

Physics 123B: Homework 5
due March 1, 5pm in the box at the PSR

1. **Fraunhofer pattern:** Here we consider the effect of a magnetic field that actually passes through a Josephson junction. Suppose a Josephson junction has thickness d (the separation between the two superconductors) in the z direction, and width L_x and L_y in the x and y directions, respectively. A magnetic field B is applied along the y direction, so we take $A_z = -Bx$. Now we can assume the phase difference $\Delta\theta$ is constant between the two superconductors, but $\gamma(x)$ will be a function of x . Then we have to generalize the Josephson equation for the current to $j(x) = j_c \sin \gamma(x)$, where j_c is the current density per unit length (x) across the junction, and $I = \int_0^{L_x} dx j(x)$.

Find the maximum current, I_c , carried by the junction. Express your result in terms of the flux through junction, Φ , the flux quantum, φ_0 , and L_x .

2. **Hund's rules:** What are the expected ground state S , L , and J for an ion in free space with 4 d electrons? What about with 3 f electrons?
3. **Brillouin Function:** Consider a free spin S moment in a magnetic field H with g -factor g .

- (a) Show that the magnetization of the moment along the direction of the field is given by $M = g\mu_B S B_S(g\mu_B S H/k_B T)$, where the Brillouin function

$$B_S(x) = \frac{2S+1}{2S} \coth \frac{2S+1}{2S} x - \frac{1}{2S} \coth \frac{1}{2S} x. \quad (1)$$

- (b) Find the zero-field susceptibility $\chi = \partial M / \partial H|_{H=0}$.