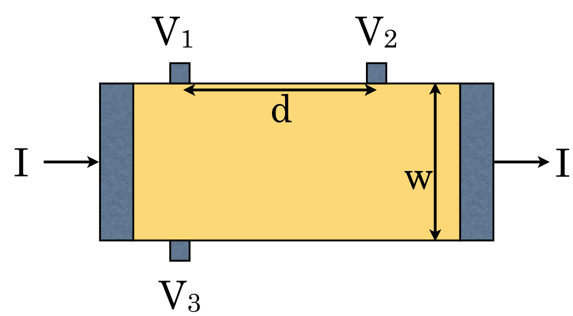
Physics 123B: Final
Due March 14, 2013, 9:30AM

1. Consider the model band structure of graphene from class. Suppose electrons are added to graphene so that the Fermi energy increases from $\epsilon_F = \epsilon_0$ to $\epsilon'_F = \epsilon_0 + \gamma$, where γ is the tight-binding matrix element introduced in class. Find and plot the Fermi surface. How many additional electrons have been added to the material (compared to neutral graphene), per carbon atom?

2. A two-dimensional electron gas is patterned into the Hall bar shown in the figure below. The gray rectangles on the ends represent current leads, and the gray squares contacts connected to a voltmeter. A large magnetic field is applied (oriented "up" out of the plane) at very low temperature, so that all electrons are spin polarized and the Fermi level lies between the lowest and first excited Landau level. What is the voltage difference $V_1 - V_3$ and $V_2 - V_3$?



3. A rotational flow is set up in a superfluid confined between two cylinders. What physical process determines the decay rate of the superflow?

4. Why is heat conduction very efficient in superfluid helium but very poor in a superconductor?

5. Two Josephson junctions are fabricated identically, except that the barrier region in junction A is twice as thick as that in junction B. Does A or B have the larger critical current? Why?

6. Find the expected magnetic states in free space for Ni^{2+}, Ni^{3+}, and Ni^{4+} ions, assuming that the 4s electrons are the first to be removed from the free Ni atom. Give the S, L, and J quantum numbers for each case.

7. Give two ways that the exchange coupling J can be measured in a ferromagnet.

8. Formulate mean field theory for the Heisenberg *antiferromagnet* on the cubic lattice, i.e. $H = +|J| \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j$, for $S = 1/2$ spins. In the first two parts, you will need to assume that $\langle \mathbf{S}_i \rangle = +\mathbf{n}$ when $x_i + y_i + z_i$ is an even multiple of the lattice spacing a, and $\langle \mathbf{S}_i \rangle = -\mathbf{n}$, when $x_i + y_i + z_i$ is an odd multiple of the lattice spacing a.

(a) Find the self-consistent equation for $n = |\mathbf{n}|$.

(b) Find the critical, or Néel, temperature T_N.

(c) Find the uniform susceptibility, $\chi = \left.\frac{\partial m^z}{\partial H_z}\right|_{H=0}$, for $T > T_N$. Here H_z is an applied uniform external field along the z axis, and $m^z = \frac{1}{N}\sum_i g\mu_B S_i^z$ is the z-component of the magnetization. You will need to assume here that the average values of the spins on the two sublattices are no longer opposite.

9. Find the energy versus wavevector dispersion relations for the *two branches* of spin wave modes of the spin-S Heisenberg ferromagnet on the honeycomb lattice.