London theory

- Once we accept that Cooper pairs form, we can study their condensation the same way we study BEC

$$\psi(r) = \sqrt{n_s^*(r)}e^{i\theta(r)}$$ Pair wavefunction

- Similar to superfluid, $\mathbf{p} = \hbar\nabla\theta - q\mathbf{A}$
  - The difference arises from the charge of Cooper pairs

London Theory

- This implies *screening*: check Maxwell eqns
- Pairs: $n_s^*=n_s/2$, $q=-2e$, $m^*=2m$
- Hence the current is

$$\mathbf{j} = -\frac{qn_s^*}{m*}\mathbf{p} = -\frac{\hbar n_s e}{2m}\left(\nabla\theta + \frac{2e}{\hbar}\mathbf{A}\right)$$

- This is often called the "London equation"
- Use with Maxwell equation to describe screening

London theory

- Maxwell

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{j}$$

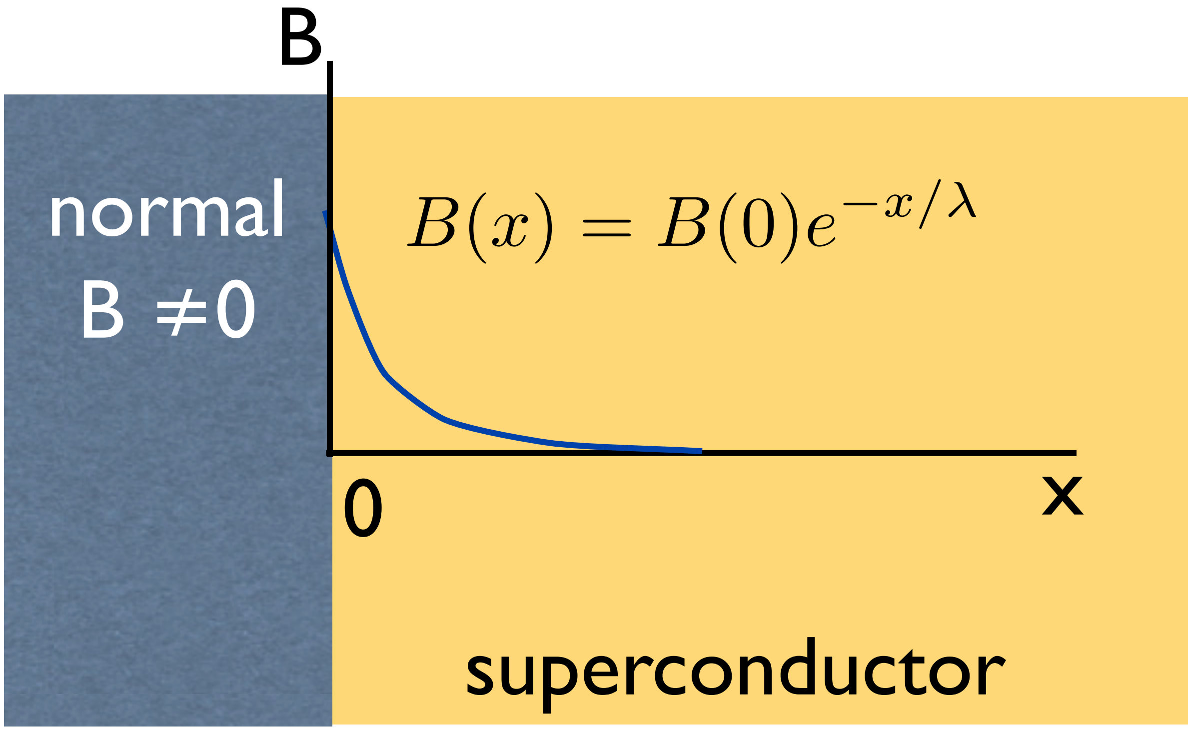
$$\nabla \times \nabla \times \mathbf{B} = \mu_0 \nabla \times \mathbf{j}$$

$$\cancelto{0}{\nabla(\nabla \cdot \mathbf{B})} - \nabla^2 \mathbf{B} = \mu_0 \left(-\frac{n_s e^2}{m} \nabla \times \mathbf{A} \right)$$

$$\boxed{\nabla^2 \mathbf{B} = \frac{1}{\lambda^2} \mathbf{B} \qquad \lambda = \left(\frac{m}{n_s e^2 \mu_0} \right)^{1/2}}$$

- These two equations describe screening

London theory



λ is called London penetration depth

λ ~ 10-100nm typically

Meissner Effect

- The above argument suggests screening of magnetic fields is due to currents which flow because of infinite conductivity
- If there was “only” infinite conductivity, then we would expect that an applied field would not penetrate, but that if we *started* an experiment with a field applied, and then cooled a material from the normal to superconducting state, the field would remain
- This is in fact not true: magnetic fields are actively *expelled* from superconductors

Meissner effect

- Expulsion of an applied field occurs because in the superconducting state, the field *costs free energy*, i.e. is thermodynamically unfavorable
- Free energy

$$F = \int d^3 r \left[n_s^* \frac{p^2}{2m^*} + a(n_s - n_s^{\mathrm{eq}})^2 + \frac{B^2}{2\mu_0} \right]$$

Meissner Effect

$$F = \int d^3r \left[\frac{n_s}{8m} |\hbar \nabla\theta + 2e\mathbf{A}|^2 + a(n_s - n_s^{\mathrm{eq}})^2 + \frac{B^2}{2\mu_0} \right]$$

gauge: we can always choose A to cancel $\nabla\theta$

$$F = \int d^3r \left[\frac{n_s e^2}{2m} |\mathbf{A}|^2 + a(n_s - n_s^{\mathrm{eq}})^2 + \frac{|\mathbf{B}|^2}{2\mu_0} \right]$$

key point: if $B \neq 0$, A must vary linearly with r, which implies $|A|^2$ diverges. Superconducting kinetic energy becomes infinite!

Meissner effect

$$F = \int d^3r \left[\frac{n_s e^2}{2m} |\mathbf{A}|^2 + a(n_s - n_s^{\mathrm{eq}})^2 + \frac{|\mathbf{B}|^2}{2\mu_0} \right]$$

- Instead, superconducting state *expels* the field.
- Eventually, if a large enough field is applied to a superconductor, the superconductivity is destroyed

Meissner effect

- To estimate the *critical field*, we need to compare the Gibbs free energy

$$G = \int d^3r \left[\frac{n_s e^2}{2m}|\mathbf{A}|^2 + a(n_s - n_s^{\mathrm{eq}})^2 + \frac{|\mathbf{B}|^2}{2\mu_0} - \mathbf{H}\cdot\mathbf{B}\right]$$

- In the SC state, $n_s = n_s^{eq}$, A=B=0

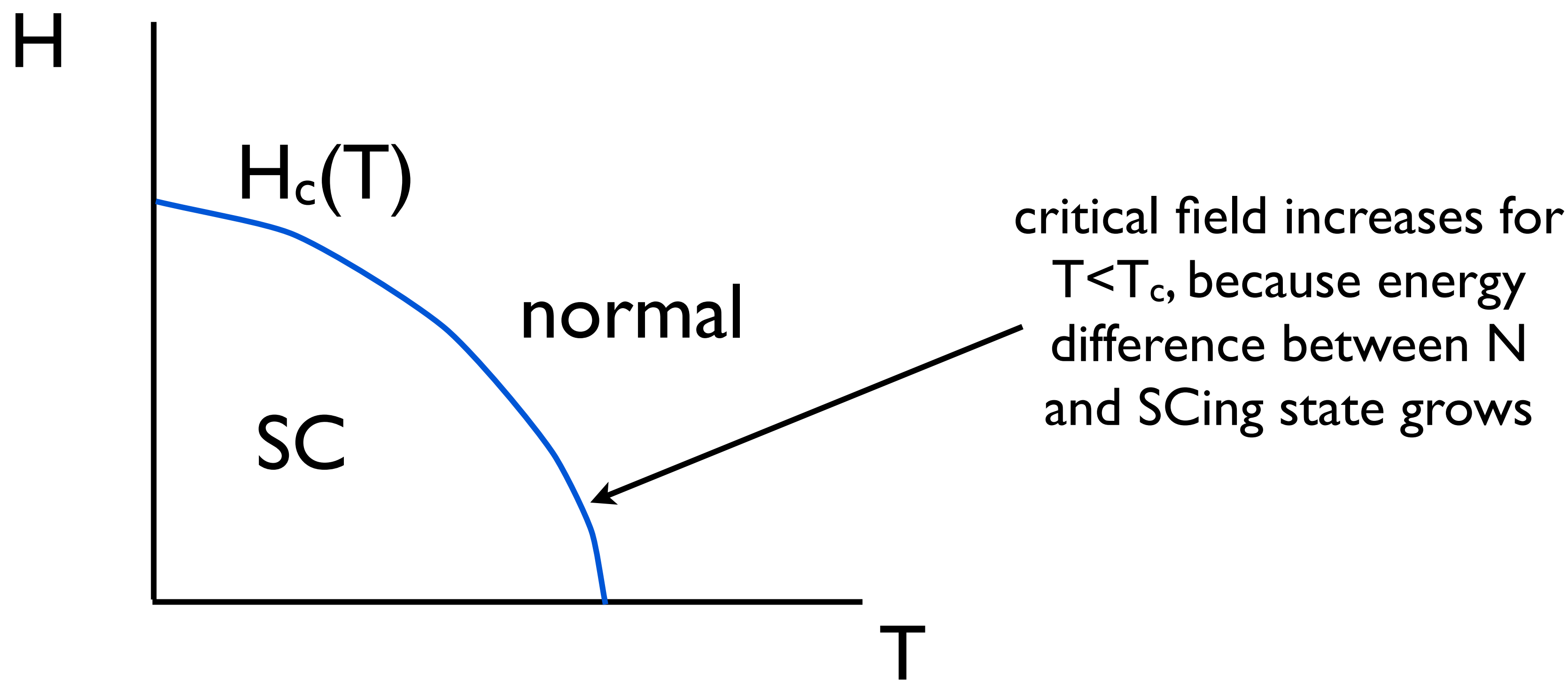
$$G_{sc} = 0$$

- In the normal state, $n_s=0$, $B=\mu_0 H$

$$G_n = V\left[a\left(n_s^{\mathrm{eq}}\right)^2 - \frac{\mu_0}{2}H^2\right]$$

- Equality $G_{sc}=G_n$ defines the critical field H_c

Meissner effect



This describes so-called “type I” superconductors

Some superconductors are “type II” and have a different phase diagram