Physics 123B: Homework 1
due January 18, 5pm in the box at the PSR

1. **Free electron Fermi surface:** Consider a hypothetical two dimensional solid with a single type of atom on a hexagonal lattice, with lattice spacing a. Suppose there are two electrons per atom.

   (a) What is the radius of the free electron circular Fermi surface in reciprocal space?

   (b) Draw the free electron Fermi surfaces in the reduced zone scheme.

   (c) Draw the same Fermi surfaces in the extended or periodic zone scheme.

   (d) What is the area of the electron and hole pockets – i.e. the density of electrons and holes?

2. **Cuprate Fermi surface:** The important energy band of the cuprate high-temperature superconducting materials can be modeled by considering a tight binding band of a single orbital per site of a square lattice, with lattice spacing a. Assuming a hopping γ between nearest neighbor sites and a hopping γ' between second-neighbor sites, which are on opposite corners of the squares (i.e. separated by $\sqrt{2}a$ distance).

   (a) Show/argue that the energy band is described by the function

   $$\epsilon(k) = -2\gamma(\cos k_x a + \cos k_y a) - 2\gamma'(\cos(k_x + k_y)a + \cos(k_x - k_y)a). \qquad (1)$$

   (b) Suppose the Fermi energy (with the above definition of the zero of energy) is zero and $\gamma' = 0$. What is the density of electrons, per site?

   (c) Take $\gamma = 1$ and $\gamma' = -0.1$. Sketch the Fermi surfaces for the Fermi energy $= 0, -0.2, -0.4, -0.6$. Lowering the Fermi energy corresponds to "hole doping".