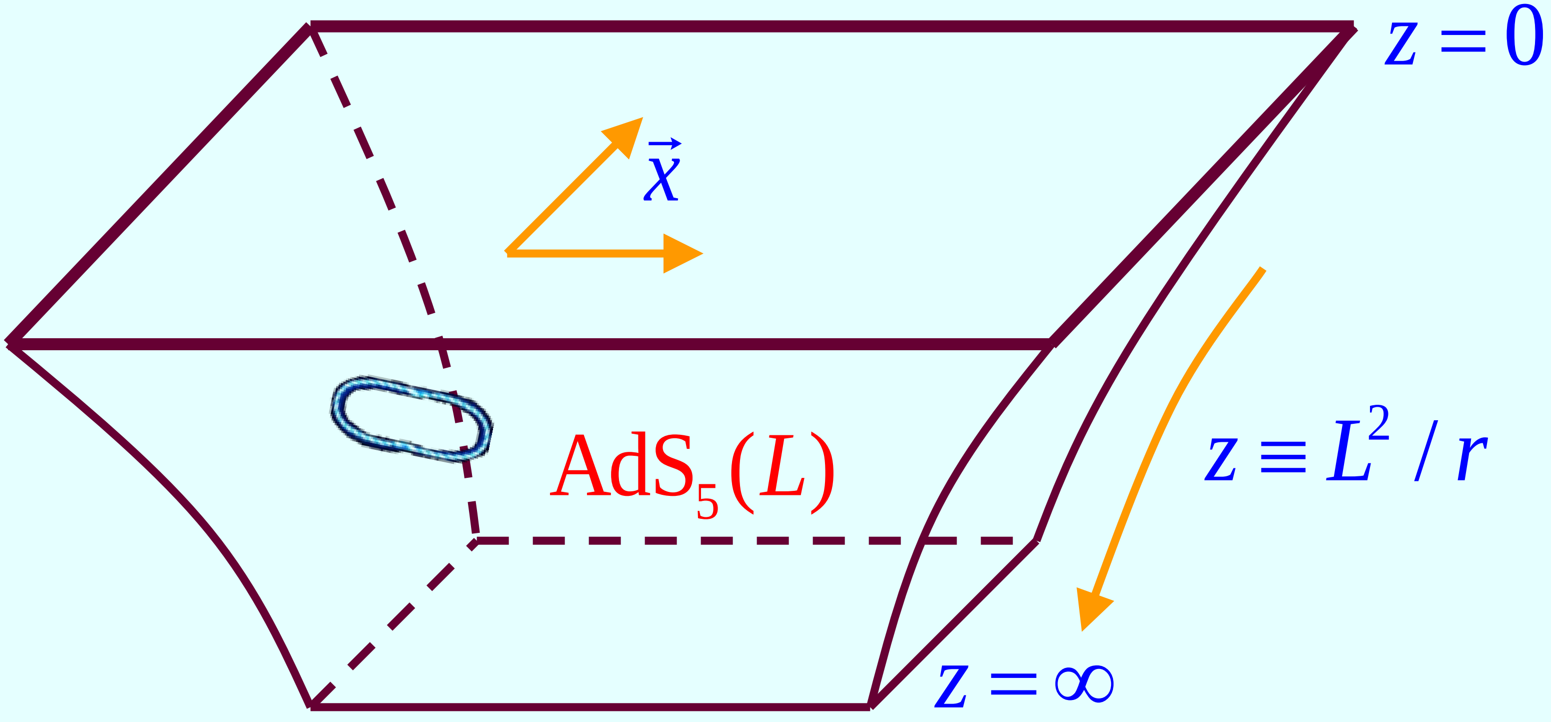
$$ds^2 = (L/z)^2(-dt^2 + d\vec{x}^2 + dz^2)$$



String Theory on AdS

Stringy fluctuations of this background can be small

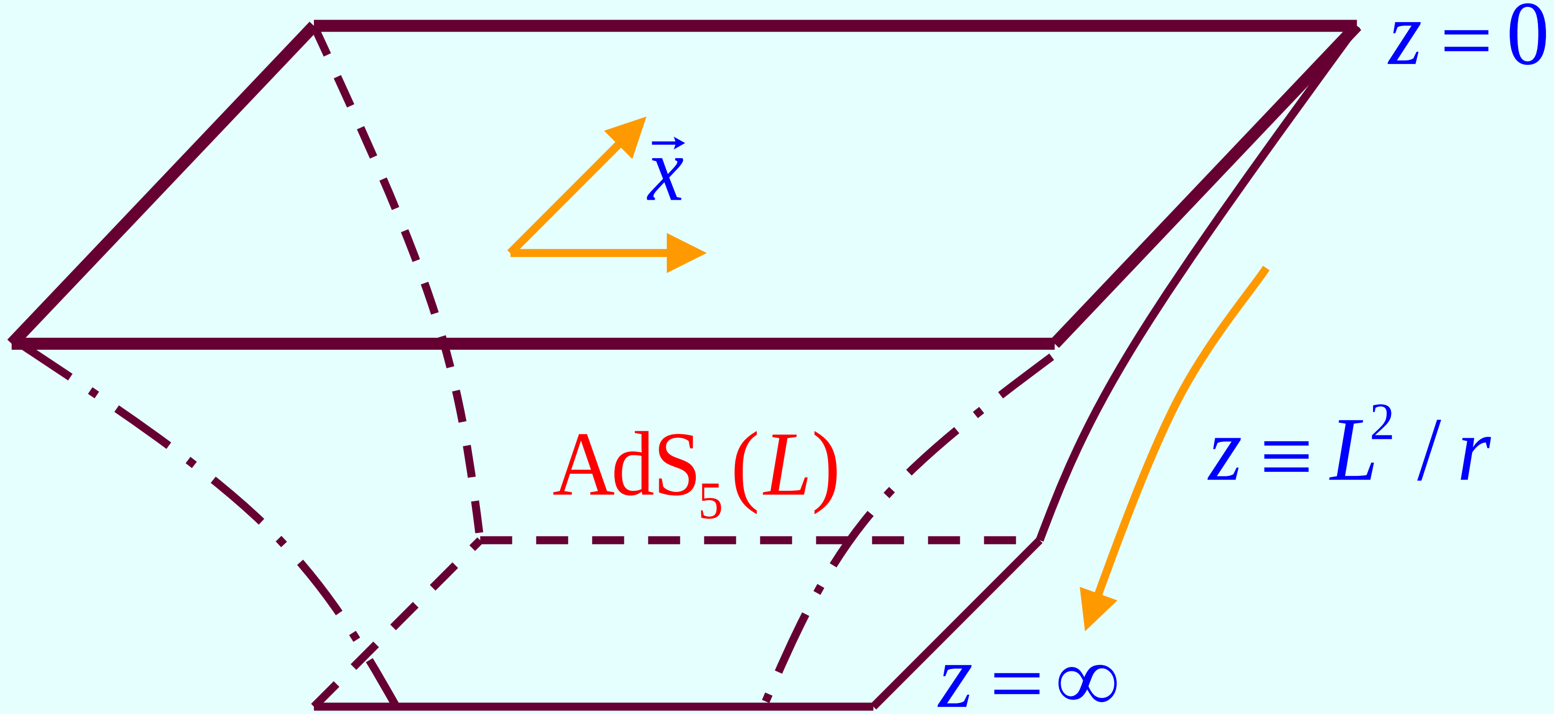
$$ds^2 = (L/z)^2(\eta_{\mu\nu}dx^\mu dx^\nu + dz^2)$$



String Theory on AdS

Stringy fluctuations of this background can be small or large

$$ds^2 = (L/z)^2 (g_{\mu\nu}(x,z) dx^\mu dx^\nu + dz^2)$$



AdS/CFT Correspondence

$SU(N_c)$ $\mathcal{N}=4$ SYM $=$ IIB String Theory on
in 3+1 dim [Maldacena] **asymptotically** $\mathrm{AdS}_5 \times \mathrm{S}^5$

Geometry on RHS is **dynamical**:

Pure AdS_5 spacetime corresponds to SYM vacuum

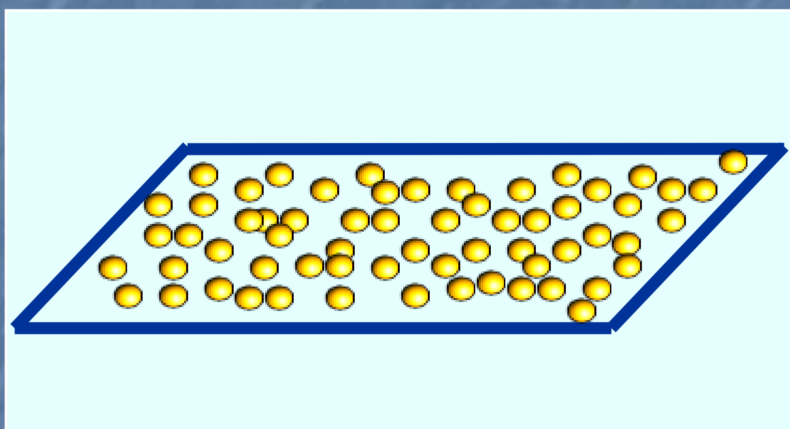
Excitations on top of AdS_5 correspond to other SYM states

e.g., black hole in AdS_5 corresponds to finite temperature

$$ds^2_{\mathrm{SchwAdS}} = \left(\frac{L}{z}\right)^2 \left[\left(-\left(1 - \frac{z^4}{z_h^4}\right) dt^2 + d\vec{x}^2 \right) + \frac{dz^2}{\left(1 - \frac{z^4}{z_h^4}\right)} \right]$$

AdS/CFT Dictionary

$SU(N_c)$ $\mathcal{N}=4$ SYM = IIB String Theory on (planar)
at finite T [Maldacena] $\mathrm{SchwAdS}_5(L, z_h)\times S^5(L)$



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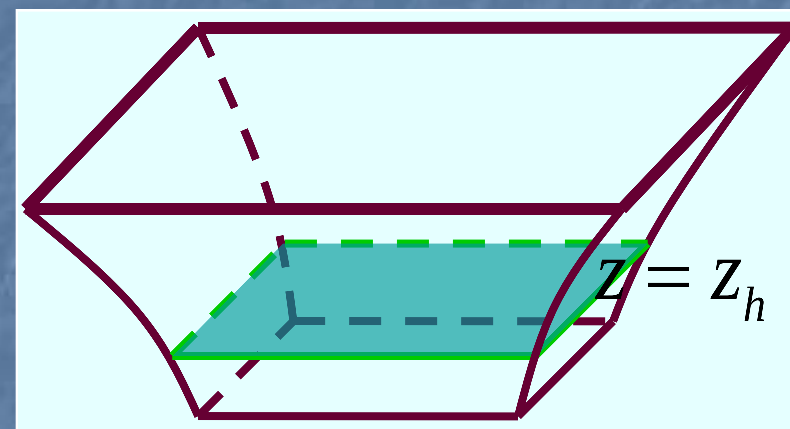


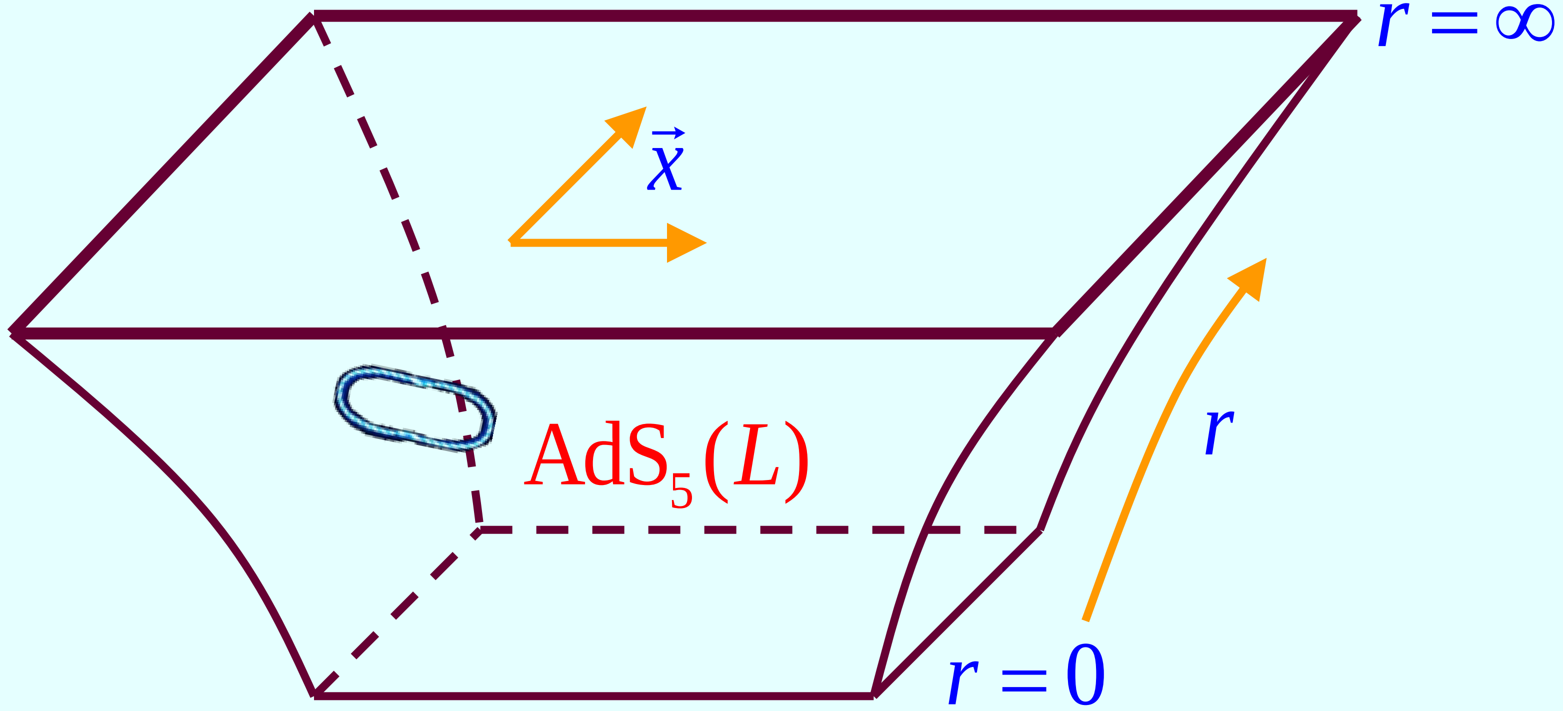
Gluon (+ adjoint scalar & fermion) plasma = Black hole (brane) in AdS

Can deduce this either by repeating AdS/CFT derivation starting w/*near*-extremal black 3-brane, OR by using AdS/CFT dictionary to reconstruct geometry dual to thermal plasma

AdS/CFT Dictionary

$SU(N_c)$ $\mathcal{N}=4$ SYM $=$ IIB String Theory on $\mathrm{aAdS}_5 \times \mathrm{S}^5$

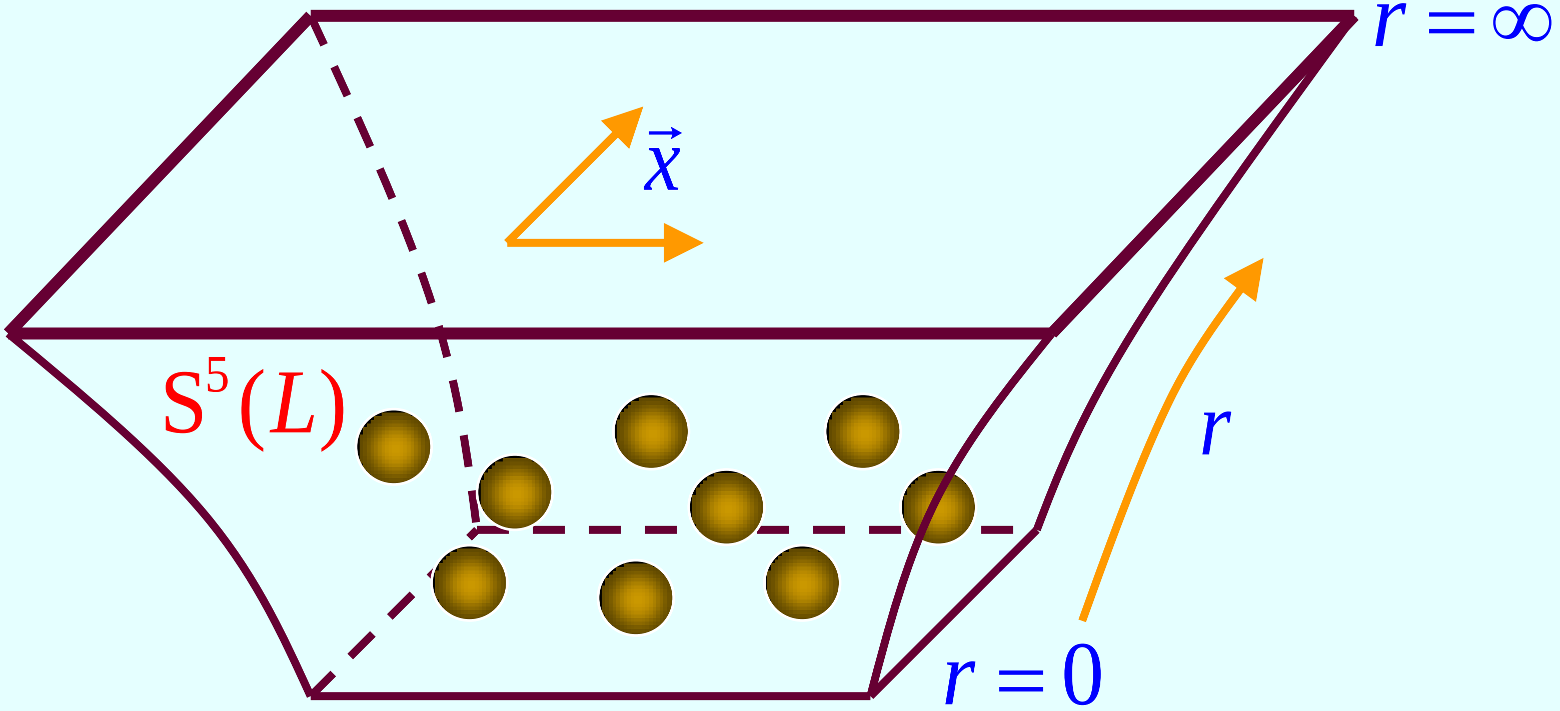
$D=3+1: \quad (t,\vec{x})$ $\qquad$ $D=9+1: \quad (t,\vec{x},r;\theta_1,\ldots,\theta_5)$



AdS/CFT Dictionary

$SU(N_c)$ $\mathcal{N}=4$ SYM $=$ IIB String Theory on $\mathrm{aAdS}_5 \times \mathrm{S}^5$

Internal space $=$ $\theta_1, \ldots, \theta_5$



AdS/CFT Dictionary

$SU(N_c)$ $\mathcal{N}=4$ SYM $=$ IIB String Theory on $\mathrm{aAdS}_5 \times \mathrm{S}^5$

Conformal group $SO(4,2)$ $=$ $SO(4,2)$ Isometries AdS_5

in particular, **dilatation**

$$(t,\vec{x}) \to (st, s\vec{x}) \quad \leftrightarrow \quad (t,\vec{x},z) \to (st, s\vec{x}, sz)$$

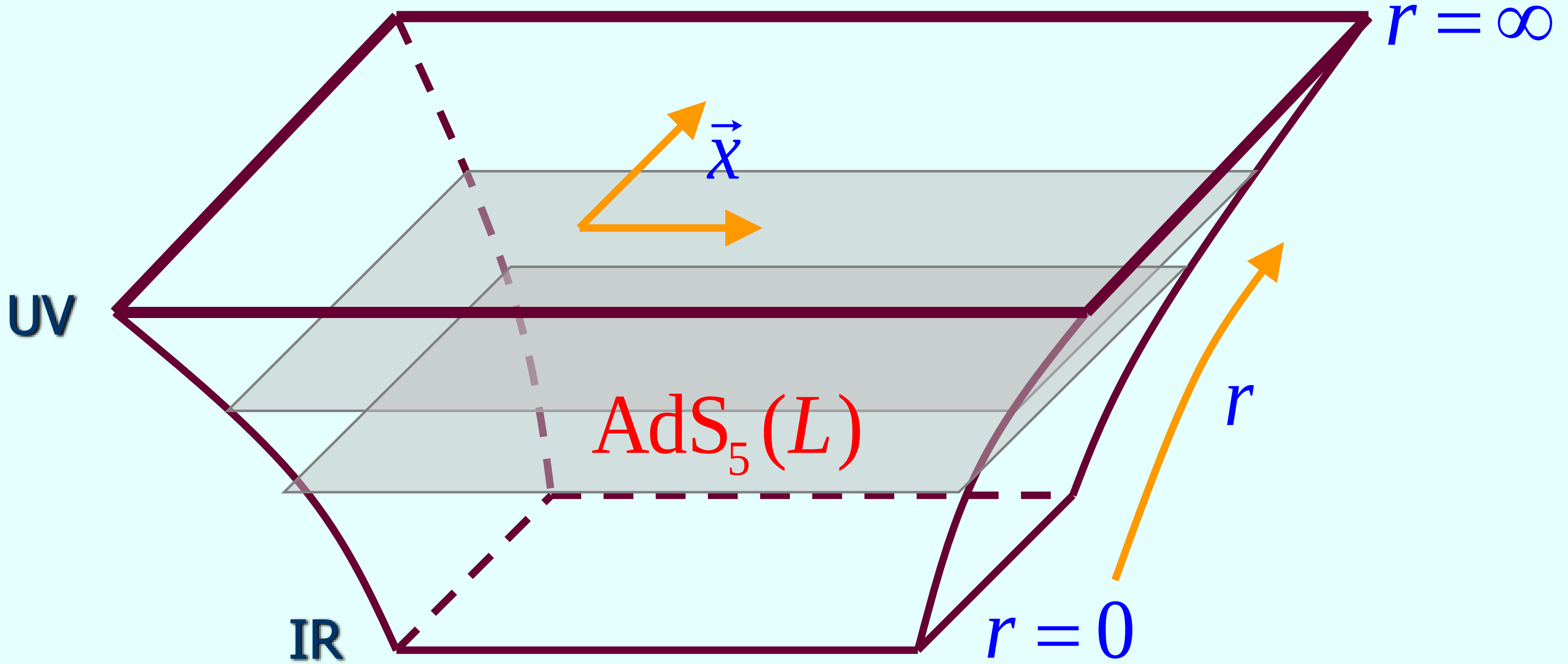
So z scales like a length,

$r = \dfrac{L^2}{z}$ scales like an energy…

AdS/CFT Dictionary

$SU(N_c)$ $\mathcal{N}=4$ SYM $=$ IIB String Theory on aAdS$_5 \times S^5$

Energy scale $E = r/L^2$ [Susskind,Witten; Polchinski,Peet]



AdS/CFT Dictionary

$SU(N_c)\ \mathcal{N}=4$ SYM $=$ IIB String Theory on $\mathrm{aAdS}_5 \times \mathrm{S}^5$

$$g^2_{YM} \quad = \quad 4\pi g_s$$

$$N_c \quad = \quad \text{Units of } F_{(5)} \text{ flux through } \mathrm{S}^5$$

$$\text{'t Hooft coupling } \lambda \equiv g^2_{YM} N_c \quad = \quad L^4 / l_s^4$$

NOTE: RHS is under calculational control only if spacetime is **weakly curved** and strings are **weakly coupled**

$$\Rightarrow \quad g^2_{YM} N_c \gg 1, \quad N_c \gg 1$$

i.e., when LHS is **strongly coupled**

Easiest: $N_c \to \infty$, $g^2_{YM} N_c \to \infty$

With more effort: $1/ g^2_{YM} N_c$ corrections

AdS/CFT Dictionary

$SU(N_c)$ $\mathcal{N}=4$ SYM $=$ IIB String Theory on $\mathrm{aAdS}_5 \times \mathrm{S}^5$

Conformal group $SO(4,2)$ $=$ $SO(4,2)$ Isometries AdS_5

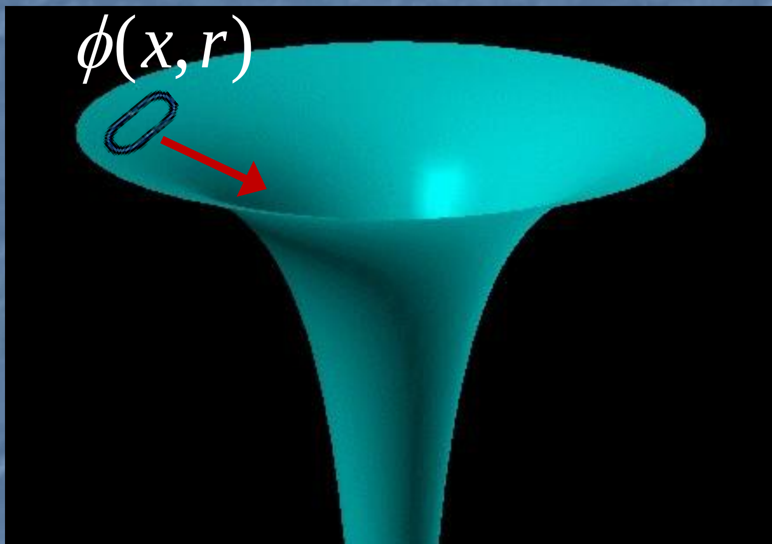
Internal symmetry $SO(6)$ $=$ $SO(6)$ Isometries S^5

Gauge group $SU(N_c) \leftrightarrow$ Nothing !

Gauge-invt. operator $O(x) \leftrightarrow \phi(x,r)$ Field (S^5 harmonic)

[Gubser,Klebanov,Polyakov;Witten]

Black Branes vs. D-branes



Black p-brane

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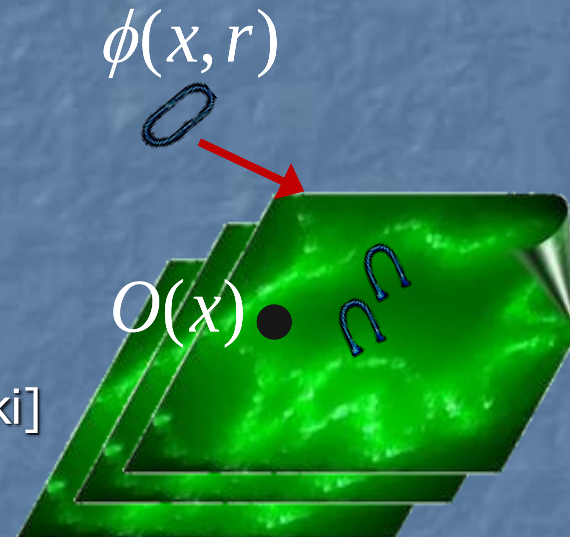
[Polchinski]



N Dp-branes
+ Flat Spacetime

AdS/CFT Dictionary

$SU(N_c)$ $\mathcal{N}=4$ SYM $=$ IIB String Theory on $\mathrm{aAdS}_5 \times \mathrm{S}^5$

Conformal group $SO(4,2) \quad = \quad SO(4,2)$ Isometries AdS_5

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Gauge group $SU(N_c) \leftrightarrow$ Nothing !

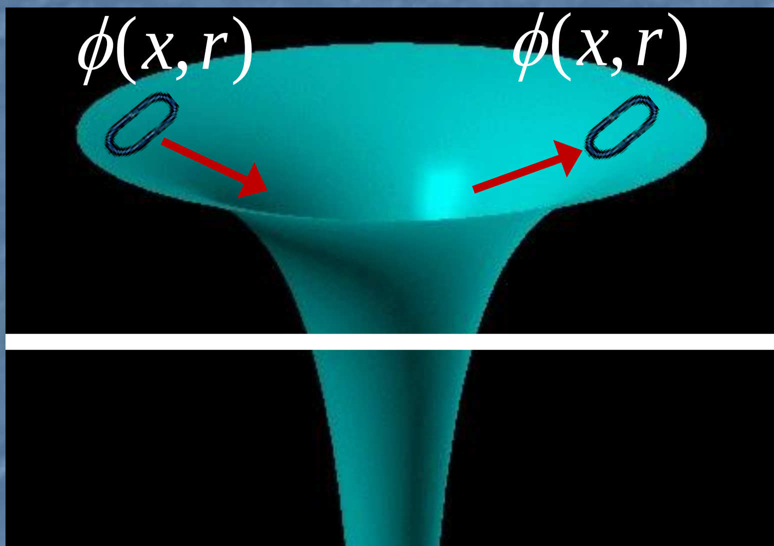
Gauge-invt. operator $O(x) \leftrightarrow \phi(x,r)$ Field (S^5 harmonic)

e.g., $\mathrm{Tr}[F^2(x)+\ldots] \leftrightarrow \varphi(x,r)$ dilaton (s wave)

$T_{\mu\nu}(x) \leftrightarrow h_{\mu\nu}(x,r)$ graviton

Find perfect match for all supergravity modes [Witten]

AdS/CFT Dictionary



Black p-brane

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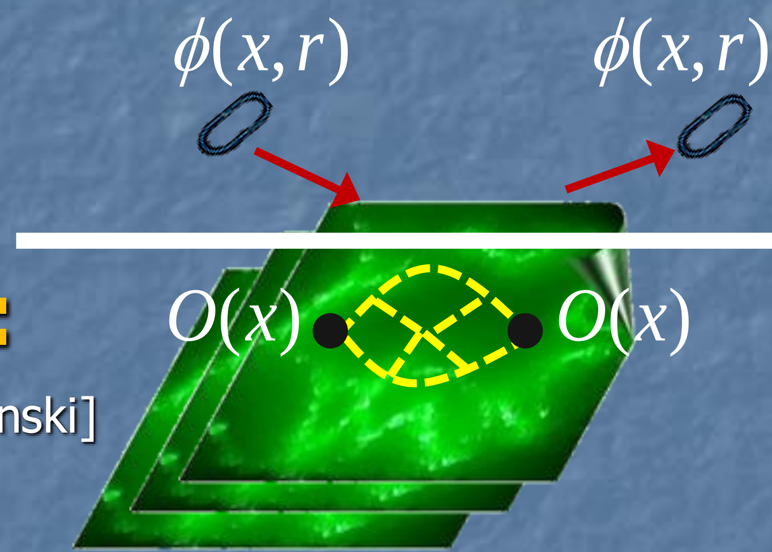
[Polchinski]



N Dp-branes
+ Flat Spacetime

AdS/CFT Dictionary

$SU(N_c)$ $\mathcal{N}=4$ SYM $=$ IIB String Theory on $\mathrm{aAdS}_5 \times \mathrm{S}^5$

Gauge-invt. operator $O(x) \leftrightarrow \phi(x,z)$ Field (S^5 harmonic)

$$\int D(\mathrm{SYM}) \exp\left[iS_{\mathrm{SYM}} + i\int d^4x\, O(x) J(x) \right] = \int D(\mathrm{ST}) \exp\left[iS_{\mathrm{ST}} \right]$$

[Gubser,Klebanov,Polyakov;Witten] $\phi(x,z{=}0){=}z^{4-\Delta}J(x)$

In $N_c \to \infty$, $g_{YM}^2 N_c \to \infty$ limit, RHS simplifies to

$$\int D(\mathrm{ST}) \exp\left[-S_{\mathrm{ST}} \right] = \exp\left[-S_{\mathrm{SUGRA}}^{\text{on-shell}} \right]$$

($1/g_{YM}^2 N_c$ corrections: higher derivative terms)

AdS/CFT Dictionary

$SU(N_c)$ $\mathcal{N}=4$ SYM = IIB String Theory on $\mathrm{aAdS}_5 \times \mathrm{S}^5$

Gauge-invt. operator $O(x) \leftrightarrow \phi(x,z)$ Field (S^5 harmonic)

$$\int D(\mathrm{SYM}) \exp\left[iS_{\mathrm{SYM}} + i\int d^4x\, O(x) J(x) \right] = \exp\left[iS_{\mathrm{SUGRA}}^{\text{on-shell}} \right]$$

[Gubser,Klebanov,Polyakov;Witten] $\phi(x,z{=}0){=}z^{4-\Delta}J(x)$

with $\Delta = 2 + \sqrt{4 + m^2L^2}$ the **conformal dimension** of the dual operator: $O(x) \to s^{\Delta} O(sx)$ under dilatations

$$\phi(x,z) = z^{4-\Delta} J(x) + z^{\Delta} \langle O(x) \rangle + \ldots$$

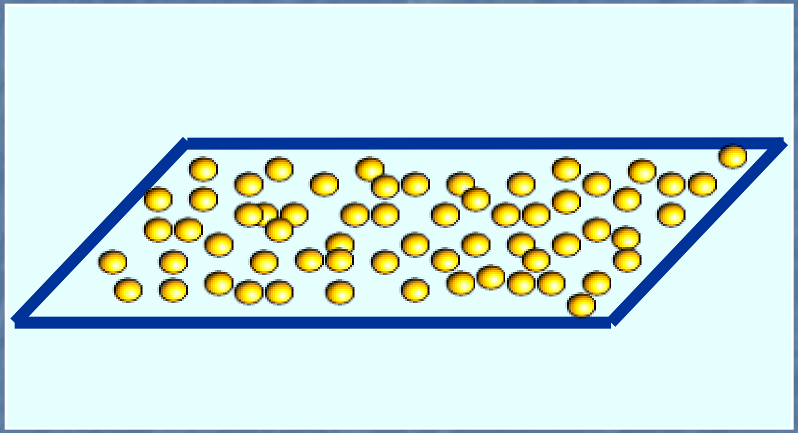
External source: determines THEORY

Expectation value: determines STATE

[Balasubramanian,Kraus,Lawrence]

AdS/CFT Dictionary

$SU(N_c)$ $\mathcal{N}=4$ SYM at finite T = IIB String Theory on (planar) $\mathrm{SchwAdS}_5(L, z_h)\times S^5(L)$



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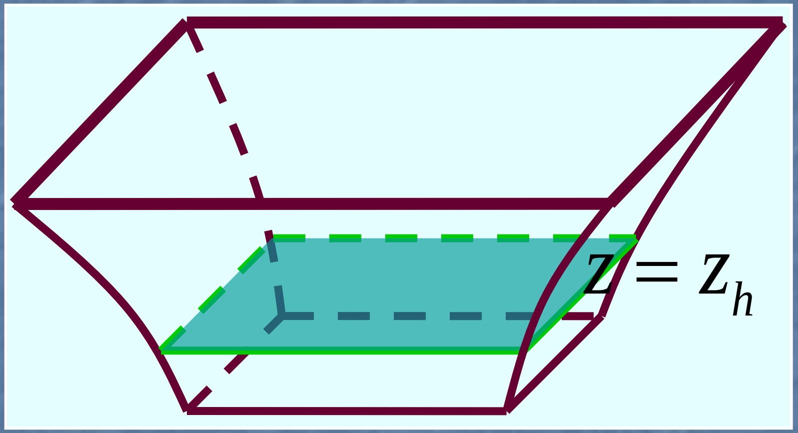
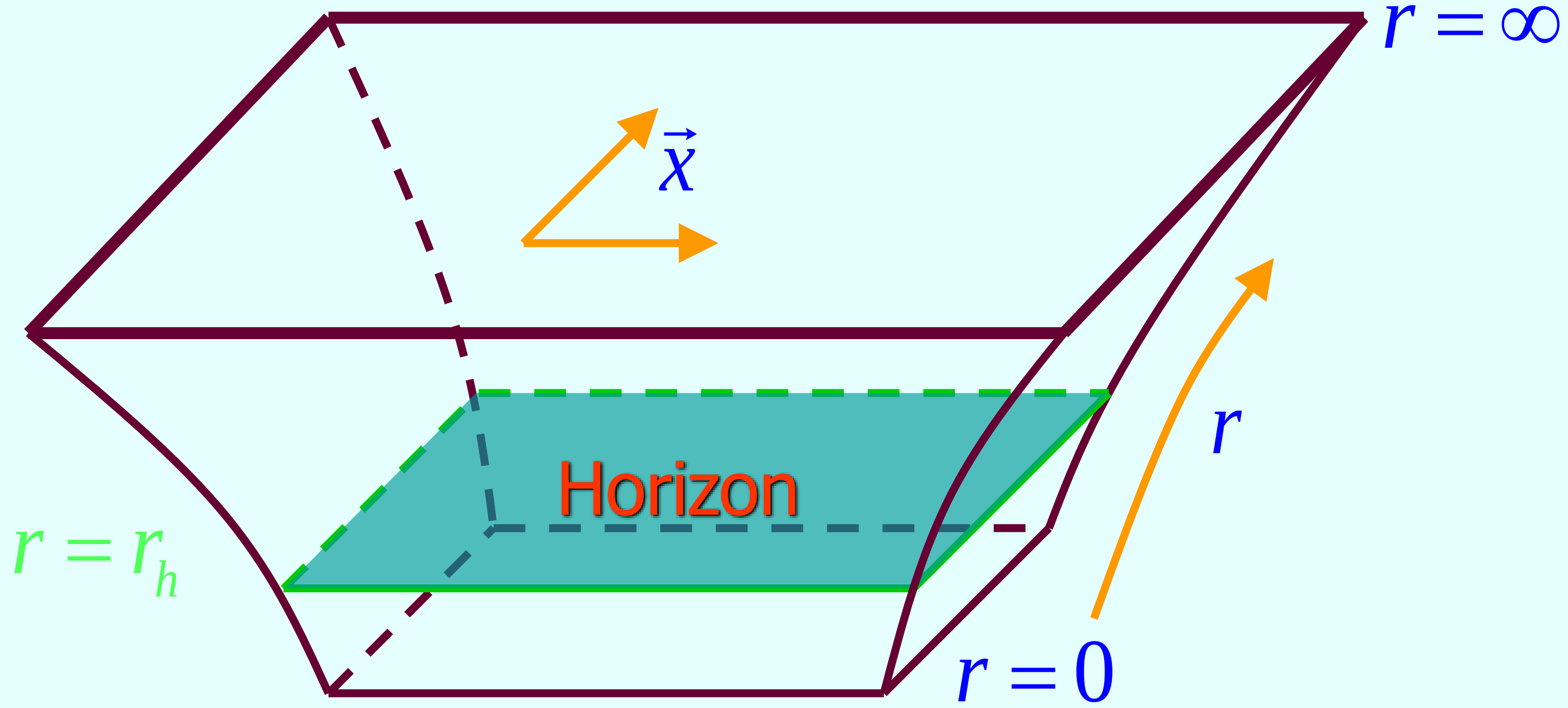


Gluon (+ adjoint scalar & fermion) plasma = Black hole (brane) in AdS

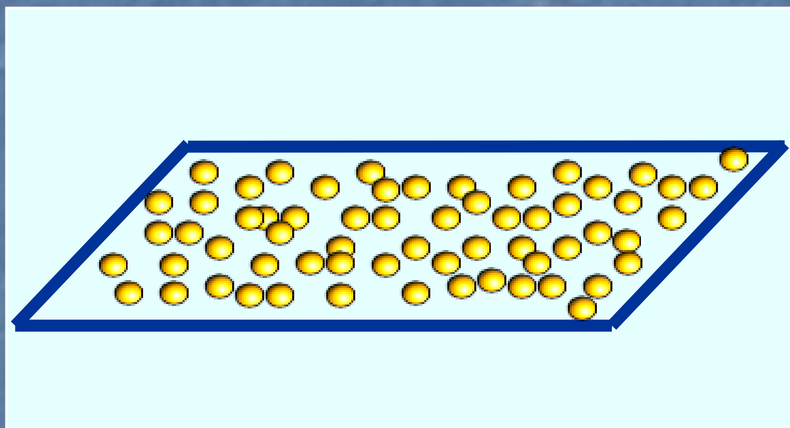
AdS/CFT Dictionary

$SU(N_c)\ \mathcal{N}=4$ SYM $=$ IIB ST on $\mathrm{SchwAdS}_5 \times S^5$

$$T = \frac{r_h}{\pi L^2} = \frac{1}{\pi z_h} = T_H$$ [Witten]



Entropy of SYM Plasma



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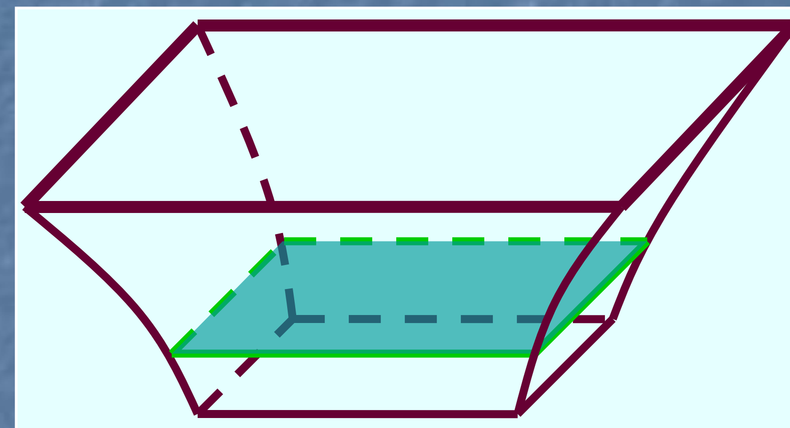
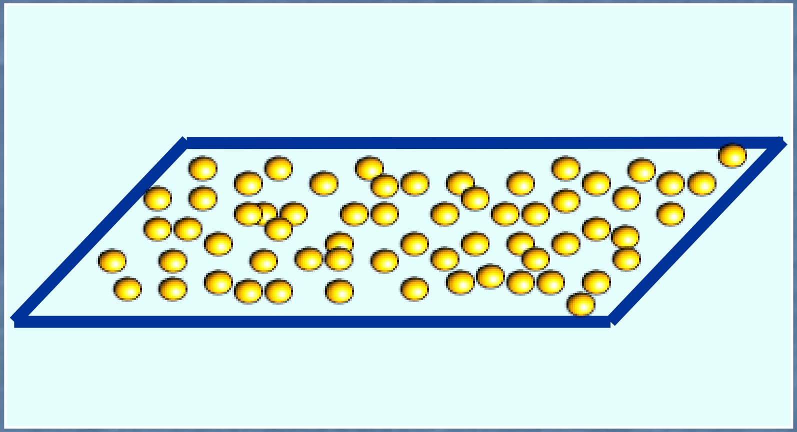


$$S_{\text{plasma}} = S_{\text{BH}} = \frac{A_H}{4G_N}$$

Entropy of SYM Plasma



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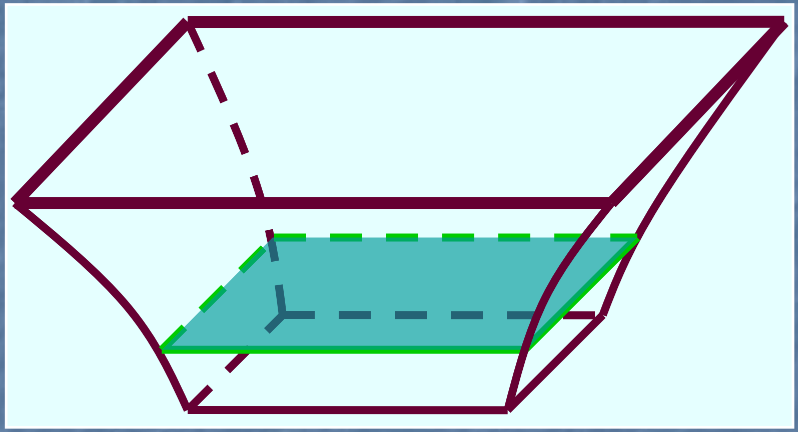


$$g_{YM}^2 N \ll 1 \quad \Rightarrow$$

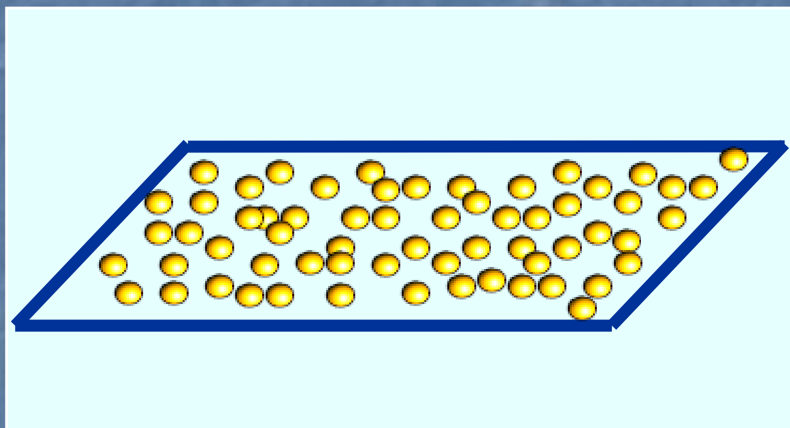
$$S_{\text{plasma}} = \frac{2\pi^2}{3} N^2 T^3 V$$

$$g_{YM}^2 N \gg 1 \quad \Rightarrow$$

$$S_{\text{BH}} = \frac{2\pi^2}{3} N^2 T^3 V \left(\frac{3}{4}\right)$$

[Gubser,Klebanov,Peet]

Entropy of SYM Plasma



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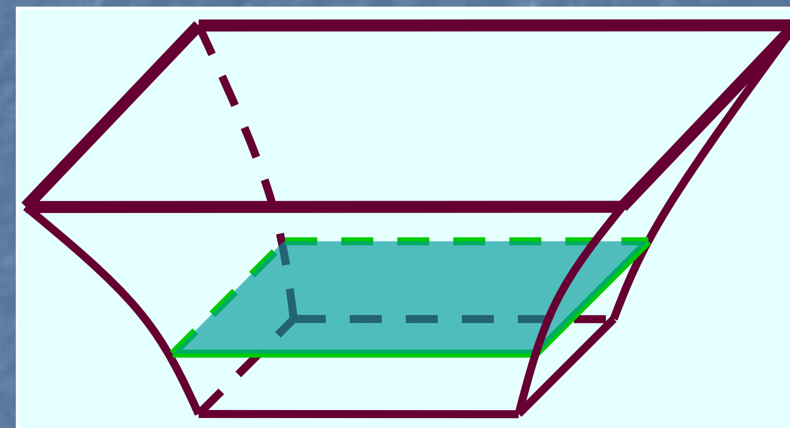


$$g_{YM}^2 N \ll 1 \quad \Rightarrow$$

$$S_{\text{plasma}} = \frac{2\pi^2}{3} N^2 T^3 V \left(1 - \frac{3 g_{YM}^2 N}{4\pi^2} + \cdots \right)$$

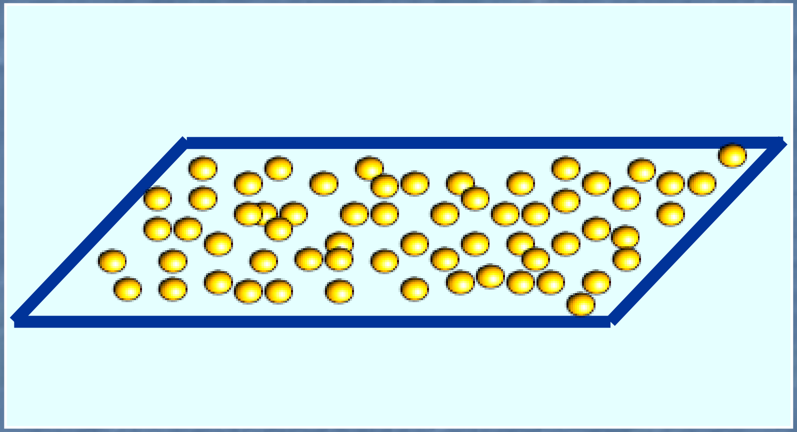
[Gubser,Klebanov,Peet;
Fotopoulos,Taylor]

$$g_{YM}^2 N \gg 1 \quad \Rightarrow$$

$$S_{\text{BH}} = \frac{2\pi^2}{3} N^2 T^3 V \left(\frac{3}{4} + \frac{45\zeta(3)}{64\sqrt{2}\left(g_{YM}^2 N\right)^{3/2}} + \cdots \right)$$

[Gubser,Klebanov,Peet;
Gubser,Klebanov,Tseytlin]

Entropy of SYM Plasma



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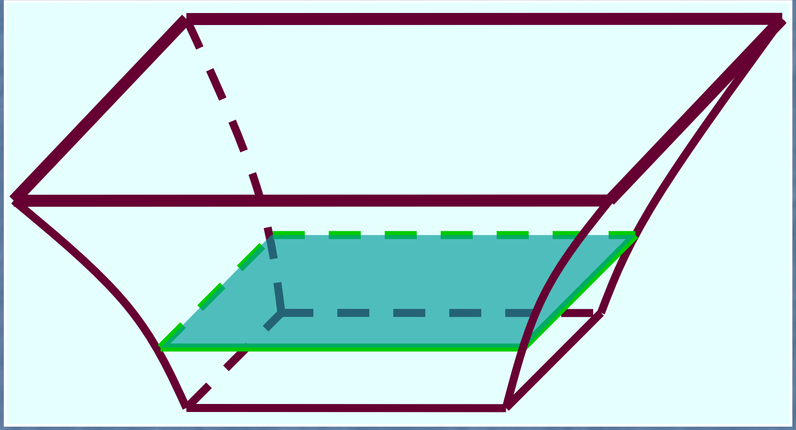

$$g_{YM}^2 N \ll 1 \quad \Rightarrow$$

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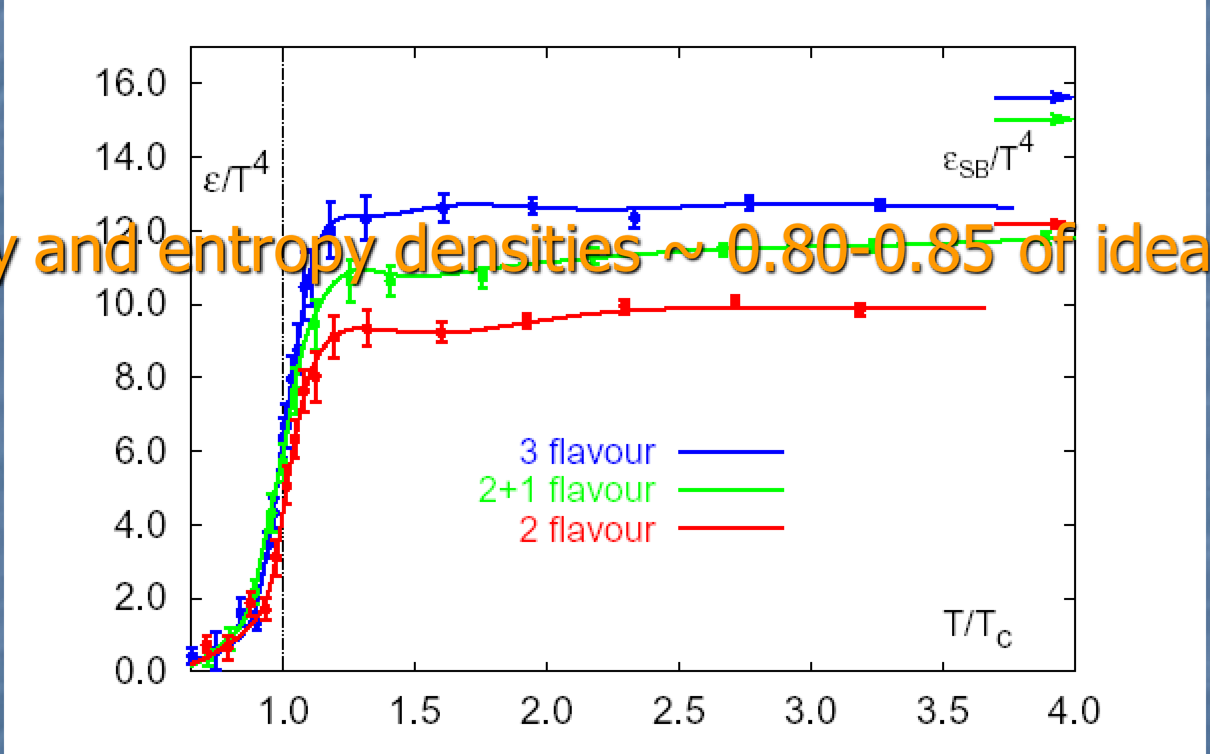
$$g_{YM}^2 N \gg 1 \quad \Rightarrow$$

$$S_{\text{BH}} = \frac{2\pi^2}{3} N^2 T^3 V \left(\frac{3}{4} + \frac{45\zeta(3)}{64\sqrt{2}\left(g_{YM}^2 N\right)^{3/2}} + \cdots \right)$$

$\dfrac{S(g_{YM}^2 N_c = \infty)}{S(g_{YM}^2 N_c = 0)} = 0.75$ close to lattice QCD @ $T \sim (1-4)T_c$

Lattice QCD

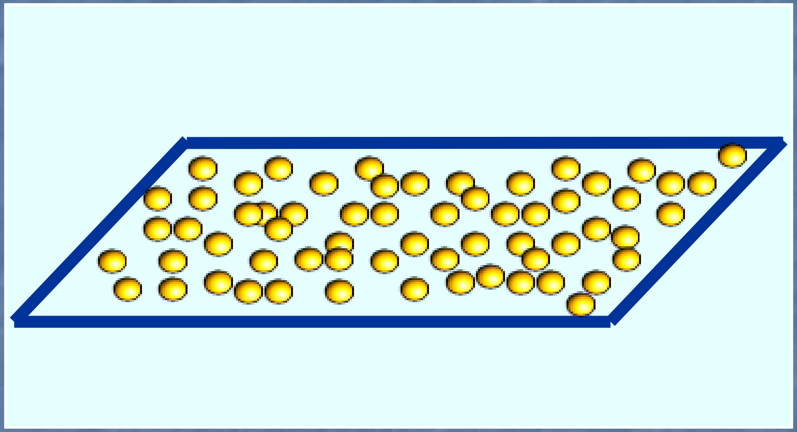
Deconfinement transition with $T_c \simeq 170-190\ \mathrm{MeV}$



Energy and entropy densities ~ 0.80-0.85 of ideal gas

From: F. Karsch, hep-lat/0106019

Shear Viscosity of SYM Plasma



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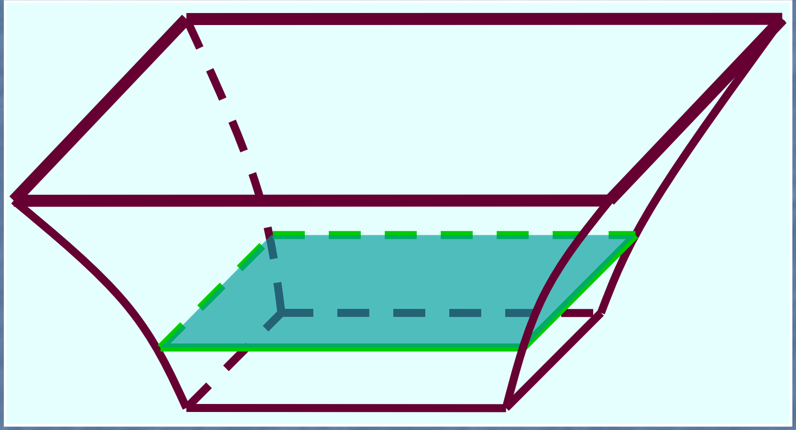
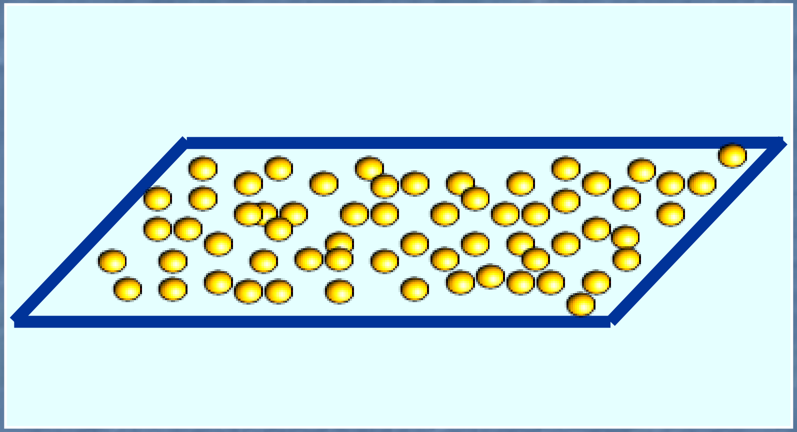


$$\eta = \lim_{\omega \to 0} \frac{1}{2\omega} \int d^4x\, e^{i\omega t} \left\langle \left[T_{xy}(x), T_{xy}(0) \right] \right\rangle \quad = \quad \lim_{\omega \to 0} \frac{1}{16\pi G_N} \sigma_{h_{\mu\nu}}(\omega)$$

[Kubo]

[Callan; Gubser, Klebanov, Polyakov; Witten]

Shear Viscosity of SYM Plasma



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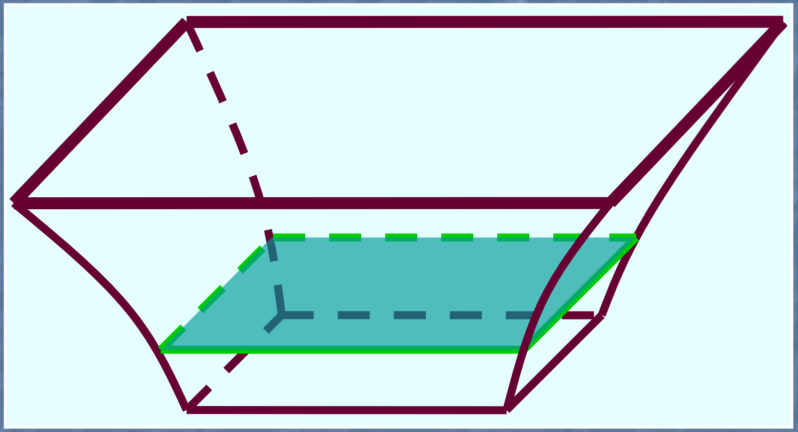


$$g_{YM}^2 N \ll 1 \quad \Rightarrow$$

$$\frac{\eta}{s} \sim \frac{1}{(g_{YM}^2 N_c)^2 \log(1/g_{YM}^2 N_c)} \gg 1$$

[Arnold,Moore,Yaffe]

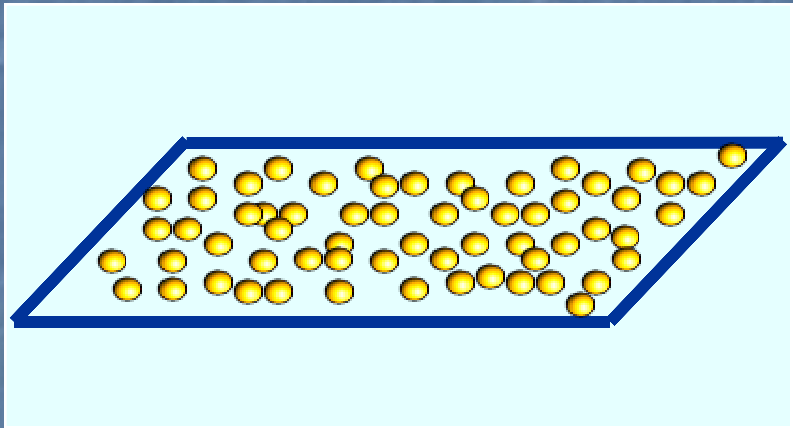
$$g_{YM}^2 N \gg 1 \quad \Rightarrow$$

$$\frac{\eta}{s} = \frac{1}{4\pi} + \frac{135\zeta(3)}{64\sqrt{2}\pi(g_{YM}^2 N_c)^{3/2}} + \ldots < 1$$

[Policastro,Son,Starinets;Buchel,Liu,Starinets]

Universal: same for all gauge theories with gravity dual!!

Shear Viscosity of SYM Plasma



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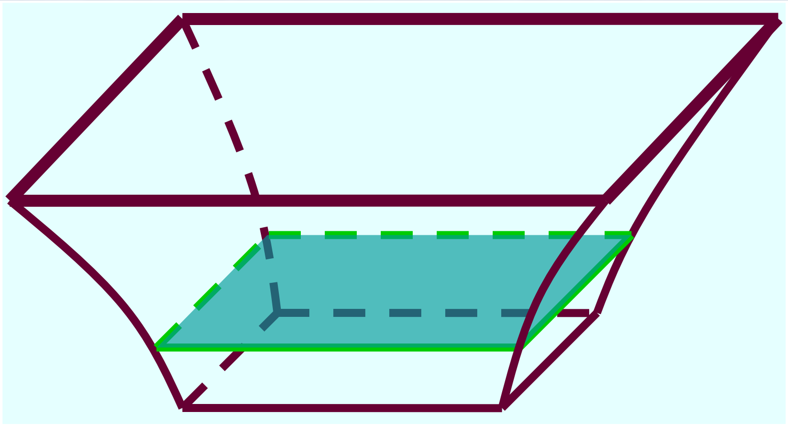

$$g_{YM}^2 N \ll 1 \quad \Rightarrow$$

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[Arnold,Moore,Yaffe]

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[Policastro,Son,Starinets;Buchel,Liu,Starinets]

... and, close to ~0.1-0.2 at RHIC!

Other Transport Coefficients

Besides the shear viscosity, there exist AdS/CFT calculations of bulk viscosity, charge diffusion constant, electrical conductivity, thermal conductivity, etc. of SYM and other plasmas.

(Can also obtain these from quasinormal modes of BH, or via fluid/gravity correspondence {V. Hubeny})

[Policastro,Son,Starinets;Buchel,Liu,Starinets; Buchel; Parnachev,Starinets; Kovtun,Starinets; Benincasa,Buchel,Starinets; Janik,Peschanski; Mas; Son,Starinets; Saremi; Buchel; Maeda,Natsuume,Okamura; Benincasa,Buchel; Benincasa,Buchel,Naryshkin; Janik; Mateos,Myers,Thomson; Liao,Shuryak; Bak,Janik; etc.],

Overall Plan

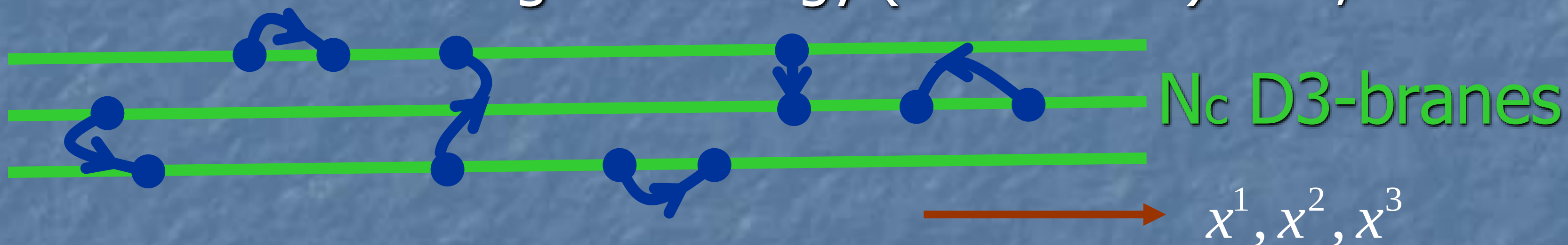
- L1: Motivation, String Theory Basics
- L2: AdS/CFT `Derivation' & Dictionary,
  Entropy, Viscosity
- L3: Energy Loss, Brownian, Unruh, Damping,
- "L4": Mesons, Screening, Limiting Velocity,
  Generalizations of AdS/CFT (Gauge/Gravity)

Plan for Lecture 3

- Adding Quarks
- Energy Loss
- Brownian Motion, Unruh, Damping

Adding Quarks to SYM

Recall: before taking low energy (Maldacena) limit, had



$\left(A_\mu\right)_{IJ}, \left(\Phi^i\right)_{IJ}$ + fermions + etc. $I, J = 1, \ldots, N_c$

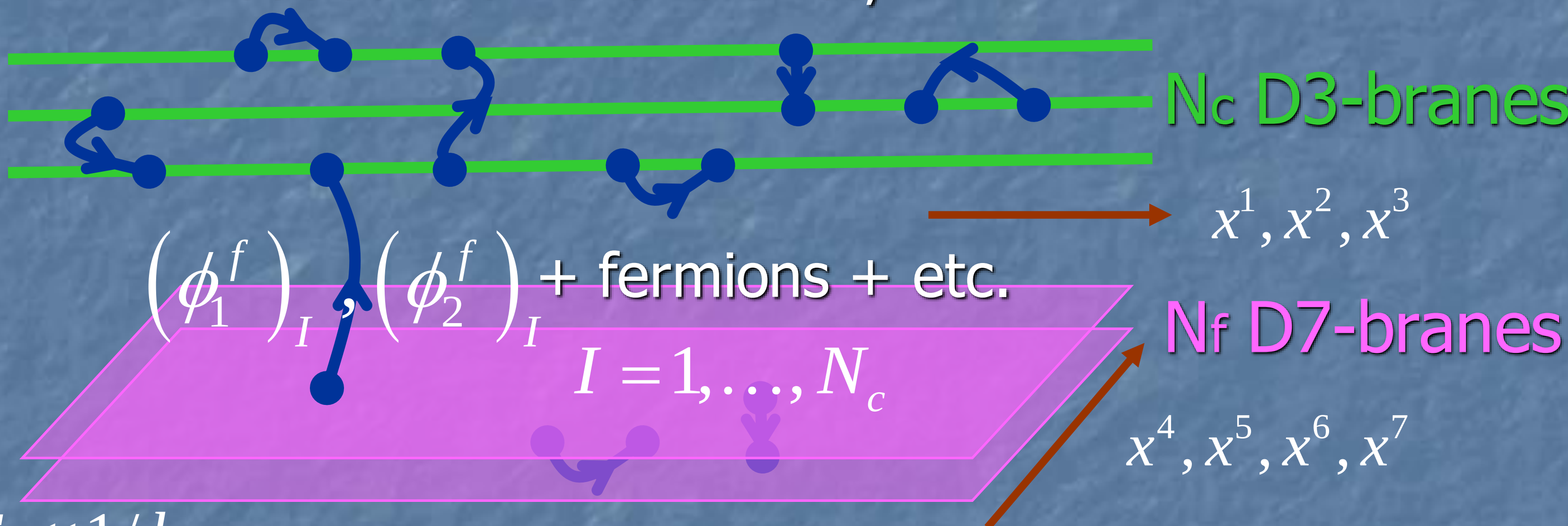
$E \ll 1/l_s$

$\Longrightarrow \quad \mathcal{N} = 4 \; U(N)$ Super-Yang-Mills in 3+1 dim

[Witten]

Adding Quarks to SYM

Now introduce additional branes,



$E \ll 1/l_s$

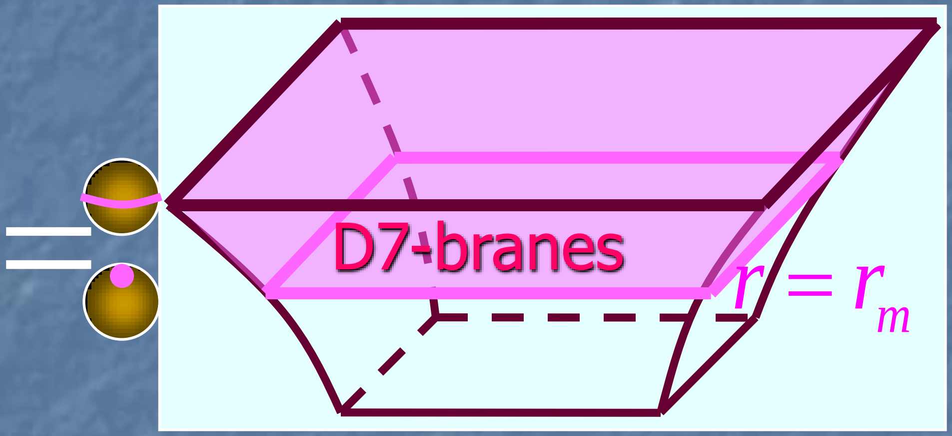
$\Longrightarrow$ $\mathcal{N}=4$ $U(N)$ Super-Yang-Mills in 3+1 dim

+ N_f sets of 4 scalars + 2 spinors ($\mathcal{N}=2$ hyperm.) in **fundamental** rep of gauge group: “quarks”

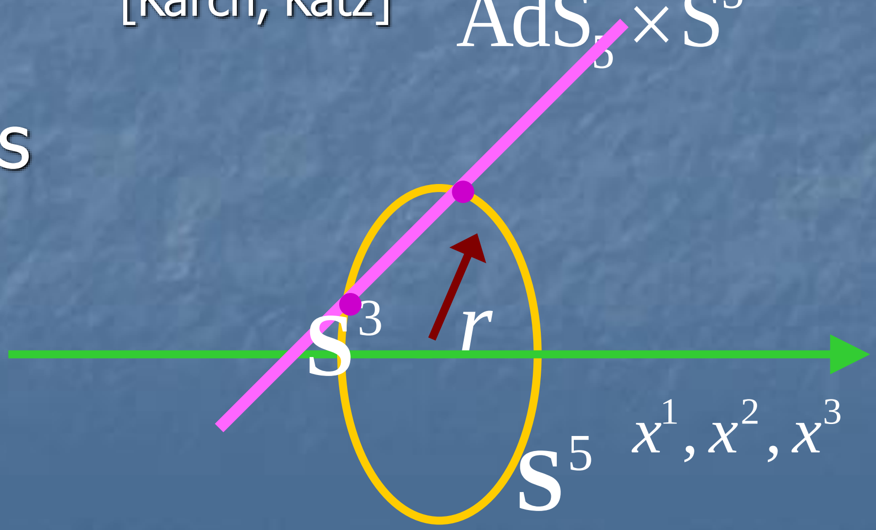
[Karch,Katz]

Adding Quarks to SYM

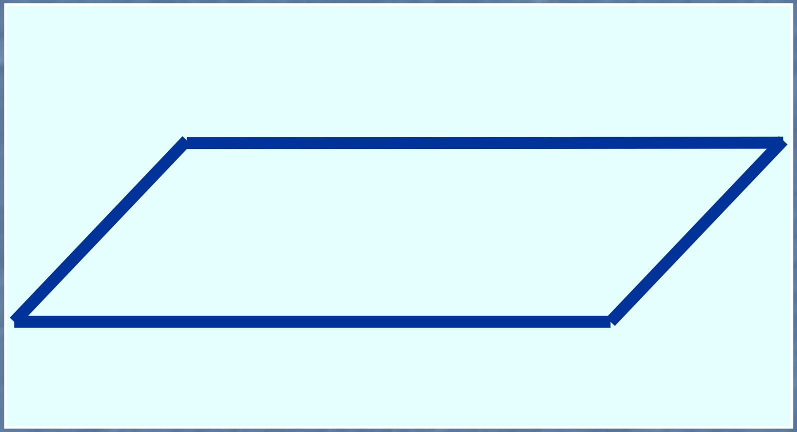
- In low-energy limit, dual description replaces D3s with AdS
- If $N_f \ll N_c$, can disregard backreaction of D7s on AdS geometry (=`quenched' approx. of lattice QCD)



[Karch, Katz]



Adding Quarks to SYM



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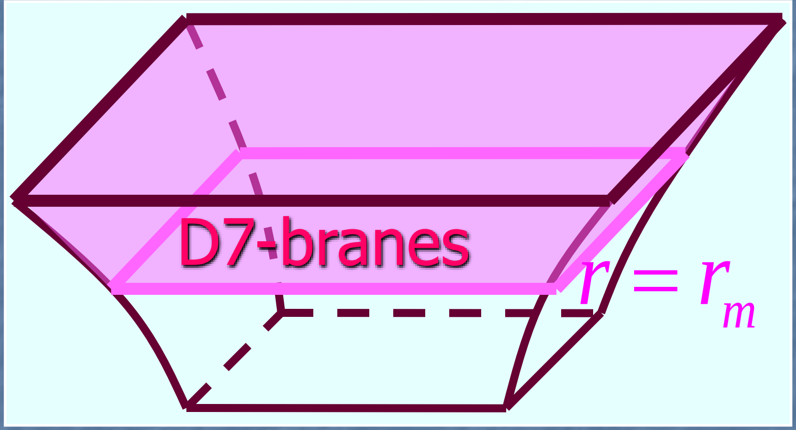
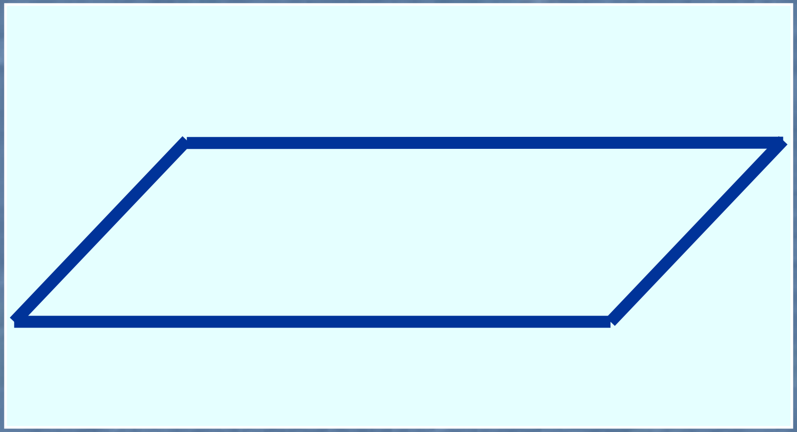


$SU(N_c)\ \mathcal{N} = 4$ SYM $=$ IIB ST on $\mathrm{AdS}_5 \times \mathrm{S}^5$

$+ N_f \ll N_c$ flavors of ($\mathcal{N} = 2$) matter in **fundamental** rep $+$ N_f **D7-branes** (wrapped on $\mathrm{S}^3 \subset \mathrm{S}^5$)

[Karch, Katz]

Adding Quarks to SYM



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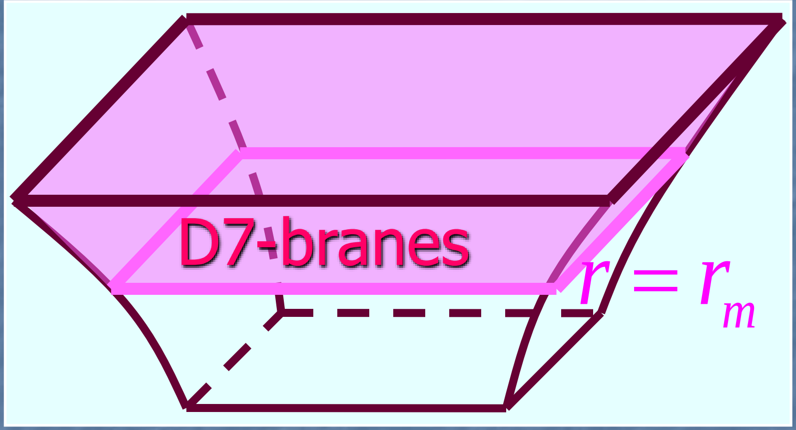


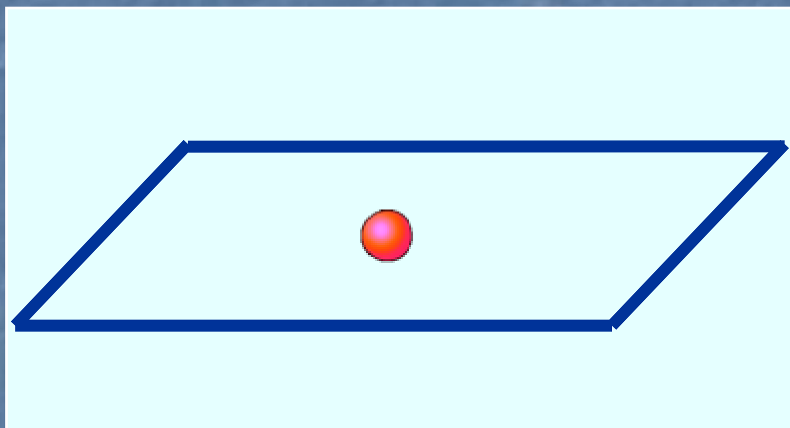
$SU(N_c)\ \mathcal{N}=4$ SYM $+ N_f \ll N_c$ flavors of $(\mathcal{N}=2)$ matter in **fundamental** rep

$=$

IIB ST on $\mathrm{AdS}_5 \times \mathrm{S}^5$ $+$ N_f **D7-branes** (wrapped on $\mathrm{S}^3 \subset \mathrm{S}^5$)

$$m = \frac{r_m}{2\pi l_s^2} = \frac{\sqrt{g_{YM}^2 N_c}}{2\pi z_m}$$

Adding Quarks to SYM



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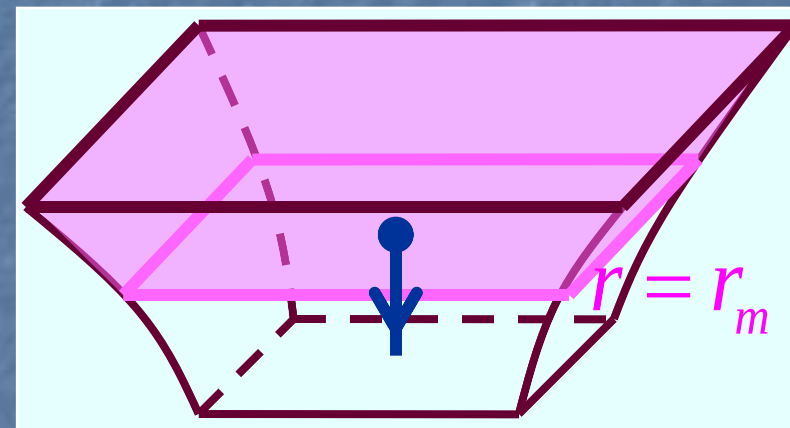
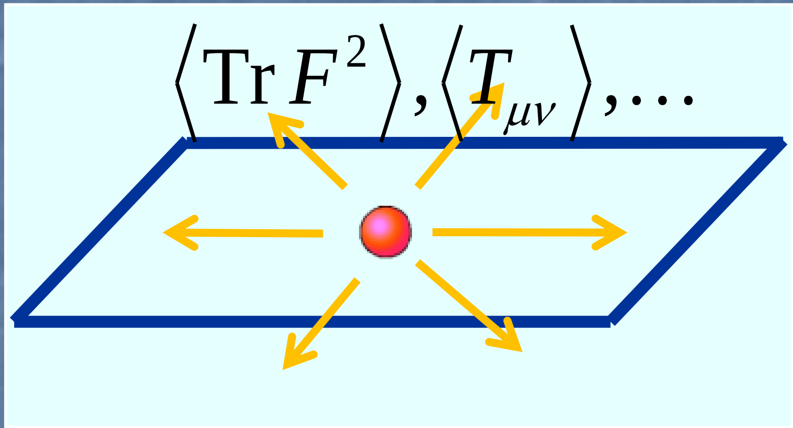


Finite-mass Quark $=$ String w/endpoint at $r_m < \infty$

Adding Quarks to SYM



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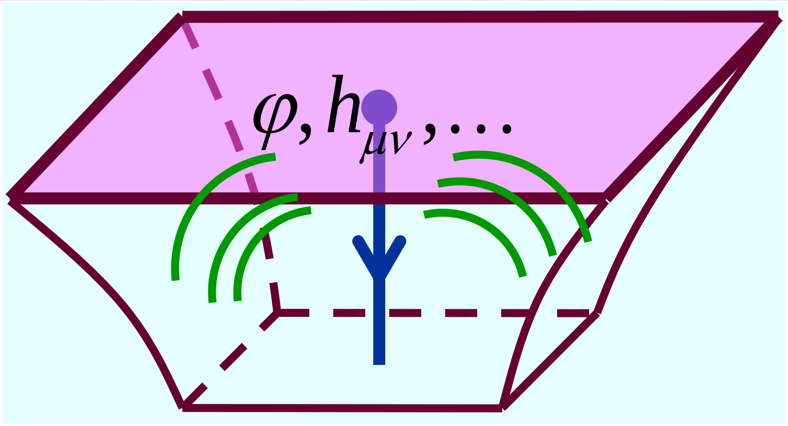


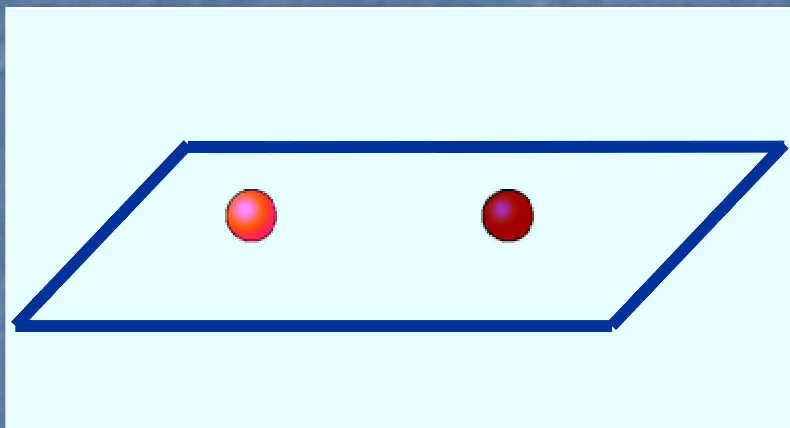
Infinitely heavy Quark = String w/endpoint at $r \to \infty$

E.g., $\left\langle \mathrm{Tr}\left[F^2(\vec{x},t) + \ldots \right] \right\rangle_q = \dfrac{\sqrt{g_{YM}^2 N_c}}{32\pi^2 |\vec{x}|^4}$

[Danielsson,Kruczenski,Keski-Vakkuri]

Coulombic profile (as expected by conformal invariance)

Quark-Antiquark Potential



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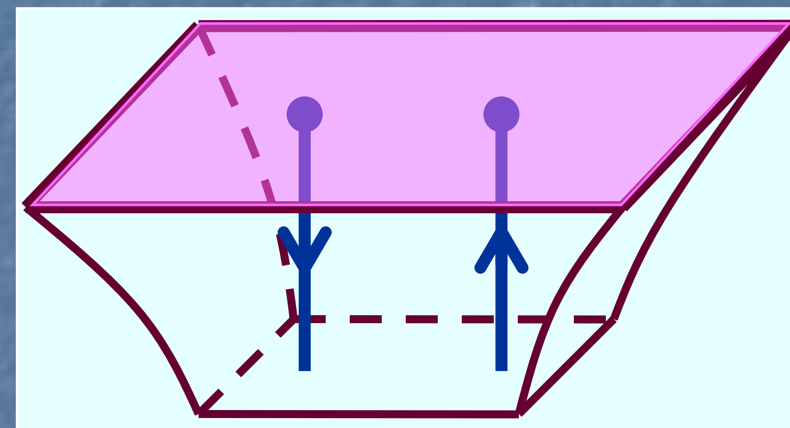
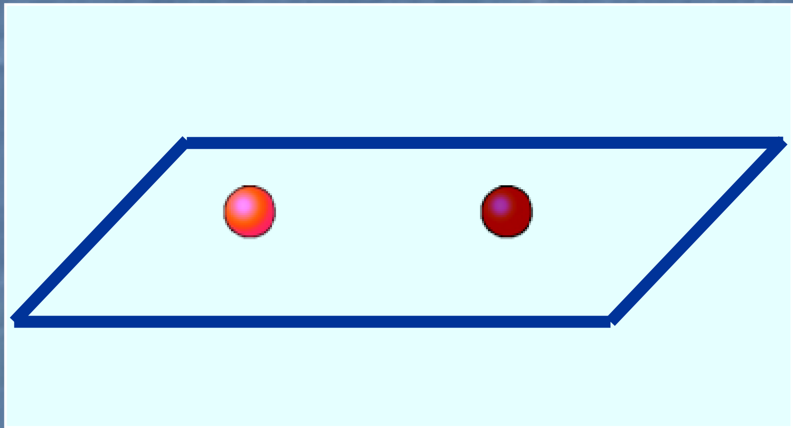
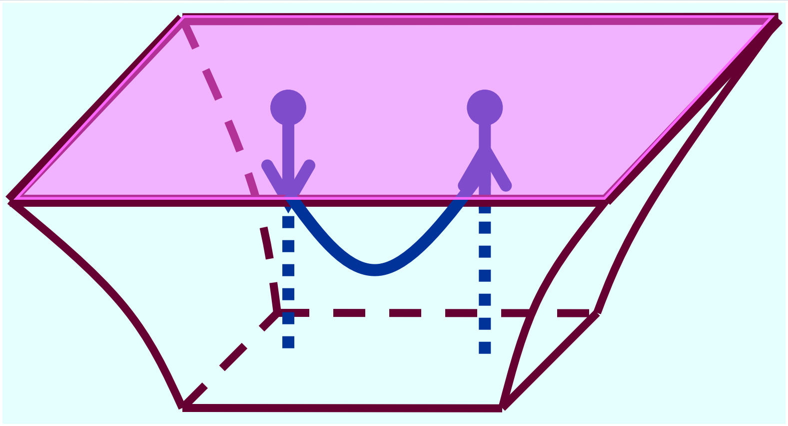


Quark and Antiquark = 2 Strings w/opposite orientations superposed

Quark-Antiquark Potential

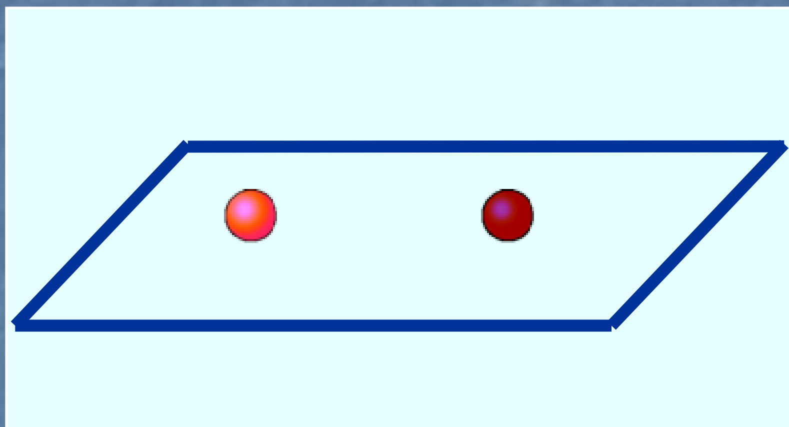
 $=$ 

Quark-Antiquark $=$ 1 String w/BOTH endpoints at $r \to \infty$

$$\left\langle \mathrm{Tr}\left[F^2(\vec{x},t)+\ldots\right]\right\rangle_{q\bar{q}} \underset{|\vec{x}|\gg L}{=} \frac{15\Gamma(\frac{1}{4})^4\sqrt{g_{YM}^2 N_c L^3}}{8(2\pi)^5\left|\vec{x}\right|^7} \quad (\text{cf. } \frac{L^2}{\left|\vec{x}\right|^6})$$

[Callan,AG] [Klebanov,Maldacena,Thorn]

Quark-Antiquark Potential



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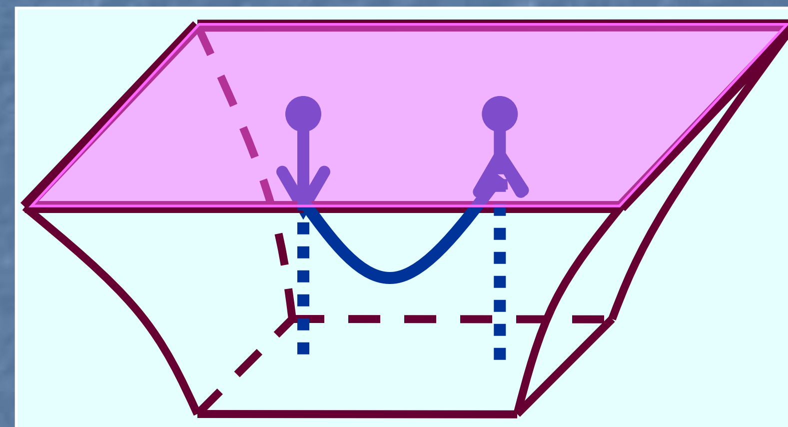
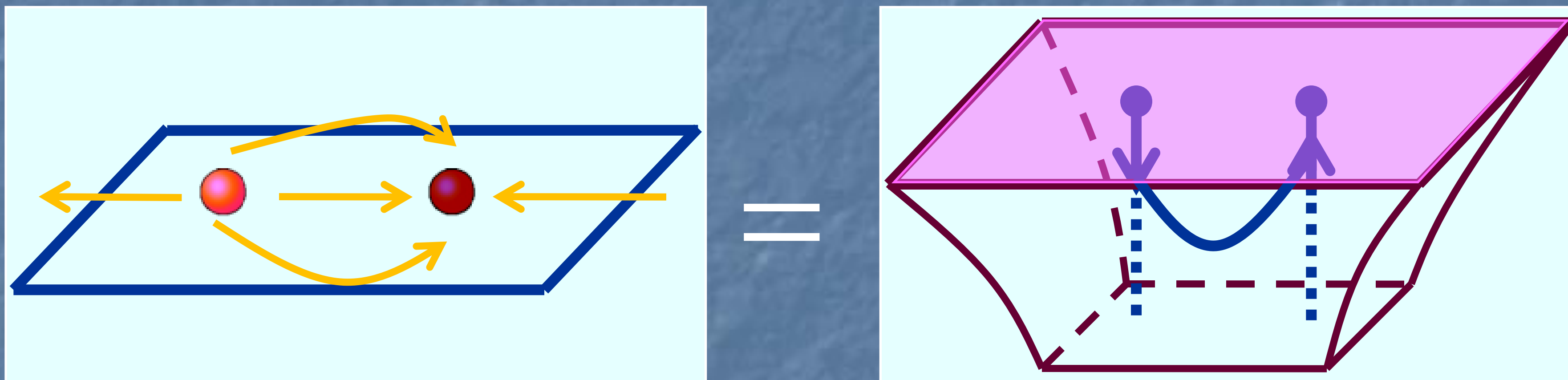


Quark-Antiquark $=$ 1 String w/BOTH endpoints at $r \to \infty$

$$V_{q\bar{q}}^{T=0}(L) = -\frac{4\pi^2\sqrt{g_{YM}^2 N}}{\Gamma(\frac{1}{4})^4 L}$$

[Rey,Yee; Maldacena]

Quark-Antiquark Potential



Quark-Antiquark $=$ 1 String w/BOTH endpoints at $r \to \infty$

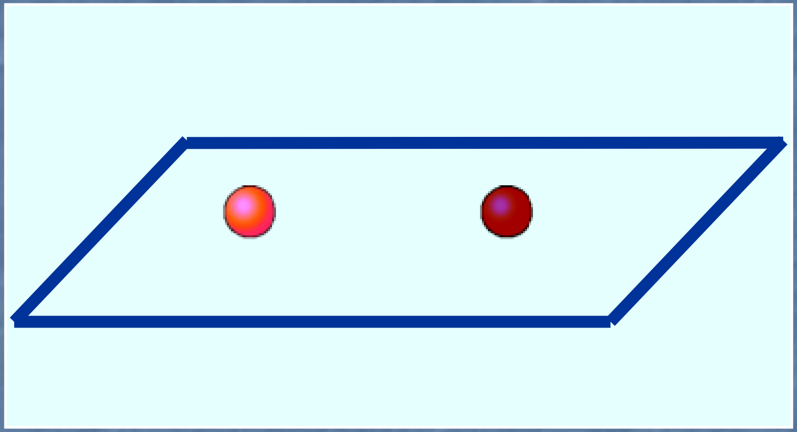
Endpoints $\leftrightarrow$ Quarks , String $\leftrightarrow$ Gluonic field

Quark

Antiquark

I.e., 'QCD string' really lives in 5 (+5) dimensions!!

Quark-Antiquark Potential



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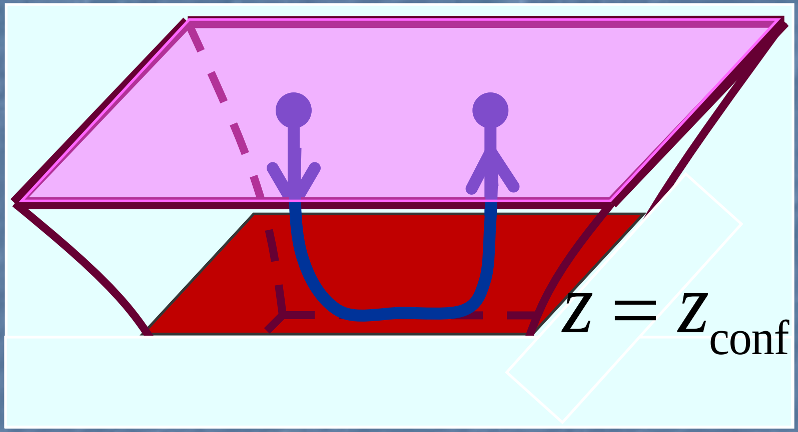
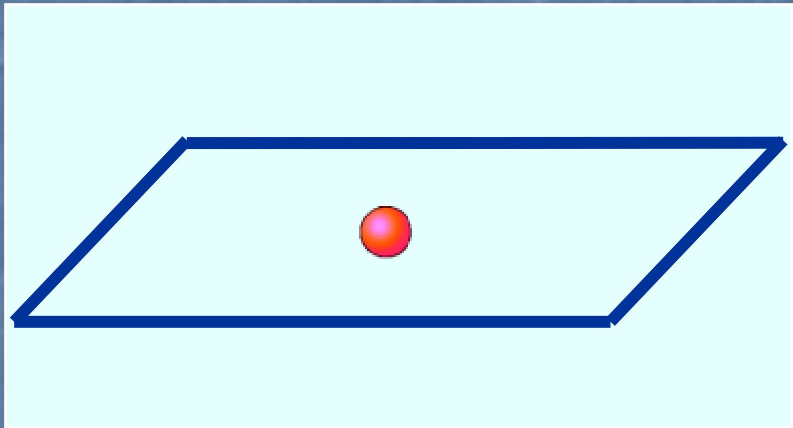


Quark-Antiquark = 1 String w/BOTH endpoints at $r \to \infty$

$$V_{q\bar{q}}^{T=0}(L) \propto L$$ [Witten; Sonnenschein et al.;etc.]

- In **confining** theories, the string is prevented from penetrating arbitrarily far into the bulk, and we then reproduce expected linear potential

Adding Quarks to SYM



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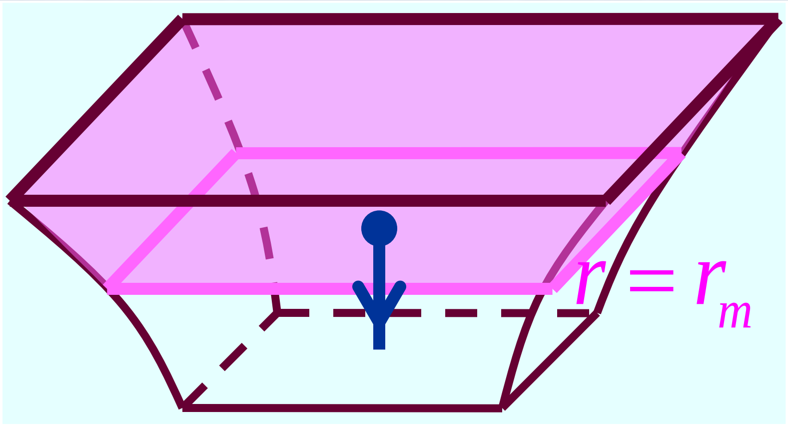
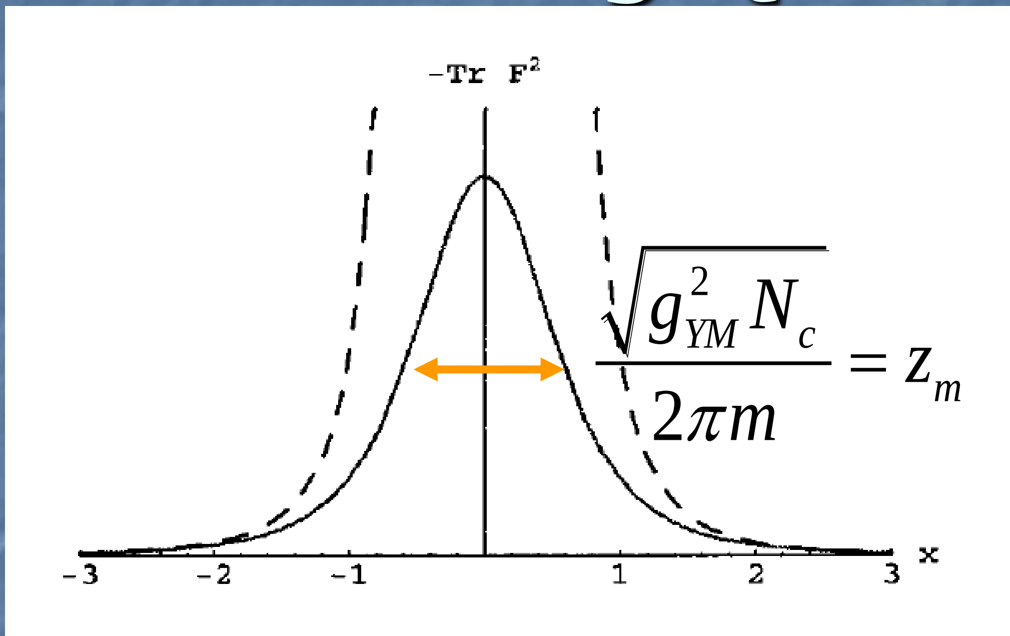


Finite-mass Quark $=$ String w/endpoint at $r_m < \infty$

Adding Quarks to SYM



$$\frac{\sqrt{g_{YM}^2 N_c}}{2\pi m} = z_m$$

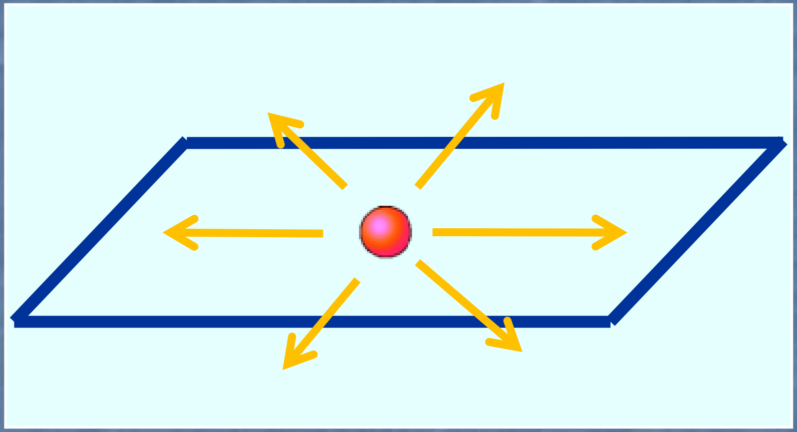
Finite mass quark is automatically "dressed" or "composite":

width of 'gluon cloud' (i.e., Compton wavelength)

$$\left\langle \mathrm{Tr}\left[F^2(\vec{x},t)+\ldots\right]\right\rangle_q = \frac{\sqrt{g_{YM}^2 N_c}}{32\pi^2 |\vec{x}|^4}\left[1-\frac{1+\frac{5}{2}\left(\frac{2\pi m|\vec{x}|}{\sqrt{g_{YM}^2 N_c}}\right)^2}{\left(1+\left(\frac{2\pi m|\vec{x}|}{\sqrt{g_{YM}^2 N_c}}\right)^2\right)^{5/2}}\right]$$

[Hovdebo,Kruczenski,Mateos,Myers,Winters]

Adding Quarks to SYM



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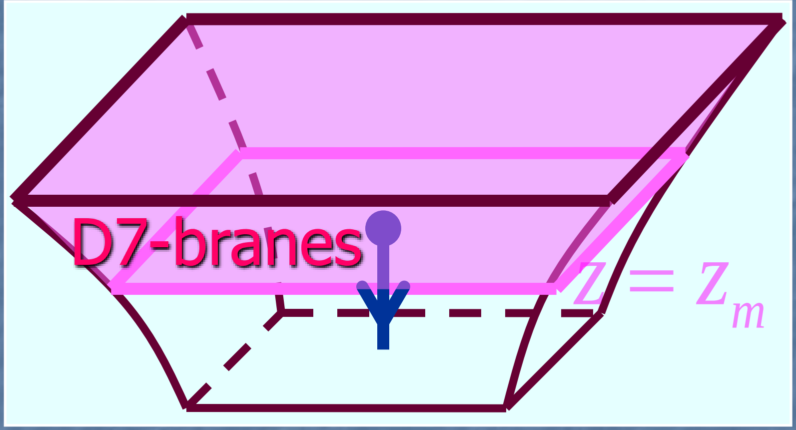


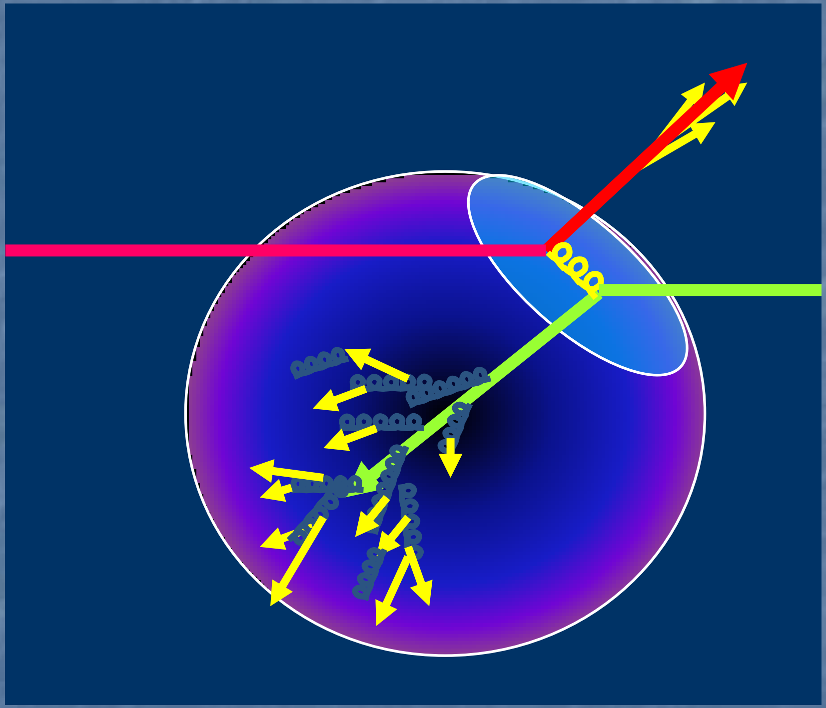
Notice we are coupling 2nd-quantized gluonic (+etc.) **fields** to **1st-quantized quark**:

$$\int Dx(\tau) DA_\mu(x') \exp\left(iS[A(x'), x(\tau)]\right)$$

Integral over $A_\mu(x')$ is done EXACTLY (with AdS) but that over $x^\mu(\tau)$ is treated in saddle point approx.
(later today: some semiclassical corrections, $\sim \lambda^{-1/2}$)

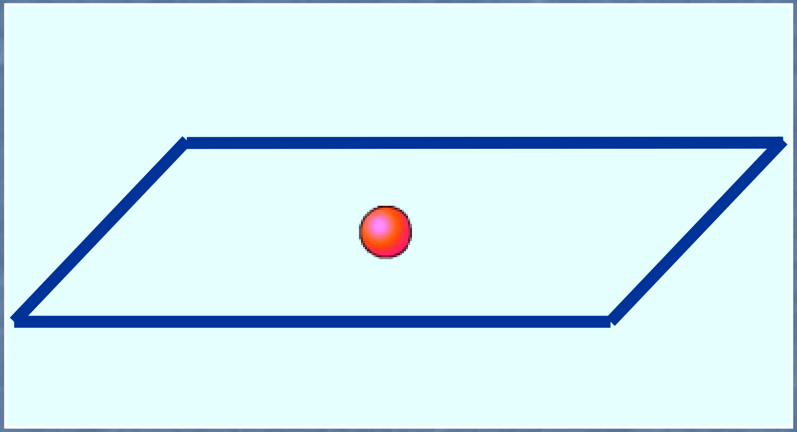
Energy Loss: Heavy Quark

RHIC finds significant energy loss for partons crossing the QGP ("jet quenching"):



How much energy is lost by quark?
Where does this energy go?

Energy Loss: Heavy Quark



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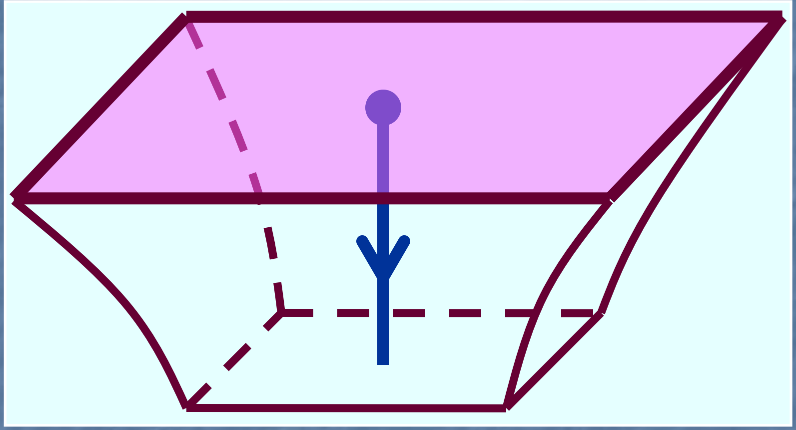
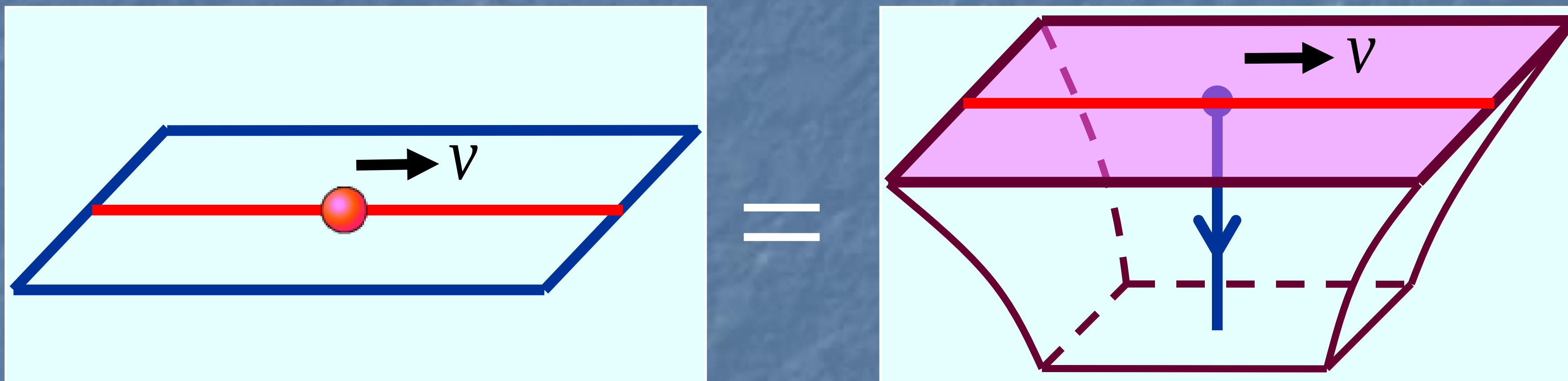


Static quark in vacuum = Vertical string on pure AdS

Energy Loss: Heavy Quark



Quark with constant v = Vertical string at constant v

No energy loss

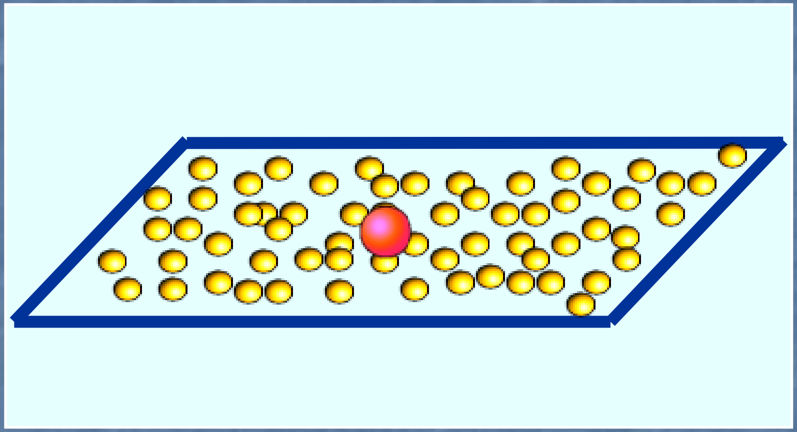
The situation changes if:

Quark accelerates

or/and

Quark is placed inside a medium ← consider this first

Energy Loss: Heavy Quark



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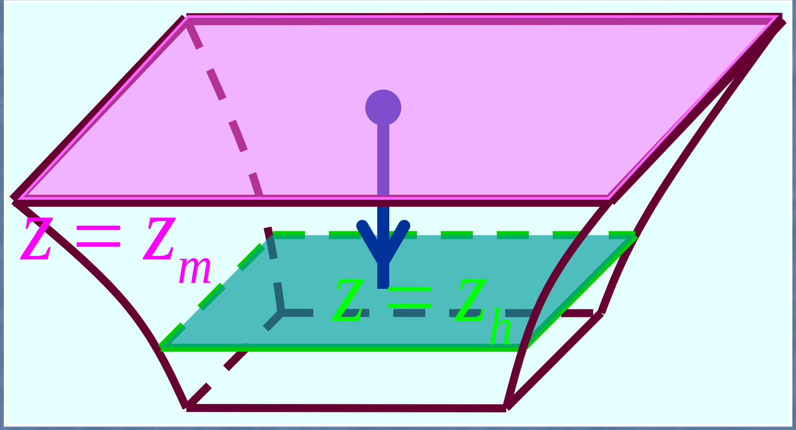


Heavy quark in thermal SYM plasma ($m \gg T$) $=$ String extending on Schw-AdS from $z = z_m$ to $z = z_h \gg z_m$

String e.o.m. follows from std. Nambu-Goto action

$$S_{\mathrm{NG}} = -\frac{1}{2\pi l_s^2}\int d^2\sigma\sqrt{-\gamma} = -\frac{1}{2\pi l_s^2}\int d^2\sigma\sqrt{-\det\left(G_{mn}\partial_a X^m \partial_b X^n\right)}$$

$$= -\frac{1}{2\pi l_s^2}\int dtdz\sqrt{\left(\dot{X}(z,t)\cdot X'(z,t)\right)^2 - \dot{X}(z,t)^2 X'(z,t)^2}$$

$$\dot{X}^m \equiv \partial_t X^m, \quad X'^m \equiv \partial_z X^m, \quad \dot{X}\cdot X' \equiv G_{mn}\dot{X}^m X'^n, \quad \text{etc.}$$

on the Schw-AdS geometry

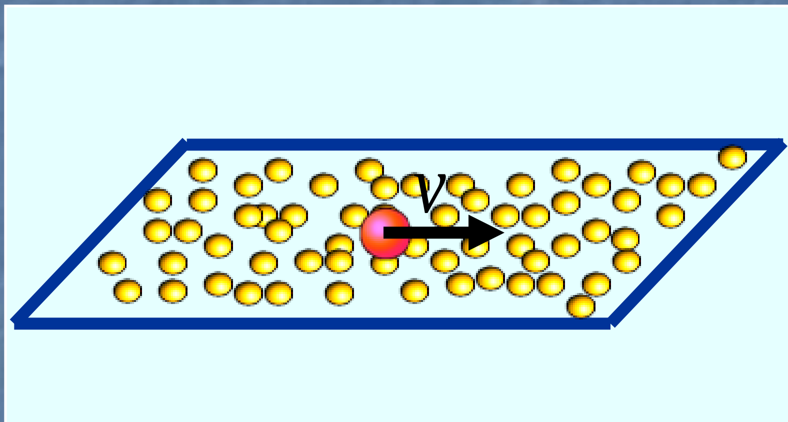
$$ds^2_{\mathrm{SchwAdS}} = \left(\frac{L}{z}\right)^2\left[\left(-h\,dt^2 + d\vec{x}^2\right) + \frac{dz^2}{h}\right], \qquad h(z) \equiv 1 - \frac{z^4}{z_h^4}$$

E.o.m. state that conjugate momentum densities are conserved currents (b/c of translation invariance),

$$\Pi_i^a \equiv \frac{\partial L_{\mathrm{NG}}}{\partial\left(\partial_a X^i\right)}, \quad \partial_t \Pi_i^t + \partial_z \Pi_i^z = 0 \qquad \Rightarrow \quad \Pi_i^z = \text{const.}$$

for stationary configurations

Energy Loss: Heavy Quark



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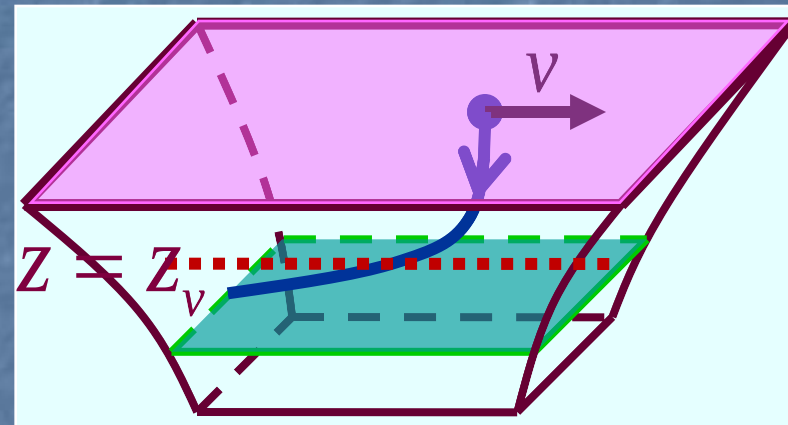


Stationary solution has $\Pi_x^z = -\frac{\pi\sqrt{\lambda}T^2}{2}\frac{v}{\sqrt{1-v^2}}$

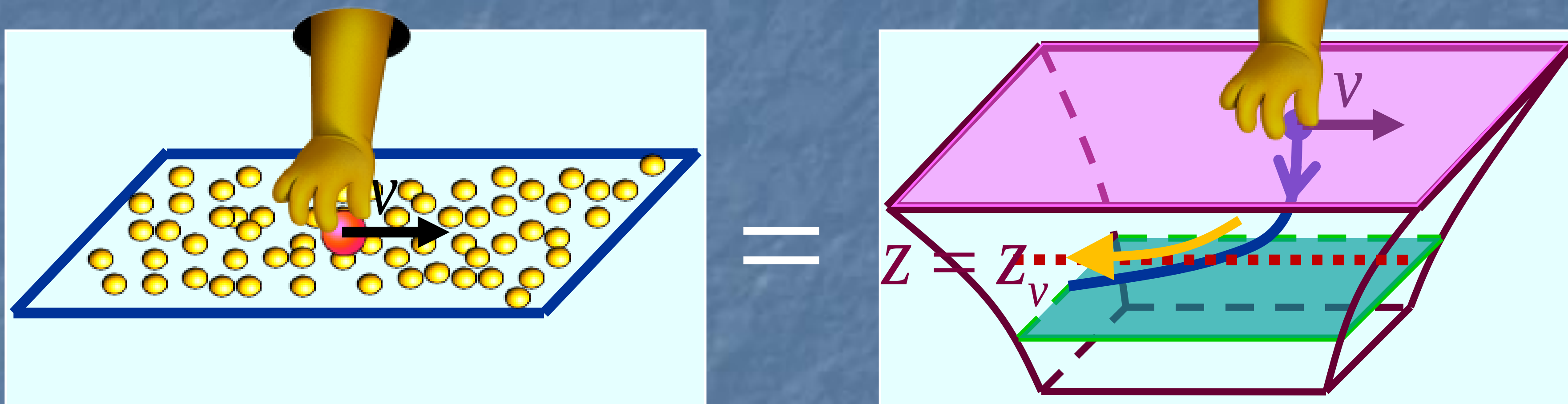
and takes the form

$$X(z,t) = v\left[t - \frac{z_h}{4}\ln\left(\frac{z_h+z}{z_h-z}\right) + \frac{z_h}{2}\tan^{-1}\left(\frac{z}{z_h}\right)\right]$$

[Herzog,Karch,Kovtun,Kozcaz,Yaffe; Gubser]

(Related work: [Casalderrey-Solana,Teaney;Liu,Rajagopal,Wiedemann])

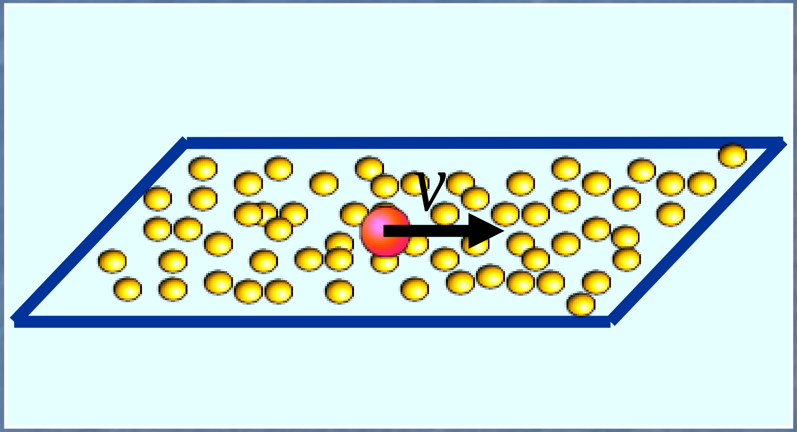
Energy Loss: Heavy Quark



Rates at which momentum and energy flow along string (supplied by external force) are given by

$$\underbrace{\Pi_x^z = -\frac{\pi\sqrt{\lambda}T^2}{2}\frac{v}{\sqrt{1-v^2}}}_{\text{Drag force on quark (rate of momentum loss)}} \equiv \frac{dp_x}{dt}, \qquad \underbrace{\Pi_t^z = -\frac{\pi\sqrt{\lambda}T^2}{2}\frac{v^2}{\sqrt{1-v^2}}}_{\text{Rate of energy loss for quark}} \equiv \frac{dE}{dt}$$

Energy Loss: Heavy Quark



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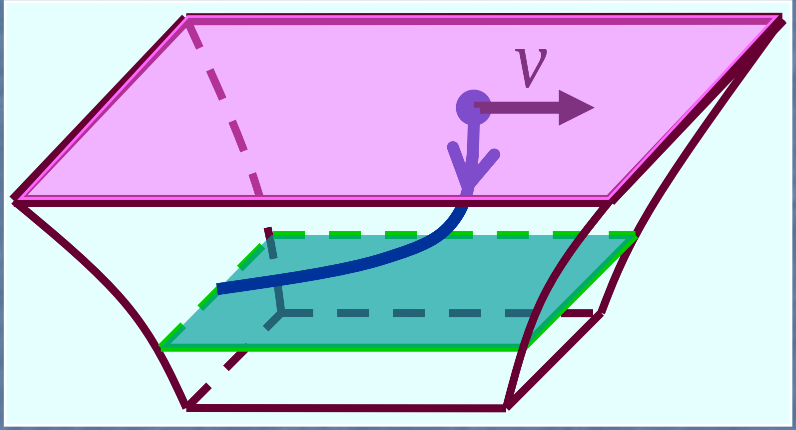
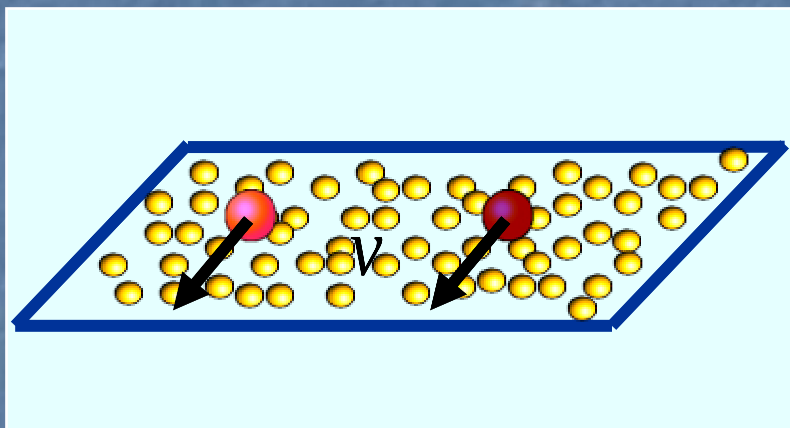


$$\Rightarrow \quad p_x(t) = p_x(0)\exp\left(-t/t_r\right) \qquad t_r = \frac{2m}{\pi\sqrt{g_{YM}^2 N}\, T^2}$$

E.g., $t_r(\text{charm}) \approx 0.6 - 2.1\ \text{fm/c}$ [Gubser]

cf. pQCD $t_r(\text{charm}) \approx 4 - 12\ \text{fm/c}$ [van Hees,Rapp]

Energy Loss: Meson and Baryon



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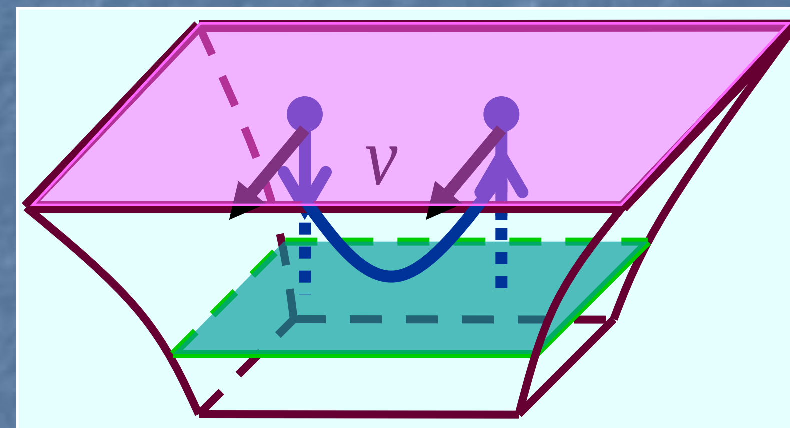


Meson in plasma feels NO drag force, as a result of its being **color-neutral**

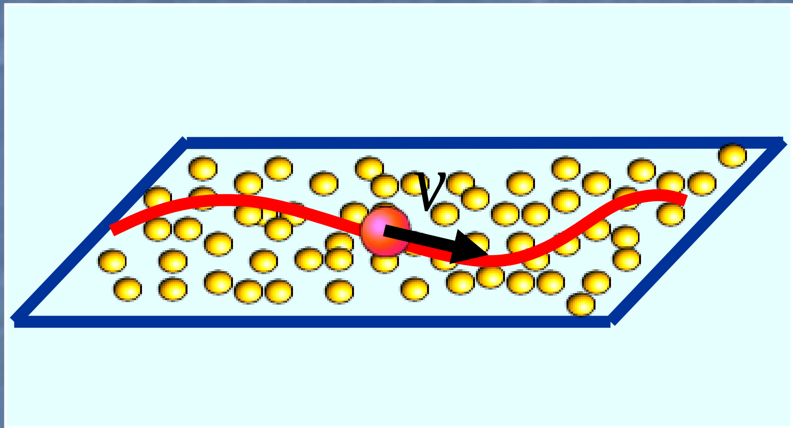
[Peeters,Sonnenschein,Zamaklar; Liu,Rajagopal,Wiedemann; Chernicoff,García,AG]

Baryon (=D5-brane [Witten;Brandhuber et al.; Callan,AG,Savvidy]) similarly feels no drag [Chernicoff,AG; Athanasiou, Liu, Rajagopal]

Drag does appear in $1/N_c^2$ corrections

[Dusling,Erdmenger,Kaminski,Rust,Teaney,Young]

Energy Loss: Heavy Quark



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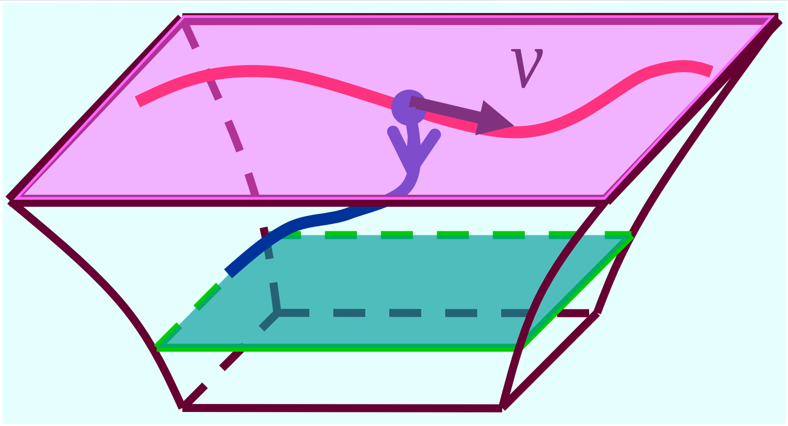


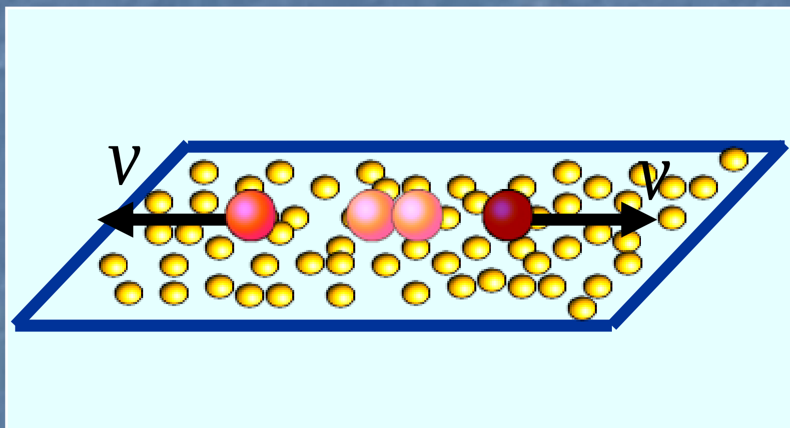
Can generalize calculation to **accelerated** quark,

[Chernicoff,AG]

and to somewhat more realistic case where quark is created within the plasma...

[Herzog,Karch,Kovtun,Kozcaz,Yaffe; Chernicoff,AG]

Energy Loss: Pair Creation



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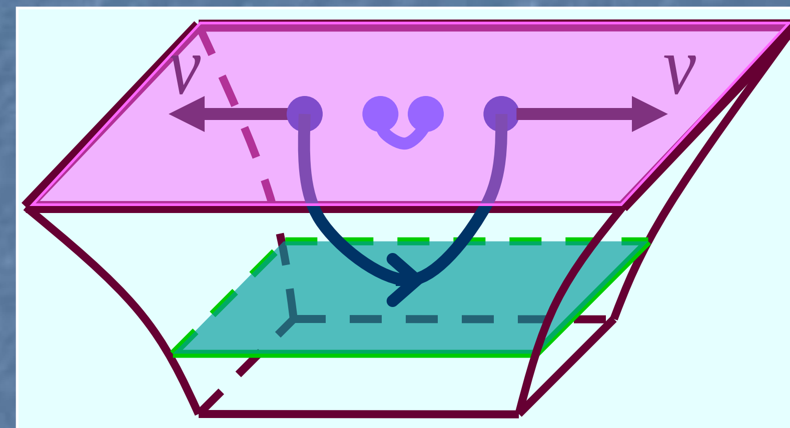


Singlet back-to-back quark-antiquark pair

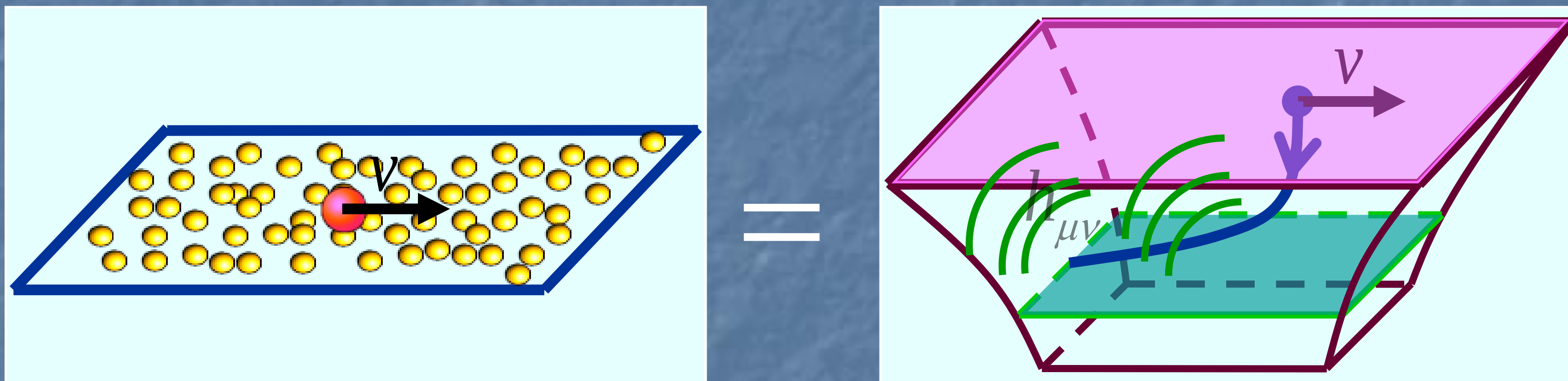
U-shaped string with initially coincident endpoints

[Herzog,Karch,Kovtun,Kozcaz,Yaffe]

Initial velocity fixed at $v_{\max}$; ‘start feeling the plasma’ according to stationary formula when q-qbar separation reaches (v-dependent) screening length

[Chernicoff, AG ;
related work: Hatta, Iancu, Mueller]

Energy Loss: Spatial Distribution



Can determine the spatial profile of dissipated energy from

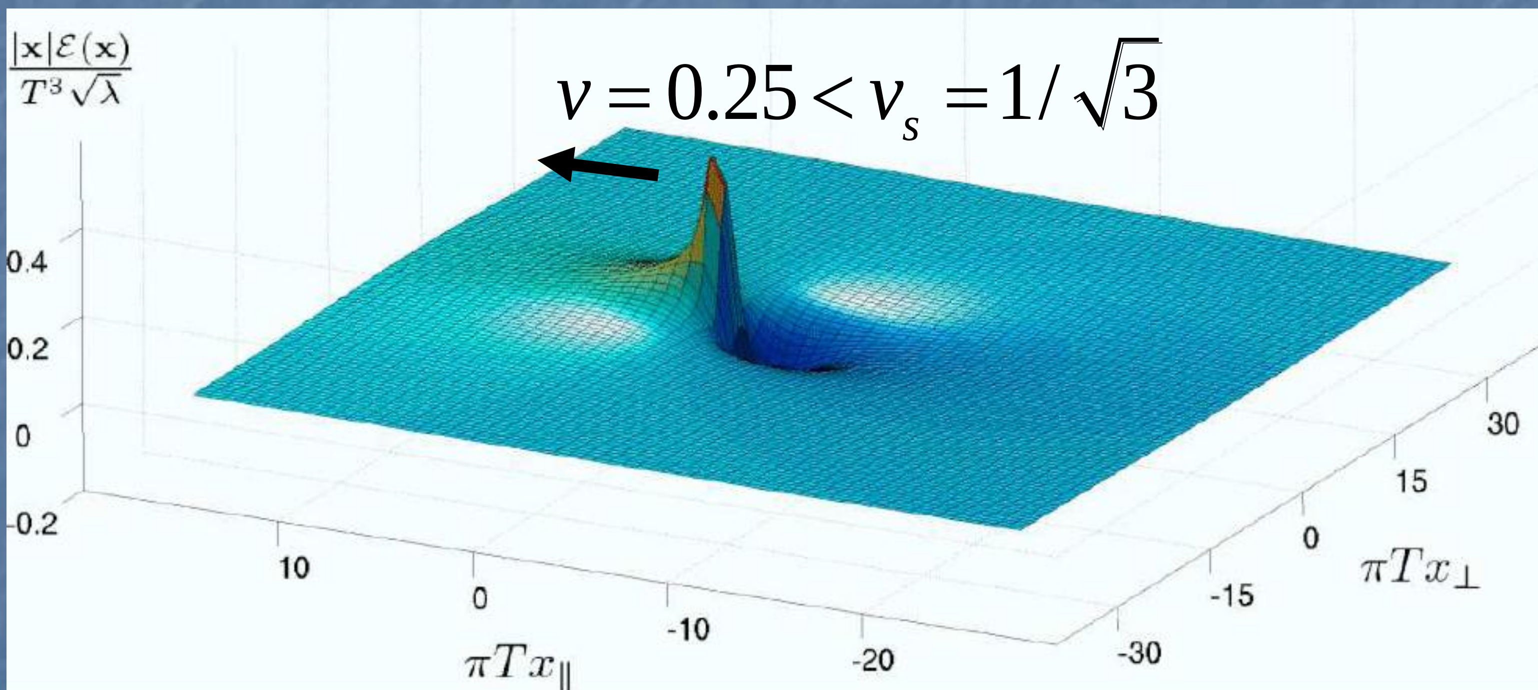
$$\left\langle T_{\mu\nu}(x) \right\rangle_{\mathrm{q},v} \quad \leftrightarrow \quad h_{\mu\nu}(x, r=\infty)$$

[Friess,Gubser,Michalogiorgakis; Friess,Gubser,Michalogiorgakis,Pufu; Yarom; Gubser,Pufu; Gubser,Pufu,Yarom; Chesler,Yaffe; Noronha,Torrieri,Gyulassy; Betz,Gyulassy,Noronha,Torrieri; etc.]

Energy Loss: Spatial Distribution

Energy density in wake generated by the quark

[Gubser,Pufu,Yarom; Chesler,Yaffe]



From: Chesler,Yaffe, arXiv:0706.0368

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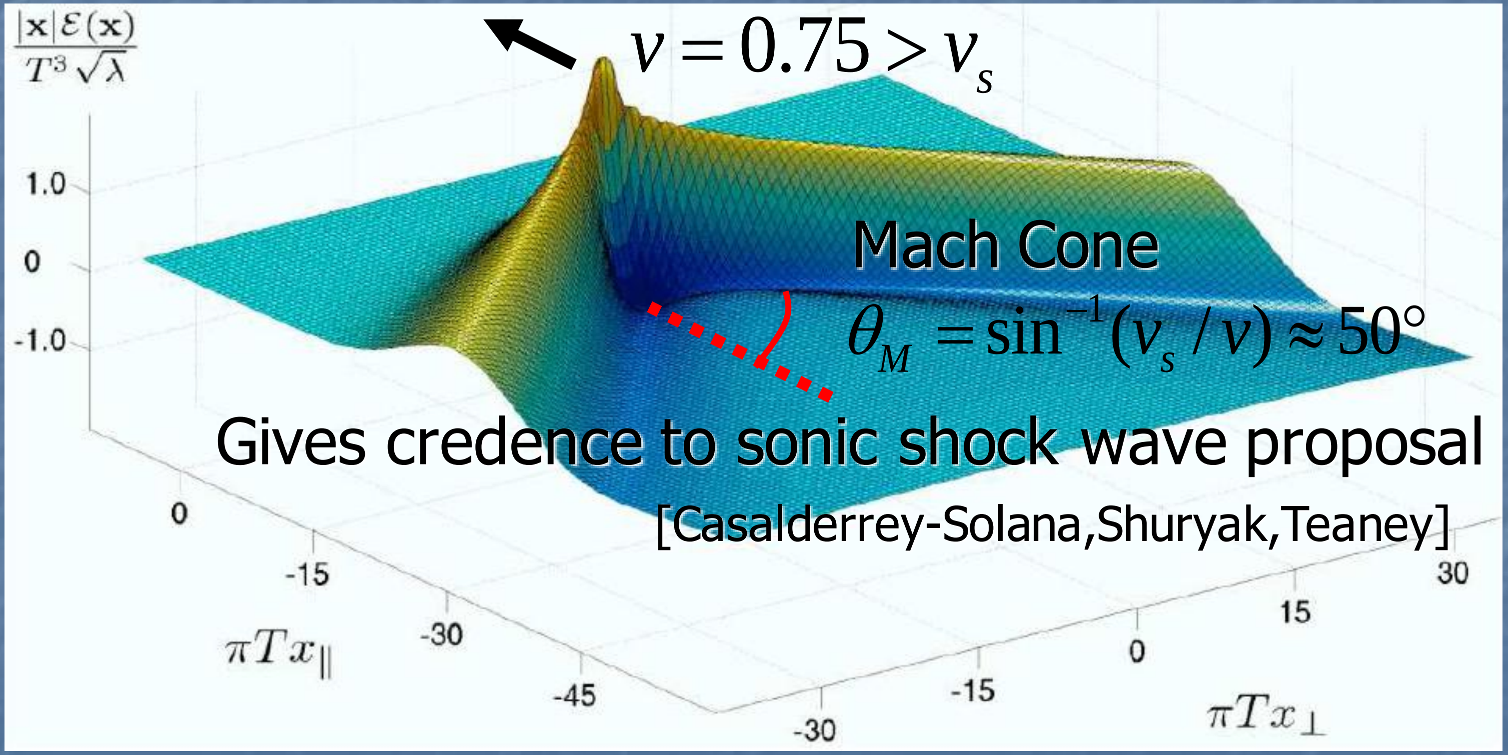

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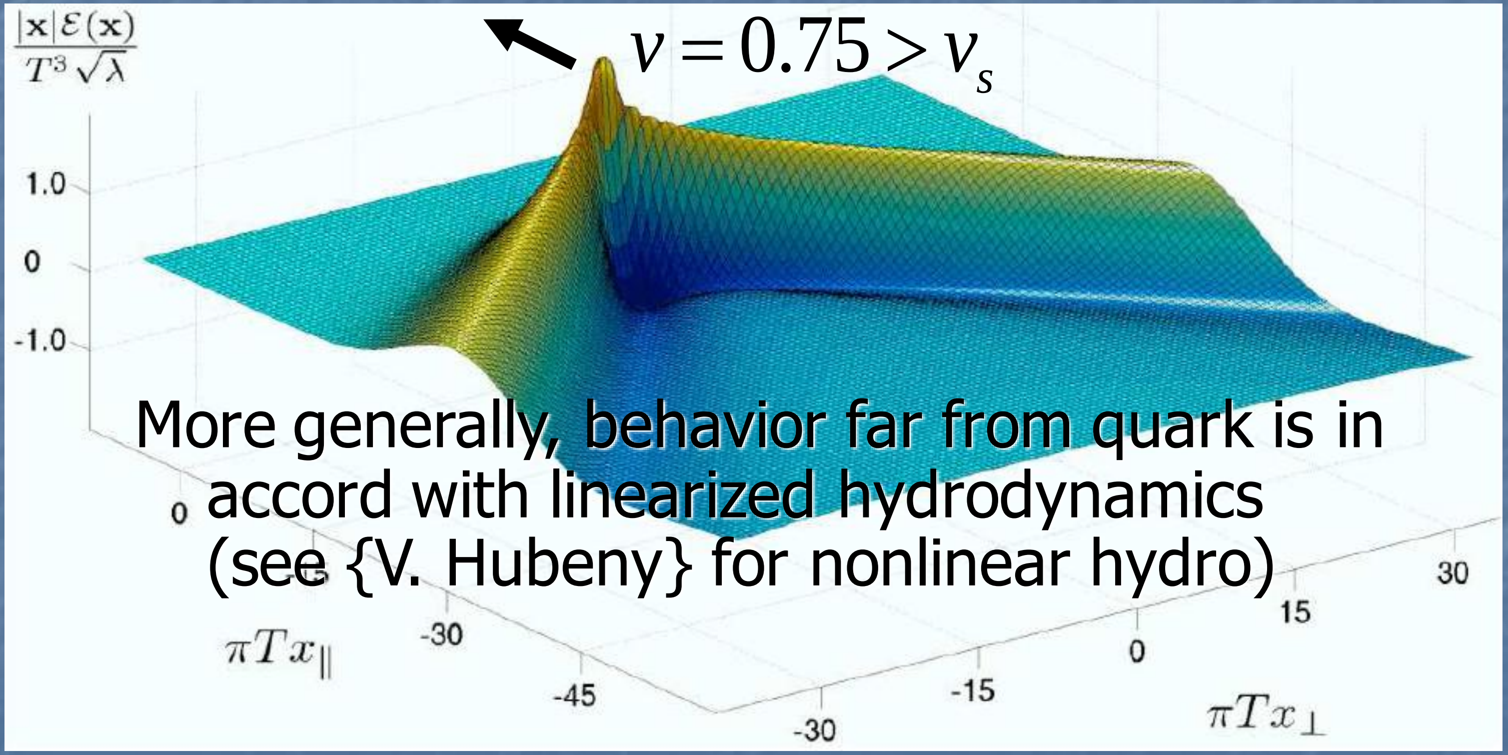
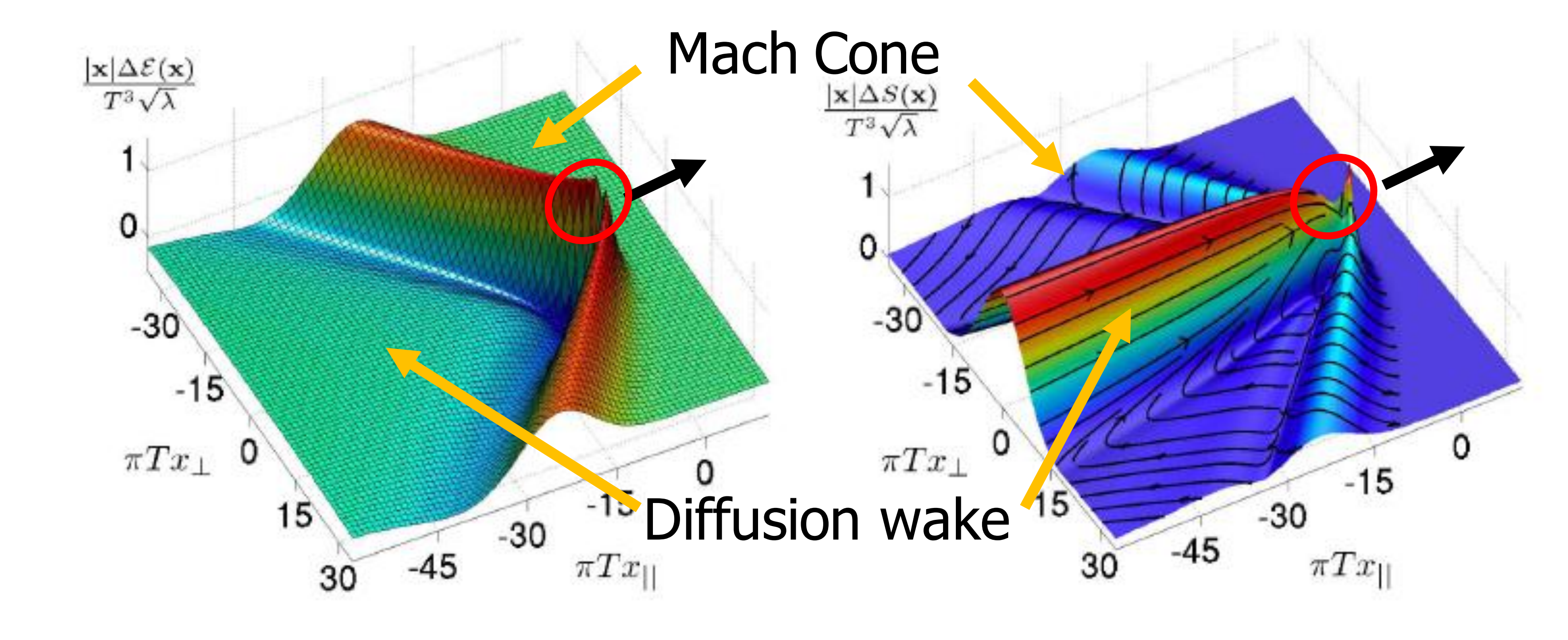


More generally, behavior far from quark is in accord with linearized hydrodynamics (see {V. Hubeny} for nonlinear hydro)

From: Chesler,Yaffe, arXiv:0706.0368

Energy Loss: Spatial Distribution

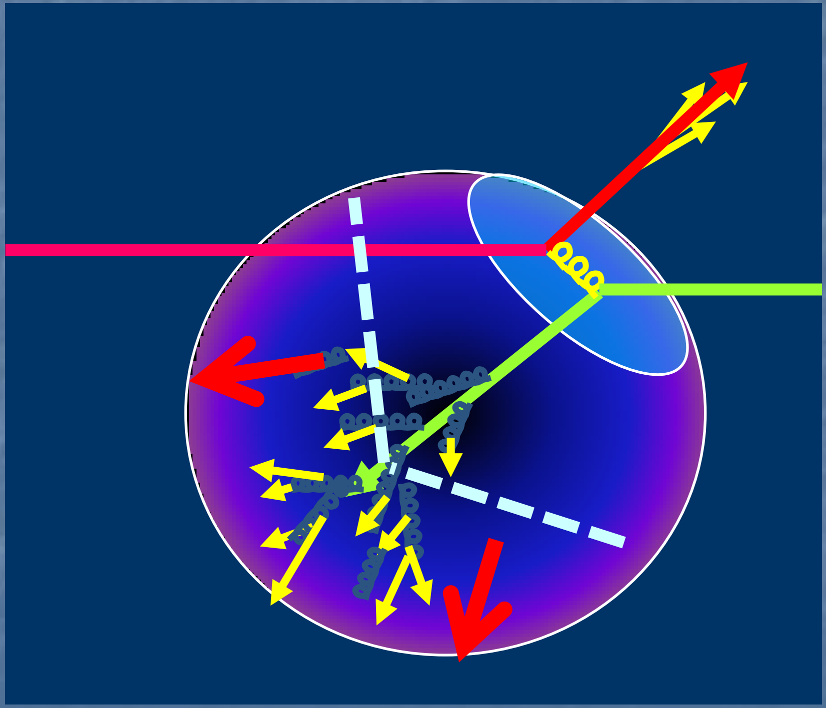


From: Chesler,Yaffe, arXiv:0712.0050

There is a "neck" region close to quark where hydro is not applicable, but AdS/CFT technology is, and gives interesting result…

Energy Loss: Spatial Distribution

Recall energy loss ("jet quenching") setup at RHIC/LHC:



Energy Loss: Spatial Distribution

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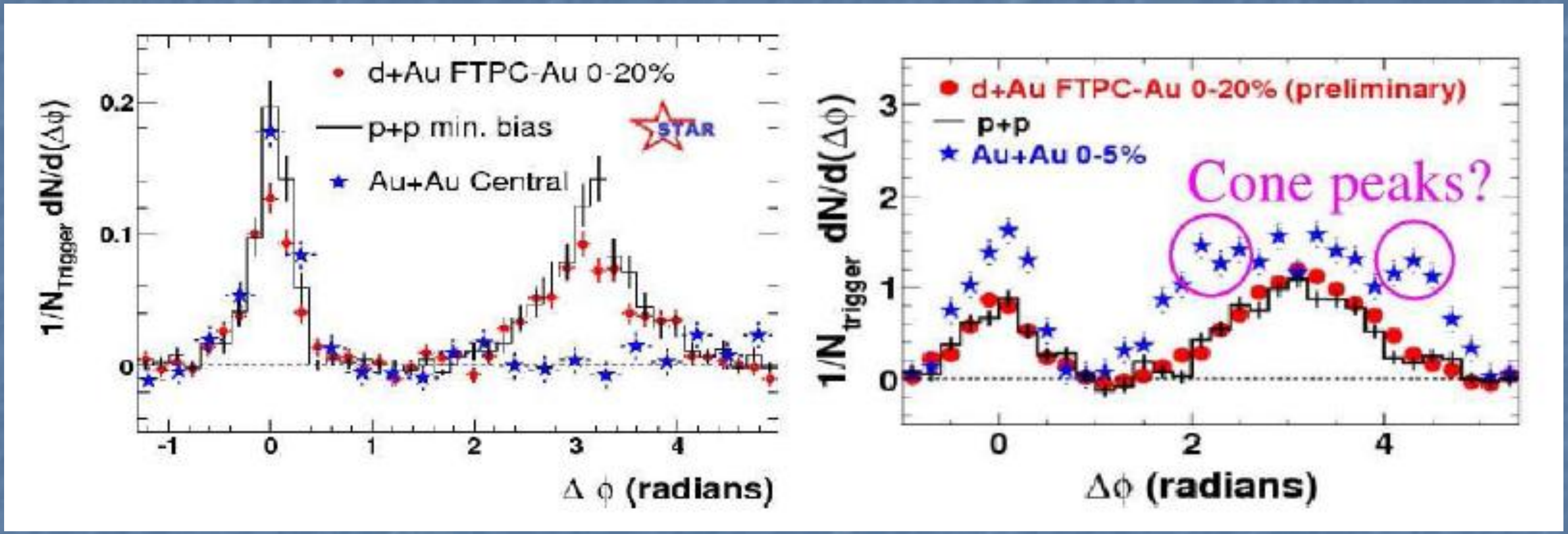
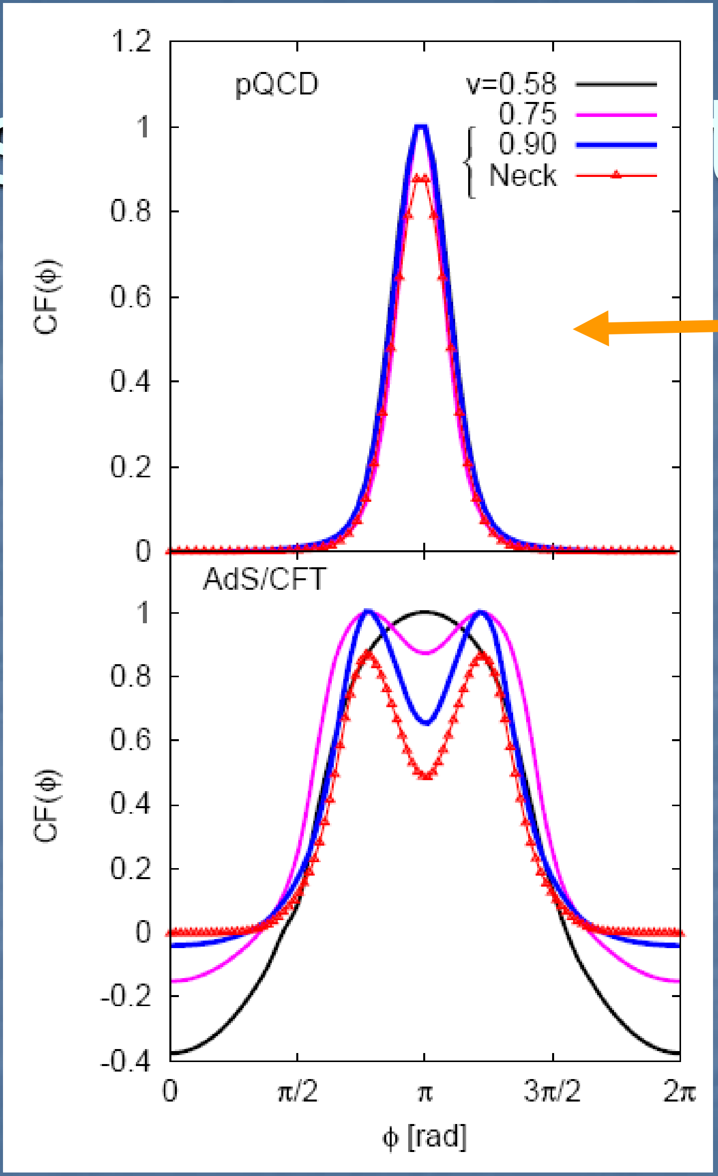


From: Torrieri,Betz,Noronha,Gyulassy, arXiv:0901.0230

AdS/CFT hydro region alone (Mach cone + diffussion wake) does not emulate this, but "neck" region does…

Energy Loss Distribution

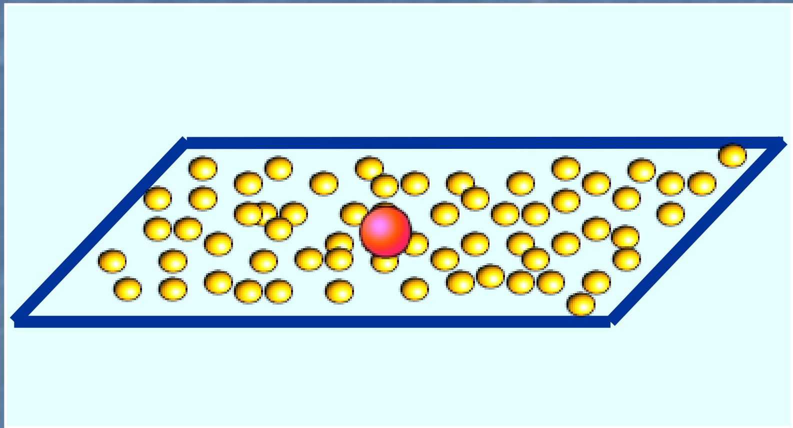


(But, see 3-jet proposal by [Ayala,Jalilian-Marian, Magnin,Ortiz,Paic, Tejeda-Yeomans])

Result from perturbative QCD vs. AdS/CFT:

From: Betz,Gyulassy,Noronha,Torrieri arXiv:0807.4526

Brownian Motion



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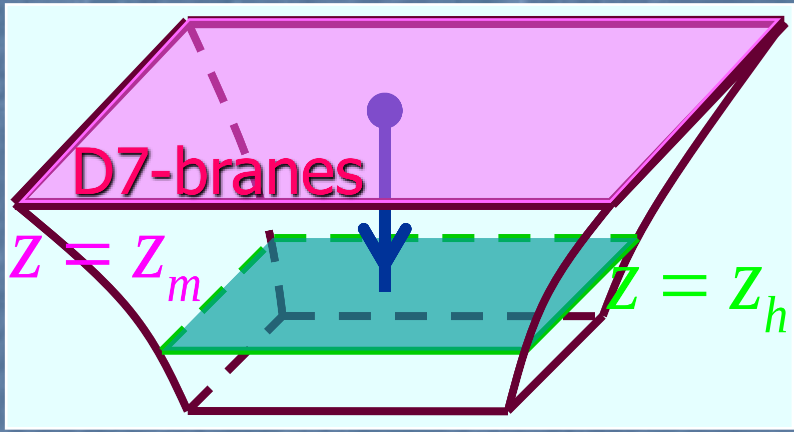


Static heavy quark in thermal $\mathcal{N} = 4$ SYM plasma $=$ Static string from $z = z_m$ to $z = z_h$ on Schw-AdS geometry

Would expect quark to undergo Brownian motion…

How does this come about in string description?

String e.o.m. follows from std. Nambu-Goto action

$$S_{\mathrm{NG}} = -\frac{1}{2\pi l_s^2}\int dtdz\sqrt{-\gamma} = -\frac{1}{2\pi l_s^2}\int dtdz\sqrt{-\det\left(G_{mn}\partial_a X^m \partial_b X^n\right)}$$

on the Schw-AdS geometry

$$ds^2_{\mathrm{SchwAdS}} = \left(\frac{L}{z}\right)^2\left[\left(-h\,dt^2 + d\vec{x}^2\right) + \frac{dz^2}{h}\right], \qquad h(z) \equiv 1 - \frac{z^4}{z_h^4}$$

Small fluctuations of embedding field around static configuration feel induced metric,

$$\vec{X}(z,t) = 0 + \delta\vec{X}(z,t)$$

$$S_{\mathrm{NG}} = -\frac{1}{2\pi l_s^2}\int dtdz\sqrt{-\gamma}\left[1 + \gamma^{ab}G_{ij}\partial_a\delta X^i \partial_b \delta X^j + O(\delta X^2)\right]$$

i.e., they are free massless fields living on 2 dim black hole geometry

Expanding into modes and quantizing,

$$X(t,z) = \sum_{\omega>0} \left[a_\omega u_\omega(t,z) + a_\omega^\dagger u_\omega(t,z)^* \right]$$

know that Hawking radiation emerging from the black hole will excite the modes of the string

Semiclassically, these modes are thermally populated,

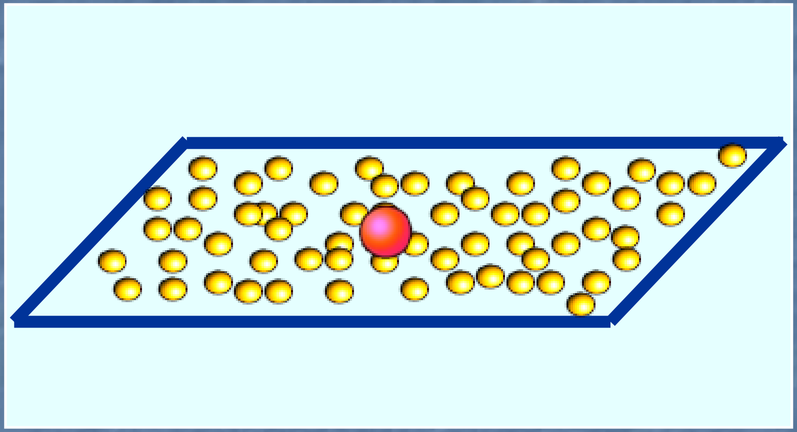
$$\left\langle a_\omega^\dagger a_{\omega'} \right\rangle = \frac{\delta_{\omega\omega'}}{e^{\beta\omega} - 1}$$

from which one can determine correlators of the fluctuating position of the string endpoint (=quark)

$$\left\langle x(t) x(0) \right\rangle$$

(Conversely, if we could compute this correlator in gauge theory, we could infer precise character of Hawking radiation!)

Brownian Motion



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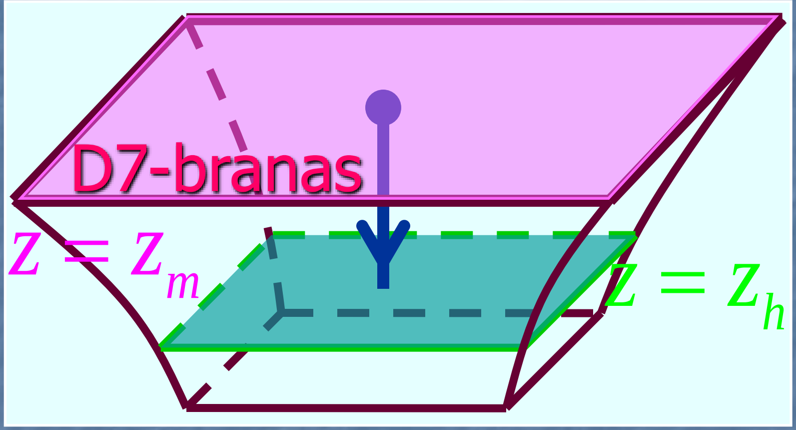
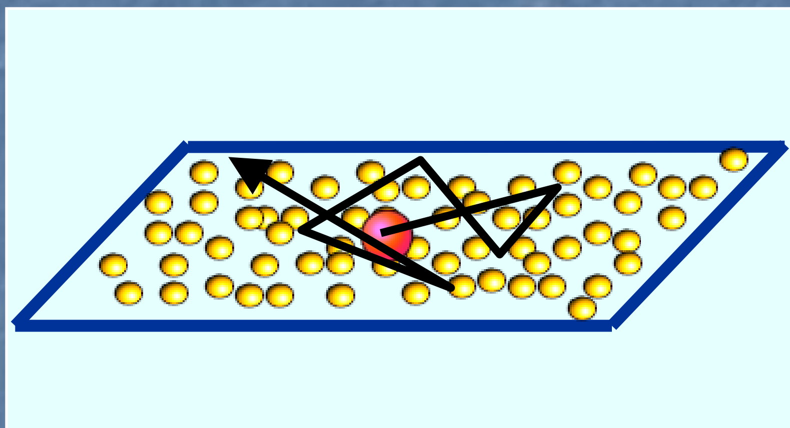


The position of the endpoint is found to obey a generalized Langevin equation

$$m\ddot{x} + \int dt'\eta(t,t')\dot{x}(t') = \xi(t), \quad \langle \xi(t)\xi(t') \rangle = \kappa(t,t')$$

[de Boer,Hubeny,Rangamani,Shigemori; Son,Teaney]

Brownian Motion



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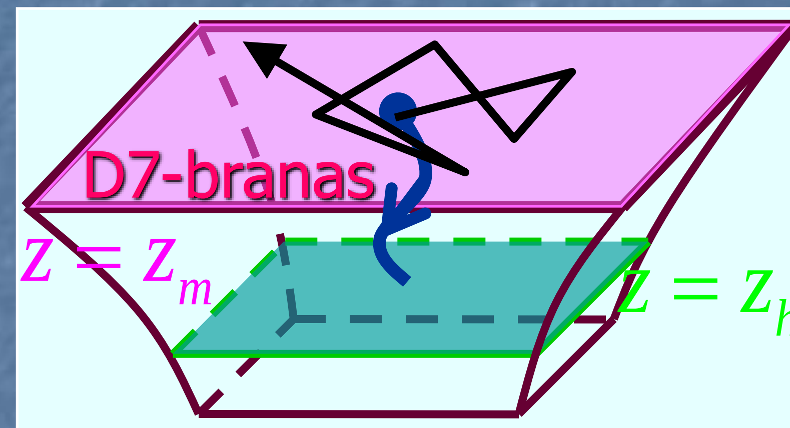

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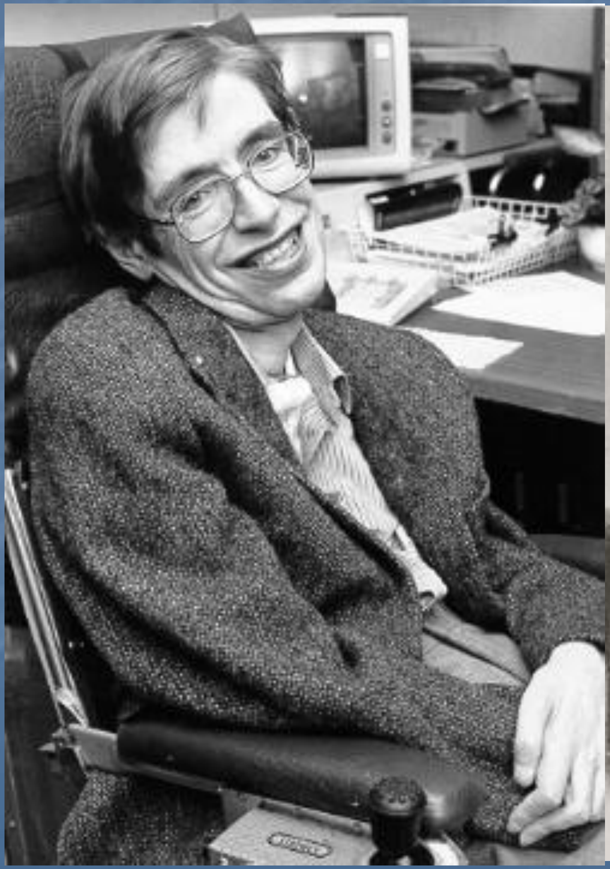
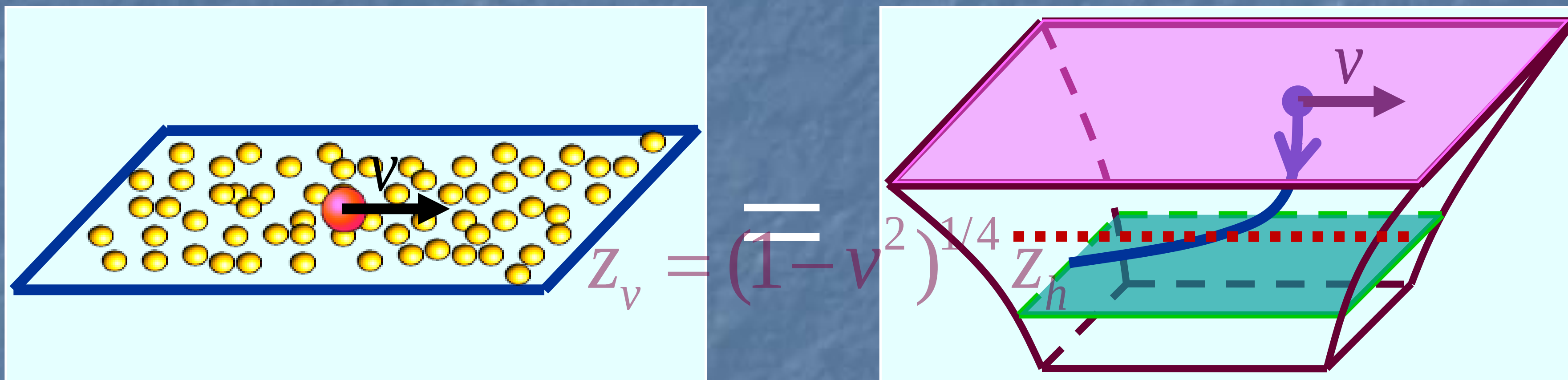
[de Boer,Hubeny,Rangamani,Shigemori; Son,Teaney]

So, in the AdS/CFT context,
Hawking = Brown!!



Brownian Motion

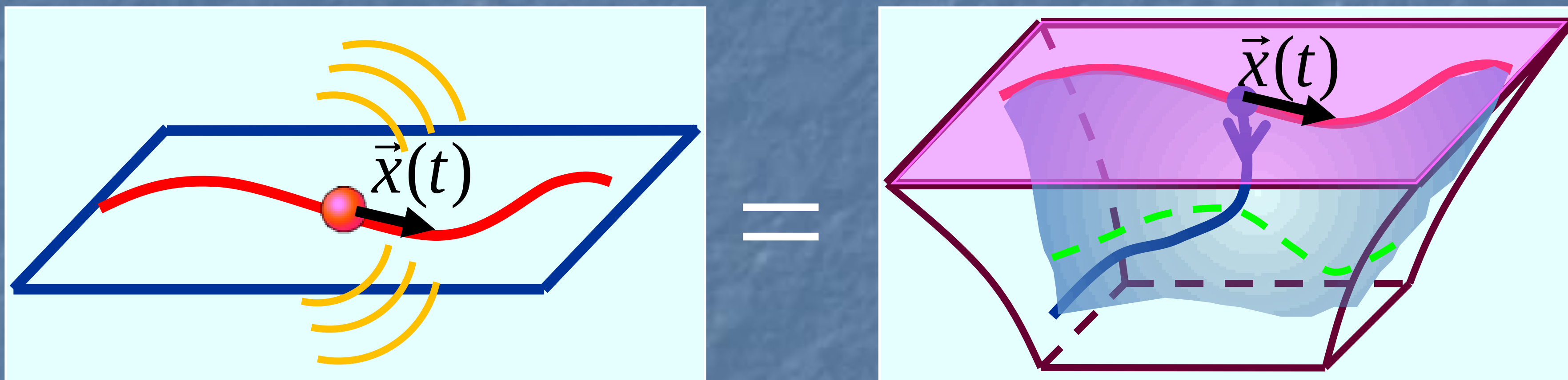


$$z_v = (1-v^2)^{1/4} z_h$$

Can generalize to case of quark moving at constant velocity (=trailing stationary string), where horizon on induced metric on the worldsheet no longer coincides with horizon of bulk metric

[Gubser; Casalderrey,Teaney; Giecold,Iancu,Mueller; Casalderrey,Teaney]

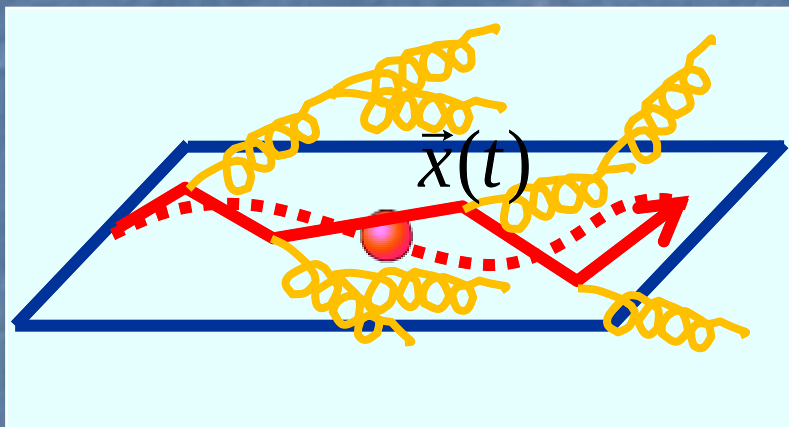
Fluctuations in Vacuum



At **zero** temperature, string embedding is known analytically for *arbitrary* string trajectory [Mikhailov]

For an accelerating (and therefore radiating) quark, an event horizon develops on string worldsheet [Chernicoff,AG]

Fluctuations in Vacuum



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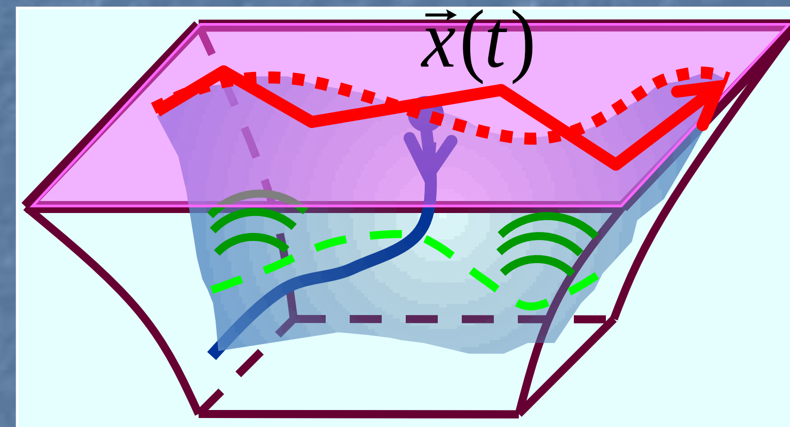
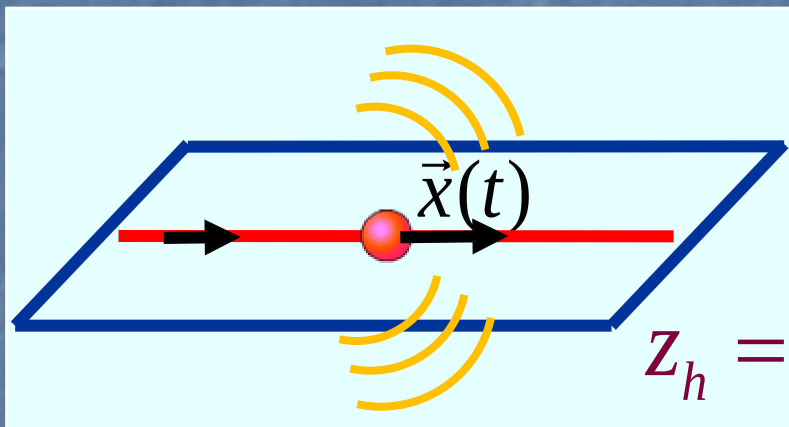
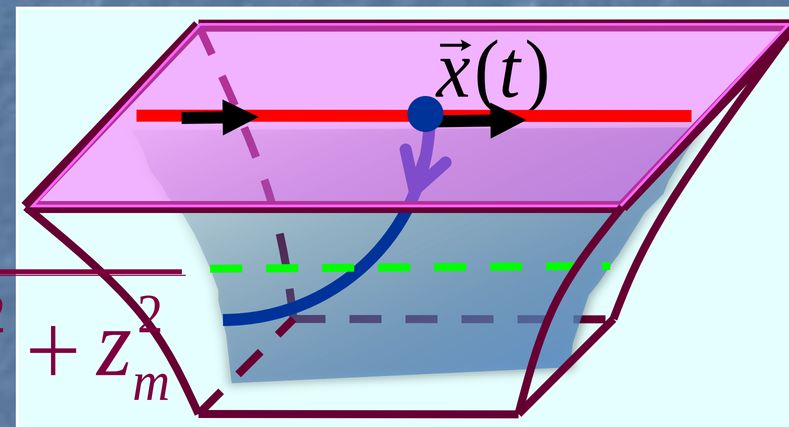


- At **zero** temperature, string embedding is known analytically for *arbitrary* string trajectory [Mikhailov]
- For an accelerating (and therefore radiating) quark, a event horizon develops on string worldsheet [Chernicoff,AG]
- Get quantum fluctuations about average quark trajectory induced by Hawking radiation in string theory description and by emitted radiation in gauge theory description

Fluctuations in Vacuum



$$= \quad z_h = \sqrt{A^{-2} + z_m^2}$$



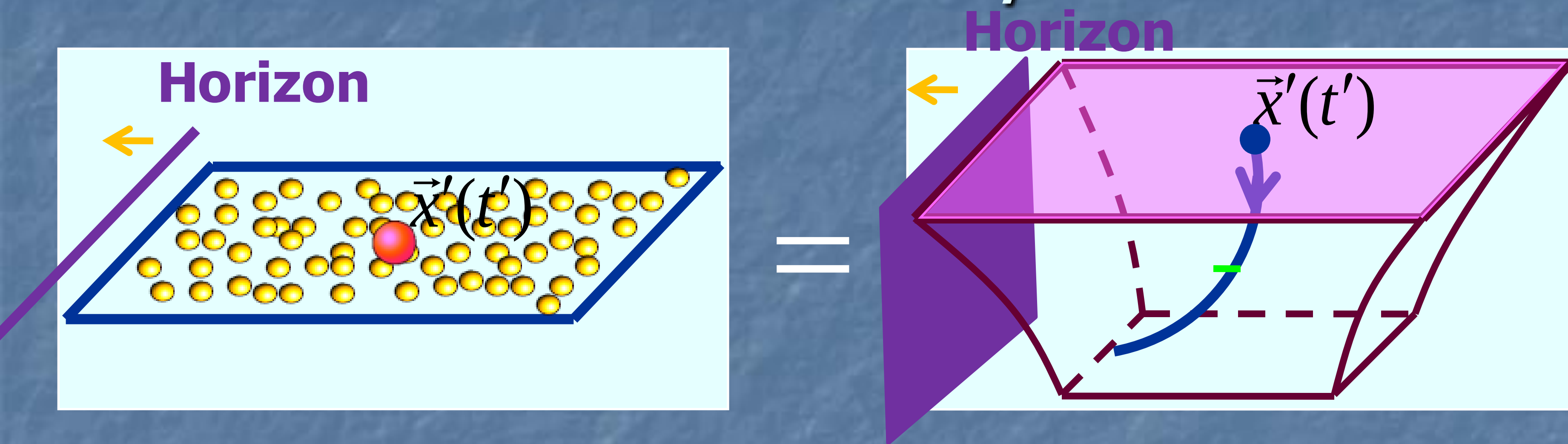
- Can work this out explicitly for quark with uniform proper acceleration A

  [Cáceres,Chernicoff,AG,Pedraza]

- The calculation makes direct contact with previous story of Brownian motion in a thermal medium, as expected from Unruh effect…

  (Related work: [Xiao; Hirayama,Kao,Kawamoto,Lin])

Unruh in AdS/CFT



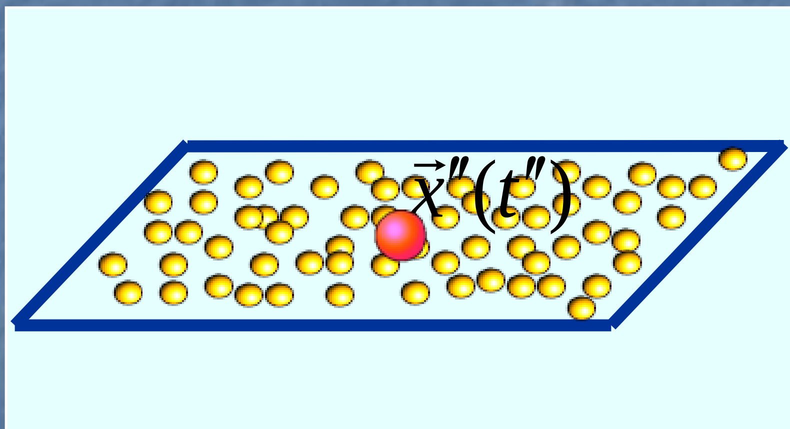
Changing to Rindler coordinates where quark is static, CFT metric takes the form

$$ds^2_{\mathrm{CFT}} = e^{2Ax'}\left(-dt'^2 + dx'^2\right) + d\vec{x}'^2_{\perp}$$

Both in CFT and AdS, find acceleration horizon at

$$x' = -\infty \quad \forall\; z', \vec{x}'_{\perp}$$

Unruh en AdS/CFT



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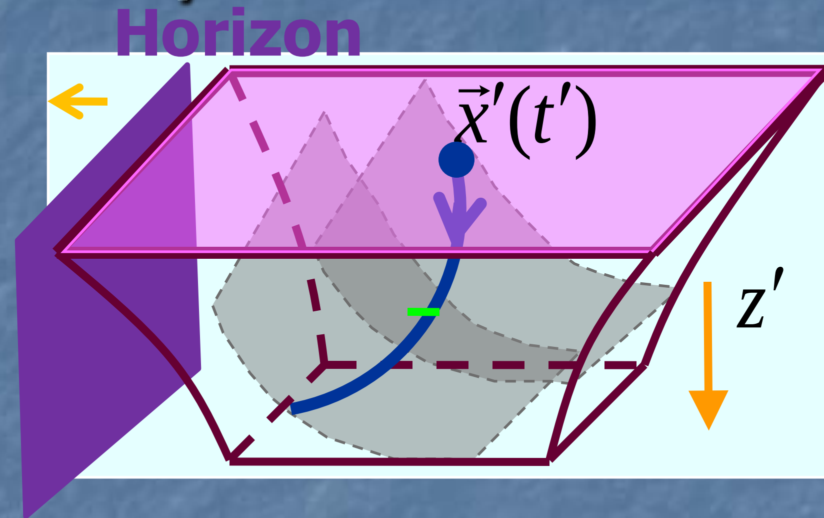


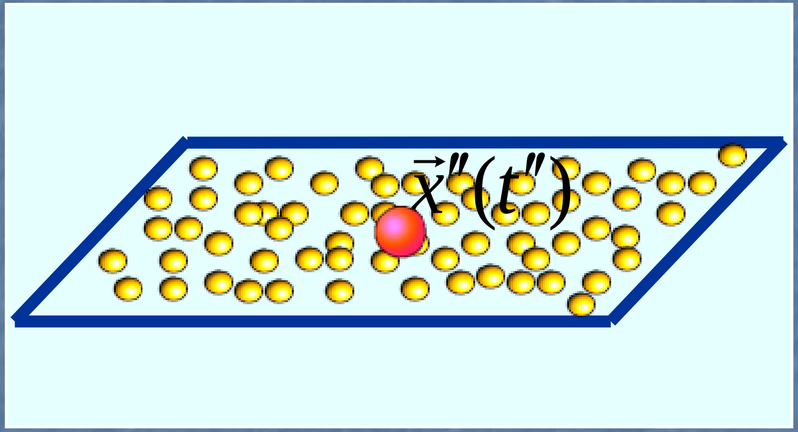
Can remove horizon from CFT by via **Weyl transformation**

$$ds^2_{\mathrm{CFT}} = e^{2Ax'}\left(-dt'^2 + dx'^2\right) + d\vec{x}'^2_{\perp} \quad \rightarrow \quad \underbrace{-dt''^2 + dx''^2 + e^{-2Ax''}d\vec{x}''^2_{\perp}}_{\text{open Einstein Universe}}$$

This corresponds to a change of radial foliation in the bulk

{Kraus}... [Imbimbo,Schwimmer,Theisen,Yankielowicz]

Unruh in AdS/CFT



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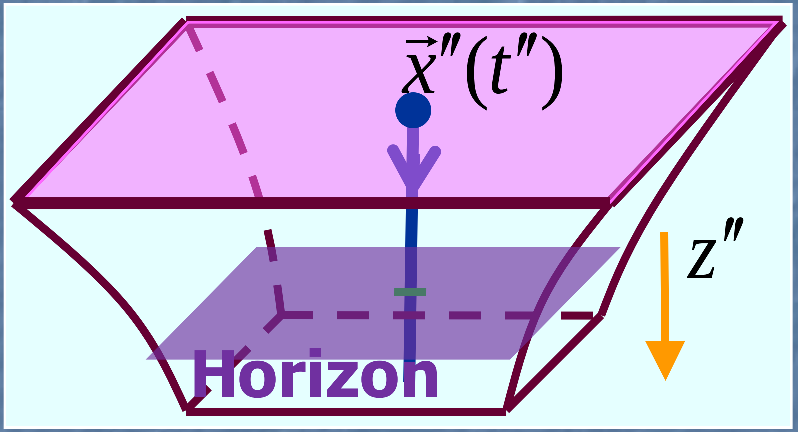


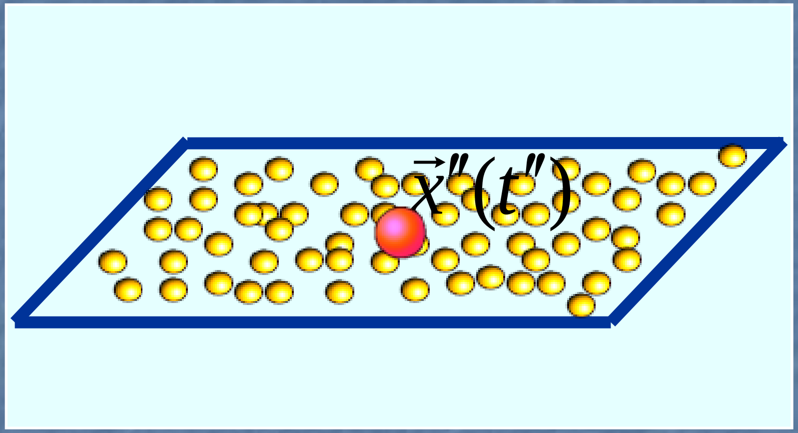
In this new presentation of the bulk, the string is vertical, and the bulk horizon lies at a fixed radius

$$z''_h = A^{-1} = 2\pi / T_U$$

indicating that thermal medium is still present

Setup coincides **precisely** with that of (3-dim) Brownian motion calculation of [de Boer,Hubeny,Rangamani,Shigemori] !

Unruh in AdS/CFT



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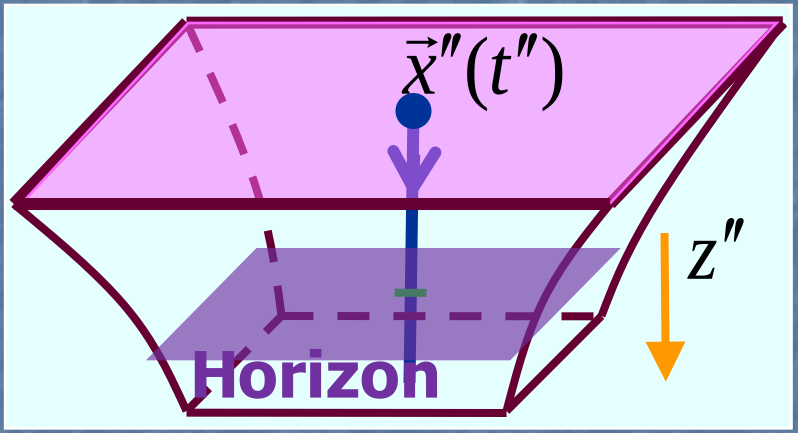

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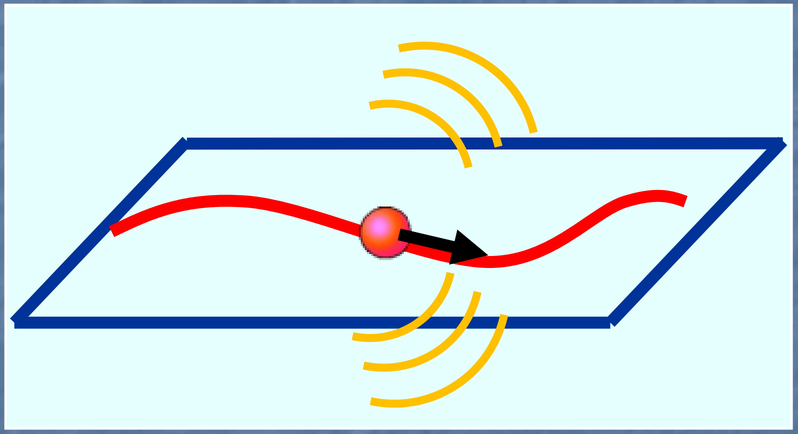
$$z''_h = A^{-1} = 2\pi / T_U$$

indicating that thermal medium is still present

- From the CFT perspective, medium now arises not from entanglement with region behind a horizon, but from direct application of a temperature (analogous to familiar Schw-AdS case)

Radiation Damping

Going back to the classical description of the quark/string, note that the question of energy loss in a **strongly-coupled non-Abelian** gauge theory is interesting already in **vacuum** (where AdS/CFT gives us full *analytic* control)



Expect accelerating quark to radiate, and experience **damping force** due to emitted radiation

Radiation Damping

In classical E&M, Abraham-Lorentz equation (NR electron)

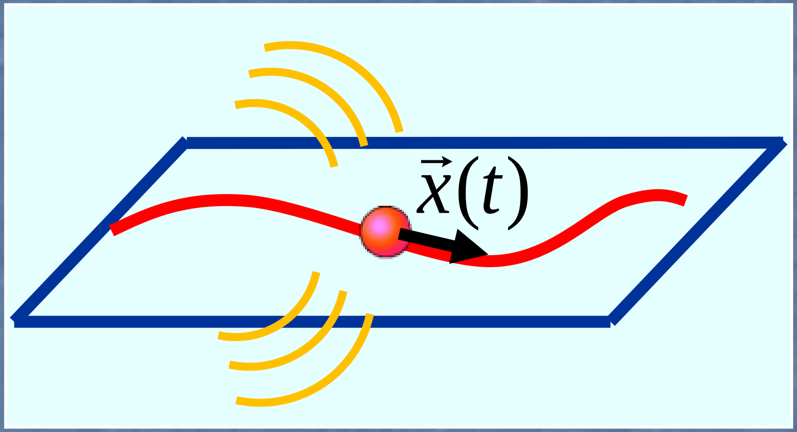
$$m\left(\frac{d^2\vec{x}}{dt^2} - \underbrace{t_e\frac{d^3\vec{x}}{dt^3}}_{\text{damping term}}\right) = \vec{F} \qquad t_e \equiv \frac{2e^2}{3mc^3}$$

~ classical electron radius

and (Abraham-)Lorentz-Dirac equation

$$m\left(\frac{d^2x^\mu}{d\tau^2} - t_e\left[\underbrace{\frac{d^3x^\mu}{d\tau^3}}_{\text{Schott (near field) term}} - \underbrace{\frac{1}{c^2}\frac{d^2x_\nu}{d\tau^2}\frac{d^2x^\nu}{d\tau^2}\frac{dx^\mu}{d\tau}}_{\text{radiation reaction term}}\right]\right) = \mathscr{F}^\mu$$

Radiation Damping



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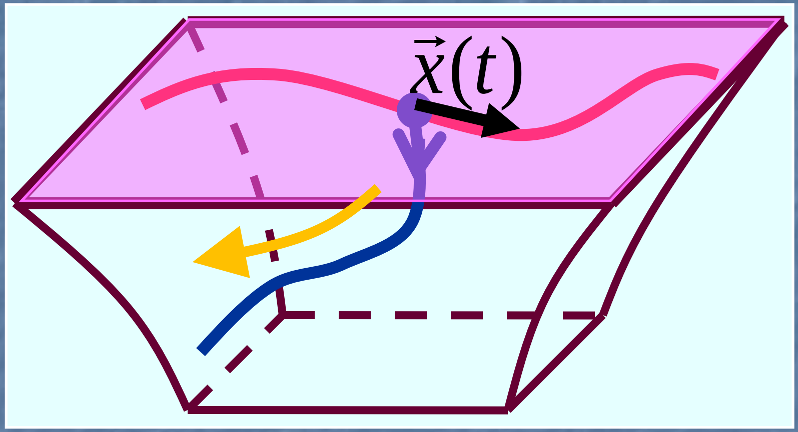

For **accelerating** quark, dual string trails behind endpoint , and acts as an energy sink

i.e., quark has a 'tail', and it is this tail that is responsible for damping effect

In fact, standard boundary condition for open string can be easily shown to take the form of an equation of motion for the quark…

Generalized Lorentz-Dirac Eqn

E.o.m. for dressed quark:

$$\frac{d}{d\tau}\left(\frac{m\dfrac{dx^\mu}{d\tau}-\dfrac{\sqrt{\lambda}}{2\pi m}\mathcal{F}^\mu}{\sqrt{1-\dfrac{\lambda}{4\pi^2 m^4}\mathcal{F}^2}}\right)=\frac{\mathcal{F}^\mu-\dfrac{\sqrt{\lambda}}{2\pi m^2}\mathcal{F}^2\dfrac{dx^\mu}{d\tau}}{1-\dfrac{\lambda}{4\pi^2 m^4}\mathcal{F}^2}$$

[Chernicoff, García, AG]

Recall $z_m=\dfrac{\sqrt{\lambda}}{2\pi m}$ plays the role of Compton wavelength

Generalized Lorentz-Dirac Eqn

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[Chernicoff, García, AG]

When $\frac{\sqrt{\lambda}}{2\pi m^2}\sqrt{|\mathcal{F}^2|} \ll 1$, can ignore denominators…

Generalized Lorentz-Dirac Eqn

To zeroth order in $\frac{\sqrt{\lambda}}{2\pi m^2}\sqrt{\left|\mathcal{F}^2\right|} << 1$, recover $m\frac{d^2x^\mu}{d\tau^2} \simeq \mathcal{F}^\mu$

To first order,

$$\frac{d}{d\tau}\left(m\frac{dx^\mu}{d\tau} - \frac{\sqrt{\lambda}}{2\pi m}\mathcal{F}^\mu\right) \simeq \mathcal{F}^\mu - \frac{\sqrt{\lambda}}{2\pi m^2}\mathcal{F}^2\frac{dx^\mu}{d\tau}$$

which is equivalent to

$$m\left(\frac{d^2x^\mu}{d\tau^2} - \frac{\sqrt{\lambda}}{2\pi m}\frac{d^3x^\mu}{d\tau^3}\right) \simeq \mathcal{F}^\mu - \frac{\sqrt{\lambda}}{2\pi}\left(\frac{d^2x^\nu}{d\tau^2}\frac{d^2x_\nu}{d\tau^2}\right)\frac{dx^\mu}{d\tau}$$

Exact same structure as Lorentz-Dirac (with $t_e \to z_m$) !

Generalized Lorentz-Dirac Eqn

To second order in $\frac{\sqrt{\lambda}}{2\pi m^2}\sqrt{\left|\mathcal{F}^2\right|} << 1$, similarly obtain

$$m\frac{d^2x^\mu}{d\tau^2}-\frac{\sqrt{\lambda}}{2\pi}\left(\frac{d^3x^\mu}{d\tau^3}-\frac{d^2x_\nu}{d\tau^2}\frac{d^2x^\nu}{d\tau^2}\frac{dx^\mu}{d\tau}\right)$$

$$+\frac{\lambda}{4\pi^2 m}\left(\frac{d^4x^\mu}{d\tau^4}-\left(1+2\right)\frac{d^2x_\nu}{d\tau^2}\frac{d^3x^\nu}{d\tau^3}\frac{dx^\mu}{d\tau}\right)$$

$$-\frac{\lambda}{4\pi^2 m}\left(\frac{1}{2}+1\right)\frac{d^2x_\nu}{d\tau^2}\frac{d^2x^\nu}{d\tau^2}\frac{d^2x^\mu}{d\tau^2}\simeq\mathcal{F}^\mu$$

Generalized Lorentz-Dirac Eqn

To second order in $\frac{\sqrt{\lambda}}{2\pi m^2}\sqrt{\left|\mathcal{F}^2\right|} << 1$, similarly obtain

$$m\frac{d^2x^\mu}{d\tau^2} - \frac{\sqrt{\lambda}}{2\pi}\left(\frac{d^3x^\mu}{d\tau^3} - \frac{d^2x_\nu}{d\tau^2}\frac{d^2x^\nu}{d\tau^2}\frac{dx^\mu}{d\tau}\right)$$

$$+\frac{\lambda}{4\pi^2 m}\left(\frac{d^4x^\mu}{d\tau^4} - (1+2)\frac{d^2x_\nu}{d\tau^2}\frac{d^3x^\nu}{d\tau^3}\frac{dx^\mu}{d\tau}\right)$$

$$-\frac{\lambda}{4\pi^2 m}\left(\frac{1}{2}+1\right)\frac{d^2x_\nu}{d\tau^2}\frac{d^2x^\nu}{d\tau^2}\frac{d^2x^\mu}{d\tau^2} \simeq \mathcal{F}^\mu$$

radiation reaction terms

Generalized Lorentz-Dirac Eqn

To second order in $\dfrac{\sqrt{\lambda}}{2\pi m^2}\sqrt{\left|\mathcal{F}^2\right|} << 1$, similarly obtain

$$m\frac{d^2x^\mu}{d\tau^2}-\frac{\sqrt{\lambda}}{2\pi}\left(\frac{d^3x^\mu}{d\tau^3}-\frac{d^2x_\nu}{d\tau^2}\frac{d^2x^\nu}{d\tau^2}\frac{dx^\mu}{d\tau}\right)$$

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$$-\frac{\lambda}{4\pi^2 m}\left(\frac{1}{2}+1\right)\frac{d^2x_\nu}{d\tau^2}\frac{d^2x^\nu}{d\tau^2}\frac{d^2x^\mu}{d\tau^2}\simeq\mathcal{F}^\mu$$

near field terms

Generalized Lorentz-Dirac Eqn

Full e.o.m. for dressed quark

$$\frac{d}{d\tau}\left(\frac{m\frac{dx^\mu}{d\tau}-\frac{\sqrt{\lambda}}{2\pi m}\mathcal{F}^\mu}{\sqrt{1-\frac{\lambda}{4\pi^2 m^4}\mathcal{F}^2}}\right)=\frac{\mathcal{F}^\mu-\frac{\sqrt{\lambda}}{2\pi m^2}\mathcal{F}^2\frac{dx^\mu}{d\tau}}{1-\frac{\lambda}{4\pi^2 m^4}\mathcal{F}^2}$$

is non-linear generalization of Lorentz-Dirac eqn

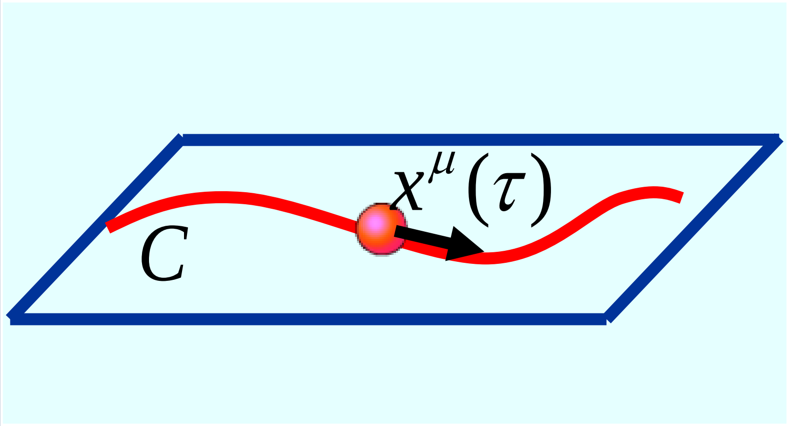
(And, unlike Lorentz-Dirac, has no self-accelerating solutions)

Wilson Loops

The quark=string entry leads to a recipe for computing Wilson loops (*non-local* gauge-invariant operators):

$$\left\langle \mathrm{Tr}\left[P\exp\left(i\oint_C dx^\mu A_\mu(x) + \ldots \right)\right]\right\rangle_{\mathrm{SYM}} = \int_{X(r=\infty)=C} DX \exp\left[iS_{\mathrm{string}}[X]\right]$$

[Rey,Yee; Maldacena; Drukker,Gross,Ooguri]



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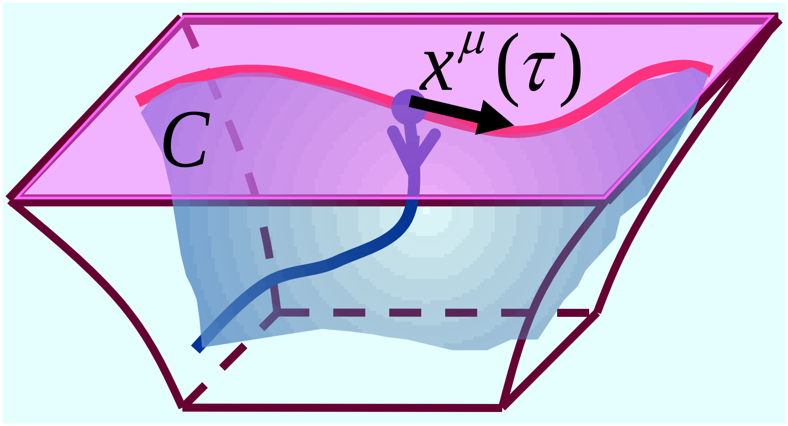

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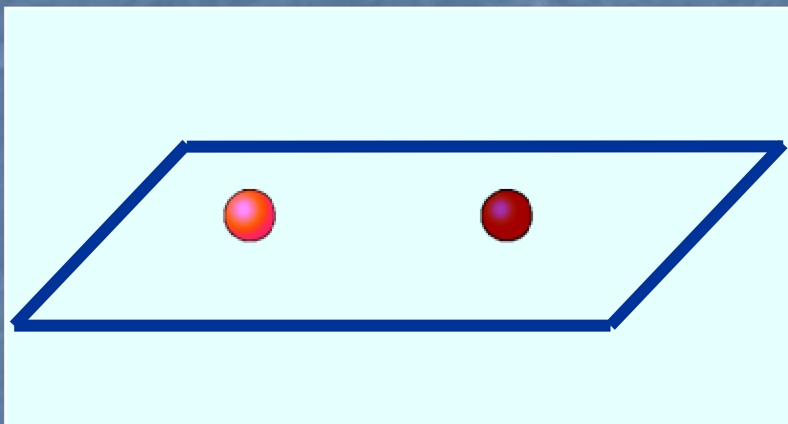
$$= \exp\left[iS^{\text{on-shell}}_{\mathrm{string}}[X]\right]$$

in $N_c \to \infty$, $g^2_{YM} N_c \to \infty$ limit

(The trace in LHS above is taken in the **fundamental** rep of the gauge group. For traces in other reps, RHS involves strings bound to D3- or D5-branes)

[Drukker,Fiol; Hartnoll,Prem Kumar; Yamaguchi; Gomis,Passerini]

Quark-Antiquark Potential



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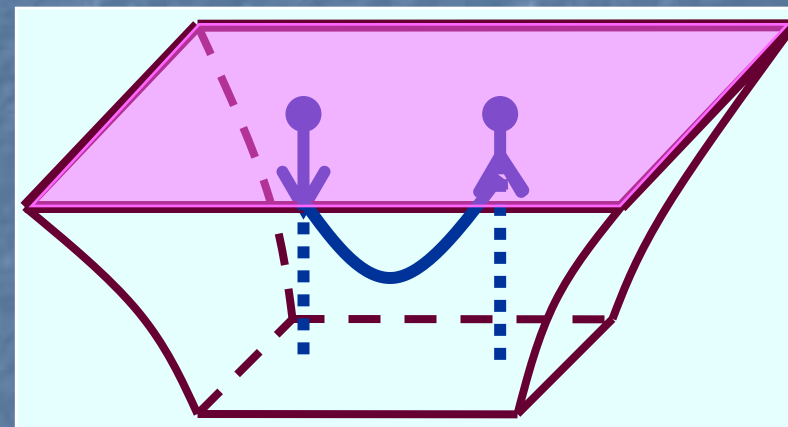
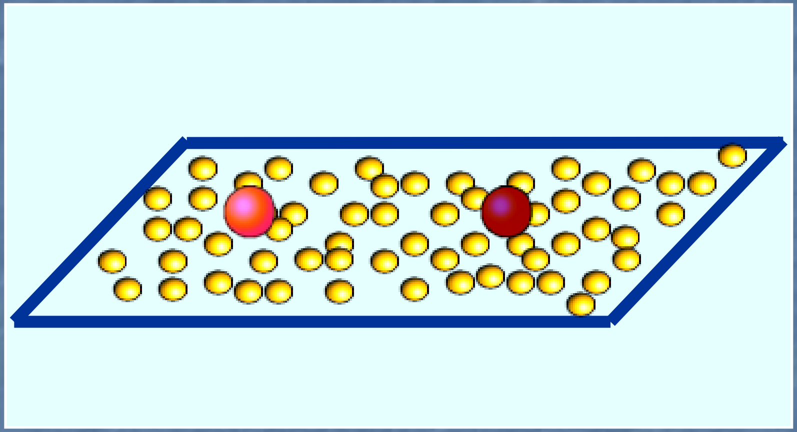


Quark-Antiquark = 1 String w/BOTH endpoints at $r \to \infty$

$$V_{q\bar{q}}^{T=0}(L) = -\frac{4\pi^2\sqrt{g_{YM}^2 N}}{\Gamma(\frac{1}{4})^4 L}$$

[Rey,Yee; Maldacena]

Screening from AdS/CFT



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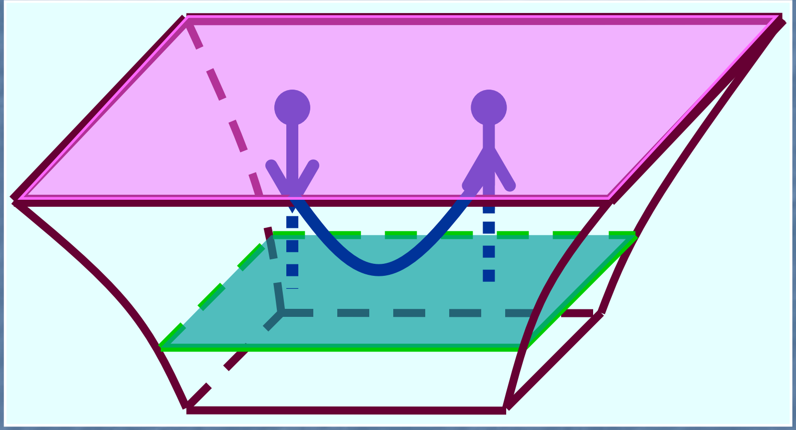
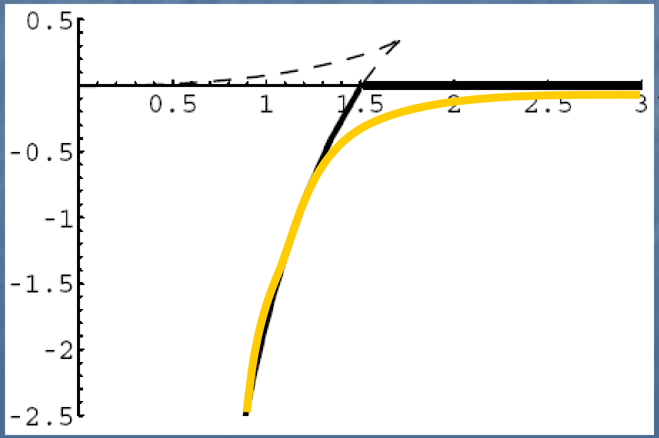


$V_{q\bar{q}}^{T}(L)\left[\sqrt{g_{YM}^{2}N}T/4\right]$

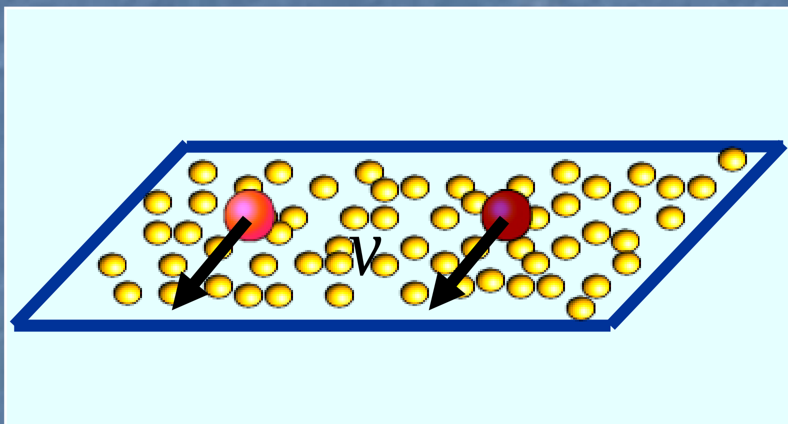


$L\left[1/2\pi T\right]$

[Rey,Theisen,Yee; Brandhuber,Itzhaki,Sonnenschein,Yankielowicz]

[Bak,Karch,Yaffe]

Screening from AdS/CFT



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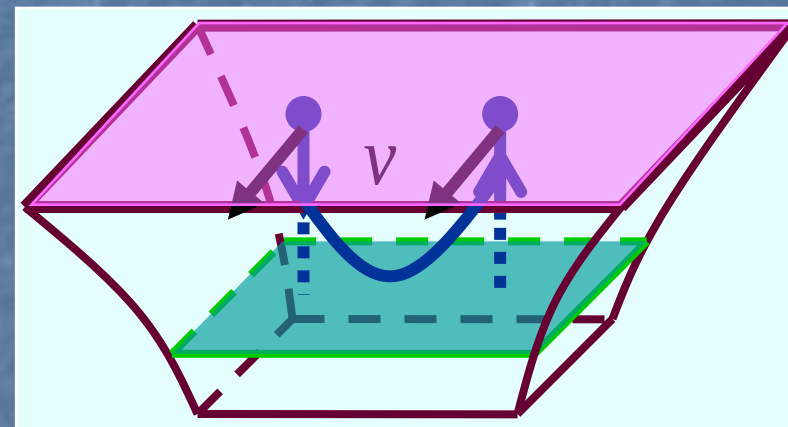


Meson moving through SYM plasma

U-shaped string moving on AdS black hole geometry

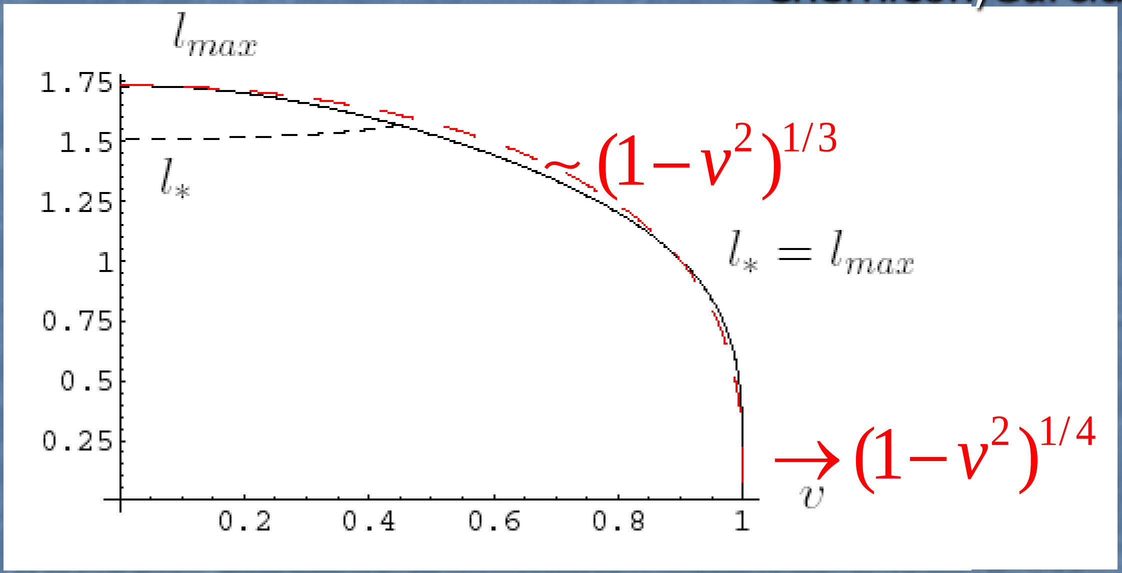
$$V_{q\bar{q}}^{T}(L,v)$$

[Chernicoff,García,AG; Liu,Rajagopal,Wiedemann]

Screening Length

$L_s(T,v)\ [1/2\pi T]$

[Liu,Rajagopal,Wiedemann;
Chernicoff,García,AG]



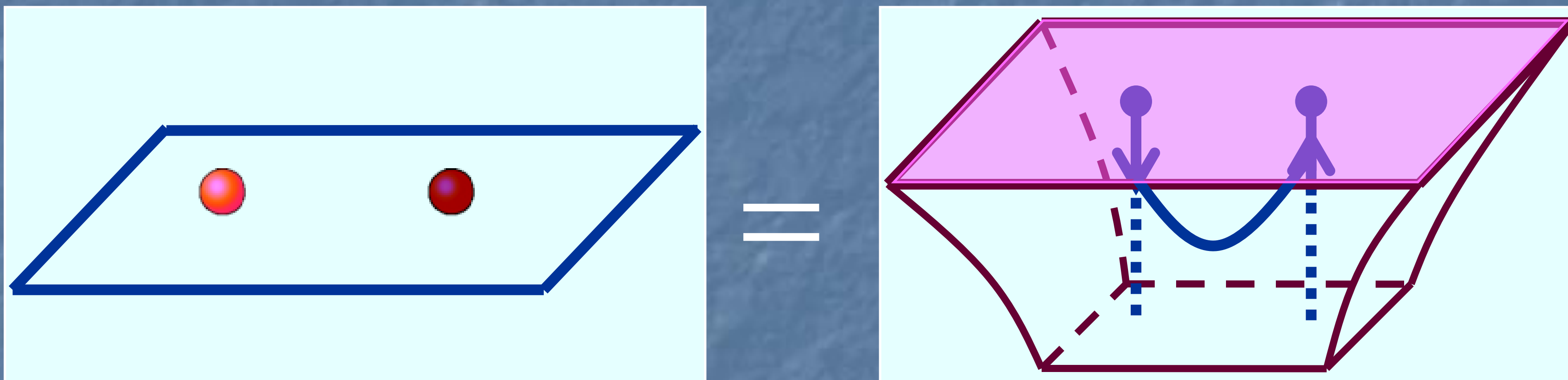
Possibly relevant for J/psi suppression [Matsui,Satz]

$$T_{\text{diss}} \propto (1-v^2)^p$$

[Liu,Rajagopal,Wiedemann;
Cáceres,Natsuume,Okamura]

(Suppression enhanced for charmonium w/larger p_T)

Meson Spectrum



Can also determine **microscopic** meson spectrum, e.g.

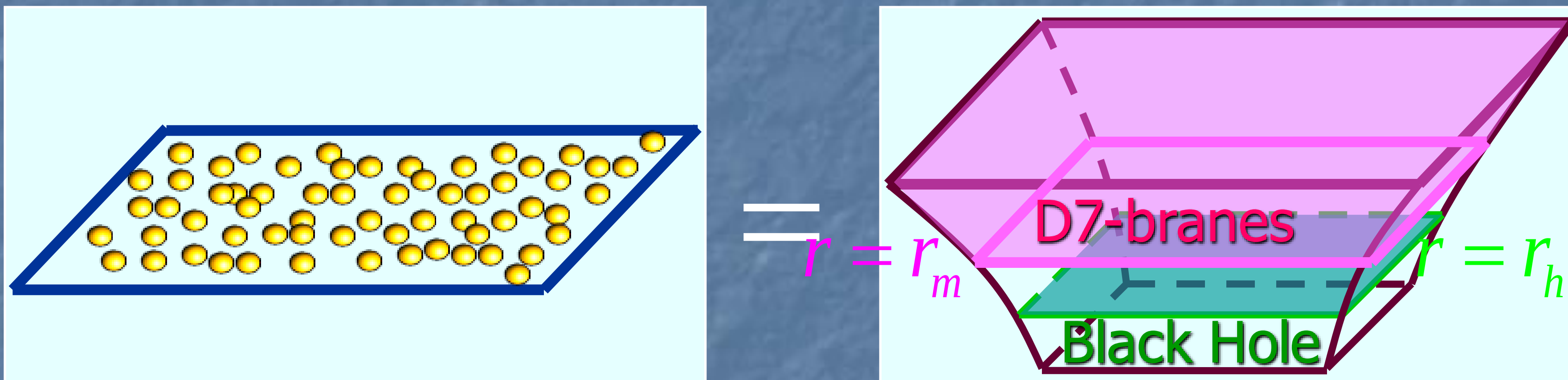
$$M_s = \frac{2\pi m_q}{\sqrt{g_{YM}^2 N}}\sqrt{(n+l+1)(n+l+2)}$$ for scalar mesons

[Kruczenski,Mateos,Myers,Winters]

Notice that $M_s \ll m_q$: **mesons** are lightest d.o.f.

(Can in fact recover quark as a soliton made of mesons!)

Meson Spectrum at Finite T



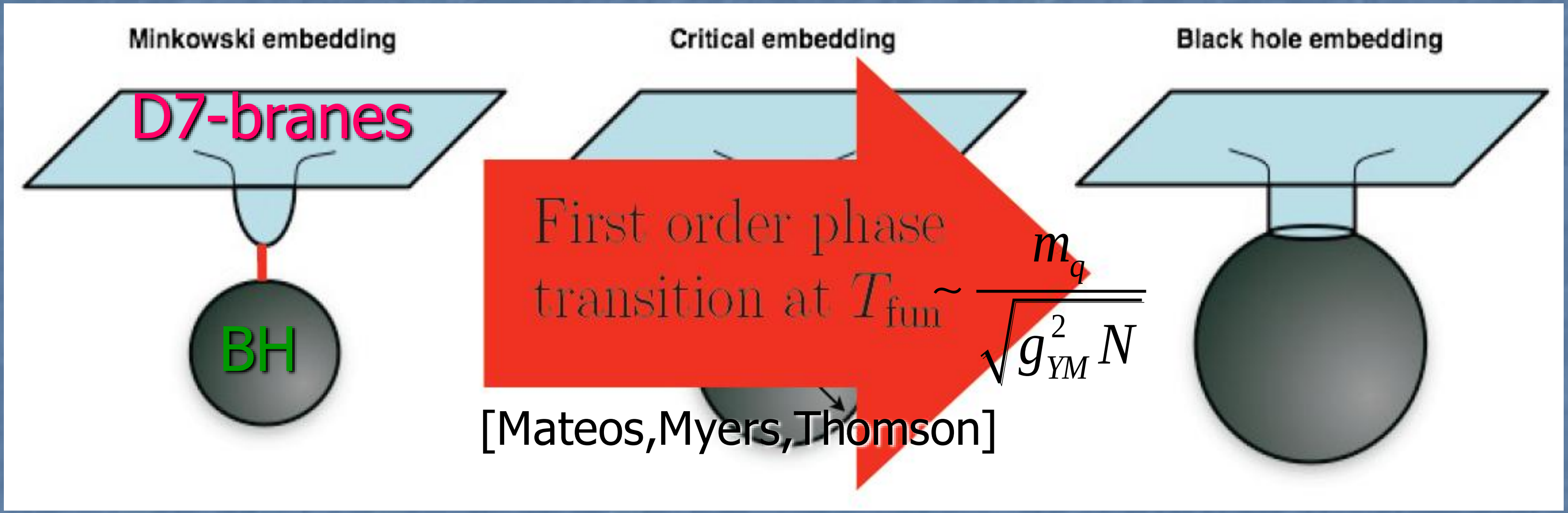
r_m related to quark mass m

r_h proportional to temperature T

For large enough T/m, the D7-branes end INSIDE BH

Meson Spectrum at Finite T

View omitting SYM directions but including S^5 :



Discrete meson spectrum

$$M_{\text{mes}} \sim T_{\text{fun}}$$

(stable: survive deconfinement)

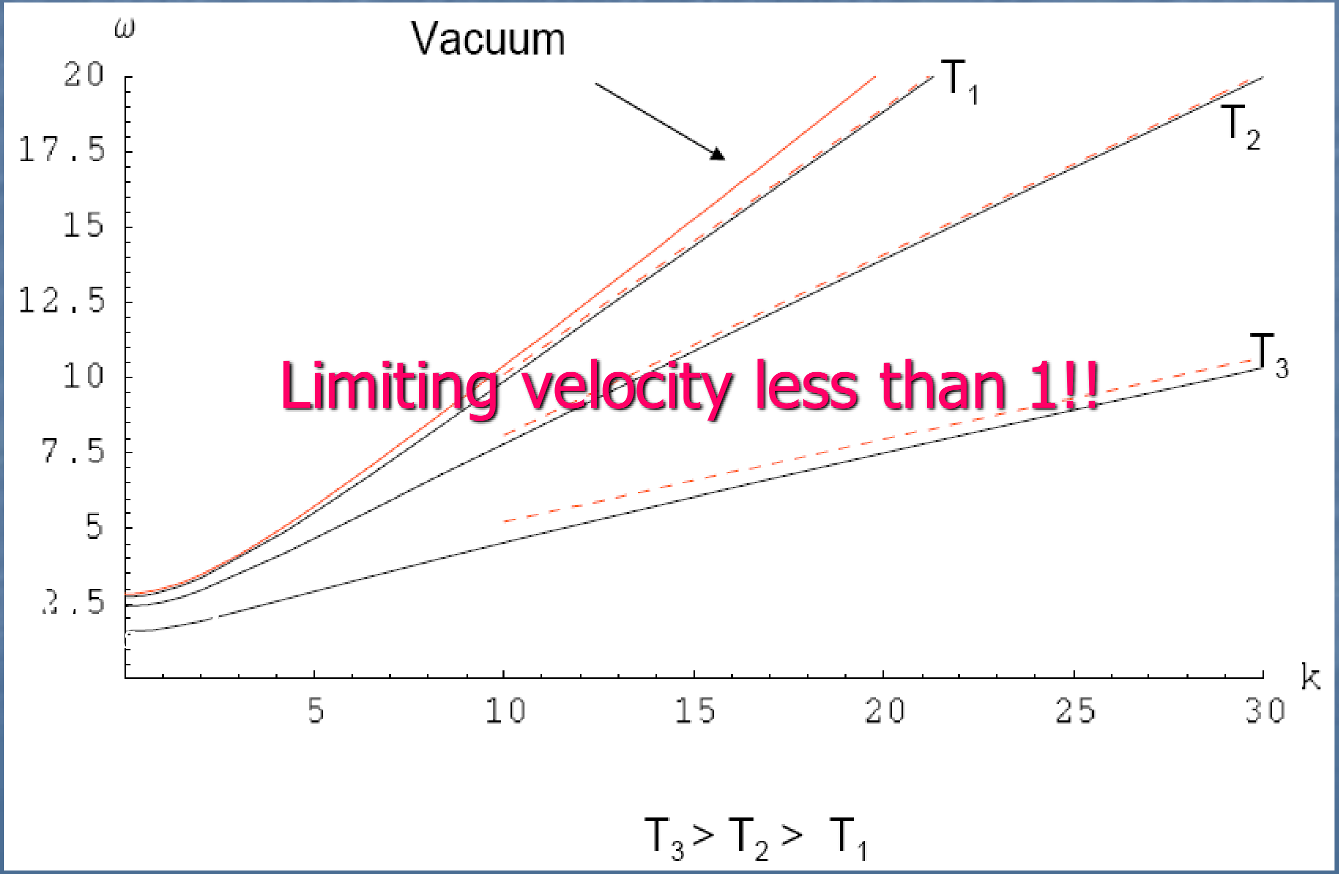
+ Massive quarks

Continuous spectrum...
with NO quasi-particles!!

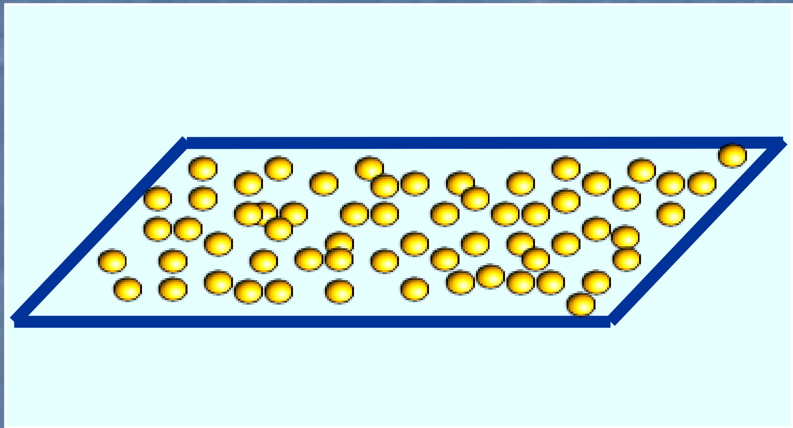
Meson Spectrum at Finite T

Meson dispersion relations at increasing $T < T_{\text{fun}}$:

[Mateos,Myers,Thomson; Ejaz,Faulkner,Liu,Rajagopal,Wiedemann]



Meson Spectrum at Finite T



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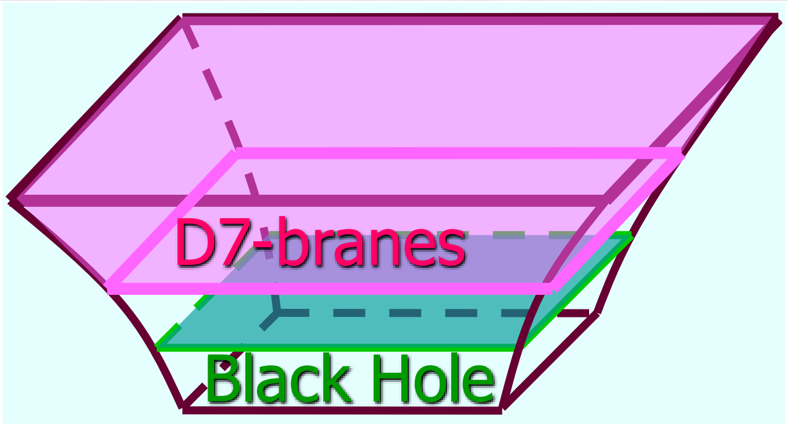
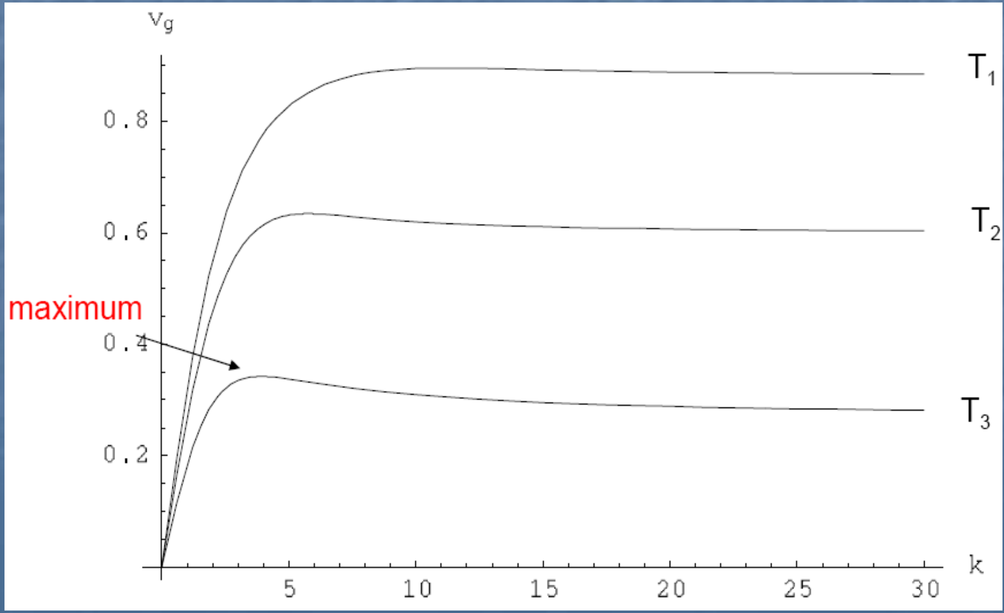




Origin: local speed of light at edge of D7-branes,

$$v_{\max} = \sqrt{1-(z_m / z_h)^4}$$

$$\approx 1-\left(\sqrt{\lambda}T / m_q\right)^4$$

[Argyres,Edalati,Vázquez-Poritz]

Generalizations

What we’ve discussed so far is the best understood example of more general **gauge/gravity (or gauge/string) correspondence** which can involve backgrounds w/different asymptotics

Various other examples are known, relating certain gauge theories (some of which are more`QCD-like’) to string theory on different curved spacetimes…

[Sakai-Sugimoto(-Witten); Klebanov-Strassler; Maldacena-Núñez; Polchinski-Strassler; Freedman-Gubser-Pilch-Warner; etc.]

Other SYM Geometries

For starters: since SYM is conformal/Weyl invariant (up to Weyl anomaly), can change from Minkowski to a different (nondynamical) 3+1 geometry via Weyl transformation

$$\eta_{\mu\nu} \to g_{\mu\nu}(x) = e^{2\omega(x)}\eta_{\mu\nu}$$

On string side, this corresponds to a bulk diffeo {Kraus}

$$z \to z'' = e^{\omega(x)} z \qquad x^{\mu} \to x''^{\mu}(x,z)$$

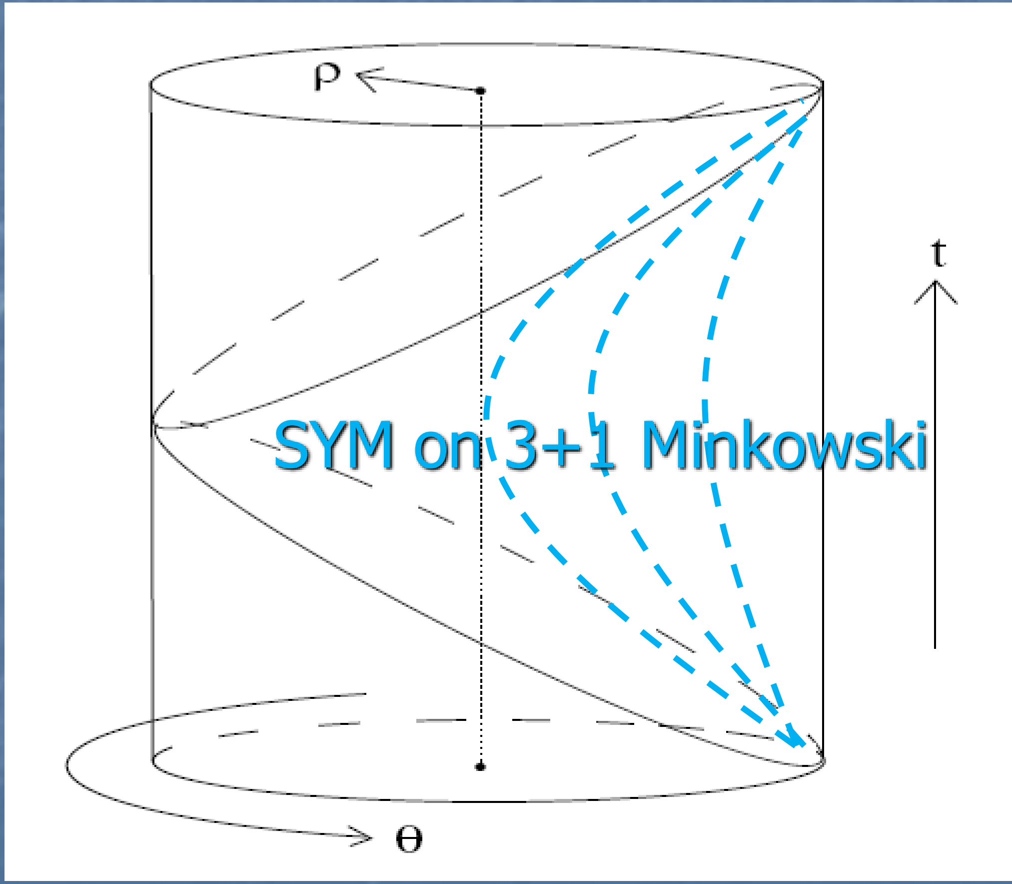
which modifies the **radial foliation**

[Witten;Imbimbo,Schwimmer,Theisen,Yankielowicz]

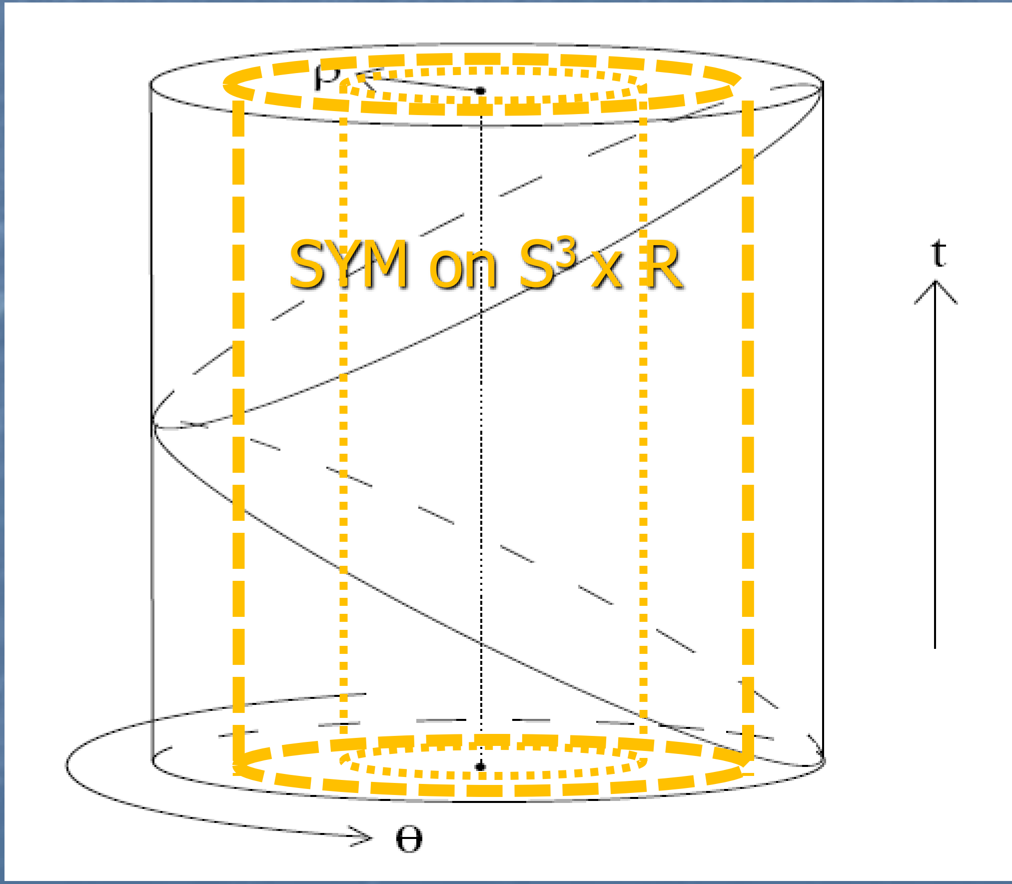
SYM on Minkowski arises from **Poincaré foliation** of AdS, but different foliations of the *same* geometry can lead, e.g., to SYM on the closed or open Einstein static universe

[Maldacena; Witten; Emparan; Birmingham; Emparan,Johnson,Myers; etc.]

Other SYM Geometries



Other SYM Geometries



Other SYM Geometries

SYM on S^3 x R at finite T has confining/deconfining phase transition, which turns out to be dual to Hawking-Page transition in AdS!
Deconfined phase = large *spherical* Schw-AdS BH

[Witten]

Weyl anomaly of SYM (quantum effect due to UV divergences) is perfectly reproduced by AdS/CFT prescription for energy-momentum tensor (classical effect due to IR divergences)!! {Kraus}

[Witten; Henningson,Skenderis; etc.]

Other AdS/CFT Examples

We got to $AdS_5 \times S^5$ by considering D3-branes on 9+1 Minkowski, $M^{9,1}$. If we start instead with D3-branes on certain other 9+1 geometries, we can arrive at Type IIB String Theory on $AdS_5 \times X^5$

finding this to be dual to a 3+1 dim theories distinct from SYM. These are CFTs (b/c of AdS factor) with fewer or no supersymmetries (b/c no longer have S^5)

[Klebanov,Witten;etc.]

It's also possible to engineer CFTs in other dimensions by using string theory objects other than D3-branes, e.g.,

e.g.,	$d=2$	$d=3$	$d=4$	$d=6$
from	D1-D5	M2	D3	M5

[Maldacena;etc.]

Gauge/Gravity Correspondence

We also know various ways to obtain "nonAdS/nonCFT" correspondences (again w/various amounts of susy):

Can consider D*p*-branes (or other branes) on Minkowski, for other values of *p*

[Itzhaki,Maldacena,Sonnenschein,Yankielowicz; Boonstra,Skenderis,Townsend;etc.]

Can deform known AdS/CFT examples adding relevant terms to the Lagrangian (=switching on non-normalizable part of dual bulk fields). In these cases CFT starting point is UV fixed point

[Polchinski-Strassler; Girardello,Petrini,Porrati,Zaffaroni; Freedman-Gubser-Pilch-Warner; etc.]

[Sakai-Sugimoto(-Witten); Klebanov-Strassler; Maldacena-Núñez; etc.]

Gauge/Gravity Correspondence

We also know various ways to obtain "nonAdS/nonCFT" correspondences (again w/various amounts of susy):

- Consider D*p*-branes (or other branes), for other values of *p*. For 3+1 theory, can start w/ *p*>3 and compactify

  [Itzhaki,Maldacena,Sonnenschein,Yankielowicz; **Maldacena-Núñez**; Kruczenski,Mateos,Myers,Winters; **Sakai-Sugimoto(-Witten);** etc.]

- Deform known AdS/CFT examples adding IR-relevant terms to the Lagrangian (=switching on non-normalizable part of dual bulk fields). Original CFT is then UV fixed point

  [**Polchinski-Strassler**; Girardello,Petrini,Porrati,Zaffaroni; Pilch-Warner; Freedman-Gubser-Pilch-Warner; Johnson,Peet,Polchinski; etc.]

- Deform previous setups to yield different asymptotics

  [**Klebanov-Strassler**; etc.]

Gauge/Gravity Correspondence

Among these generalizations we find QCD-like gauge theories, with confinement and chiral symmetry breaking

What comes closest to QCD is the **Sakai-Sugimoto(-Witten) model**, which uses D4-branes + D8-branes + anti-D8-branes to obtain a non-CFT & non-SUSY theory with quarks, confinement and non-Abelian chiral symmetry breaking

By dialing a continuous dimensionless parameter in this model, we would precisely obtain QCD

(Sakai-Sugimoto is also connected to the (pheno-menologically useful) Nambu-Jona-Lasinio model)

[Antonyan,Harvey,Jensen,Kutasov]

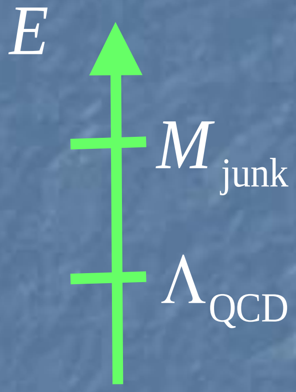
Gauge/Gravity Correspondence

Finite temperature studies show that, in the regime of SS where 4-fermion interactions become important, the confinment and chiral-symm-breaking transitions no longer coincide [Aharony,Sonnenschein,Yankielowicz; Parnachev,Sahakyan]

This separation has been later verified w/lattice calculation [Sinclair]

Unfortunately, moving towards QCD takes us outside the supergravity approximation, so we lose ability to compute (conversely, in supergravity regime where we can calculate, SS is truly 4+1 dim at scale of Λ_{QCD})

Gauge/Gravity Correspondence



This is generic: in all cases we get QCD (or YM) + junk , and the latter CANNOT be ignored within supergravity description

Basic problem is that UV region in QCD is asymptotically free, which translates into strongly-curved, and therefore highly stringy, geometry

Another way to state the problem: validity of supergravity requires large separation between masses of spin J=0,1,2 vs. J>2 modes. QCD is not like that.

Gauge/Gravity Correspondence

Nevertheless, we can use the known examples as toy models to develop intuition. This is interesting and useful in its own right.

Armed with that, to move toward QCD we can:

*Build **phenomenological** models that attempt to bridge the gap between our toy models and real-world QCD,

*Search for appropriately **universal** quantities, which allow us to extrapolate to QCD

There's similar very interesting current work on possible applications to a variety of condensed matter and atomic physics systems

Gauge/Gravity Correspondence

- The approach we've discussed so far is "top-down": it starts with a known string/brane construction, and therefore gives us some control over what the theory on each side of the correspondence should be
- One can also try to follow a "bottom-up" approach, where one starts with a gauge theory of interest and tries to guess a phenomenological realization in terms of a gravity dual ("AdS/QCD")…

  [Polchinski,Strassler; Erlich,Katz,Son; Da Rold,Pomarol; Karch,Katz,Son,Stephanov; etc.]

- … or, starts with a given geometry (not necessarily string-related) and study the properties of the putative field theory dual (dS/CFT, Kerr/CFT, AdS/CMT,…)

  [Strominger; Strominger; Son; Balasubramanian,McGreevy; Kachru,Liu,Mulligan; Gubser; Hartnoll,Herzog,Horowitz; etc.]

Conclusions

1) AdS/CFT is an **efficient tool** for calculations in certain strongly-coupled gauge theories. It is an established *theoretical* tool, and already makes suggestions for phenomenological models
2) The sQGP produced at RHIC/LHC, and some strongly-coupled condensed matter / atomic physics systems, appear to be the **most promising sites** for **eventually** obtaining firm experimental predictions from AdS/CFT (& string theory)...But a lot remains to be done!
3) AdS/CFT is also been used in the opposite direction, where it gives some interesting suggestions on the problem of quantum gravity. Key idea: **Holography**

   ['t Hooft; Susskind] (See, e.g. [Horowitz,Polchinski; Horowitz])

   [Berenstein; Lin,Lunin,Maldacena; Mathur; Bena,Warner; Kraus,Larsen; Berenstein; Yamaguchi; Balasubramanian,de Boer,Jejjala,Simon; Grant et al.; Balasubramanian,Czech,Larjo,Marolf,Simon; Mandal; etc.]