Current

- Consider work done by small change of phase:

$$dF = IV\,dt = I\frac{\hbar}{2e}d\gamma$$

- But

$$dF = d(-E_J\cos\gamma) = E_J\sin\gamma d\gamma$$

$$I = \frac{2eE_J}{\hbar}\sin\gamma$$

- Hence a constant phase can produce a supercurrent, with zero voltage, for $I<I_c$, with critical current $I_c = \frac{2eE_J}{\hbar}$

Size of critical current

- The amount of current a JJ can carry is obviously dependent upon the junction
- Natural to expect that I_c is correlated with the conductance G in the normal state
- Ambegaokar/Baratoff formula (BCS theory):

$$I_c R_n = \frac{\pi \Delta}{2e} \tanh\left(\frac{\Delta}{2kT}\right)$$

dimensions make sense!

Consequences

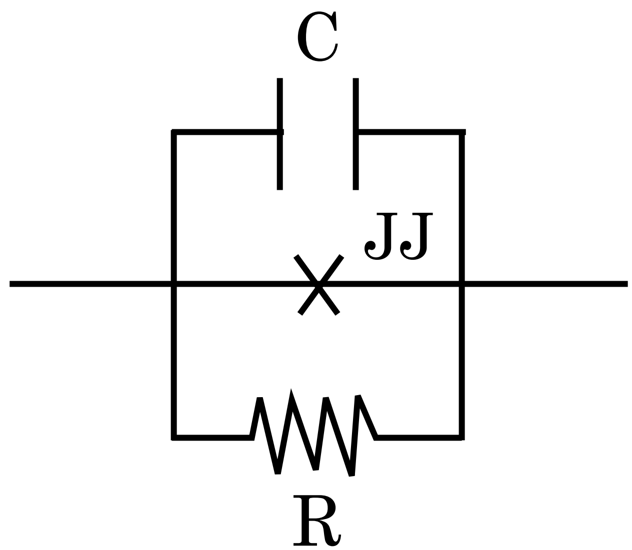
- Zero-bias (dissipationless) current
- AC Josephson effect: a voltage induces an oscillating current

Josephson frequency

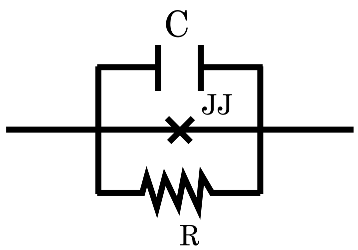
$$\gamma = \frac{2eVt}{\hbar} \quad \Rightarrow \quad I(t) = I_c \sin \frac{2eV}{\hbar} t$$

RCSJ model

- A simple model for the IV curve of a JJ



$$I = I_c \sin\gamma + \frac{V}{R} + C\frac{dV}{dt} \qquad\qquad V = \frac{\hbar}{2e}\frac{d\gamma}{dt}$$

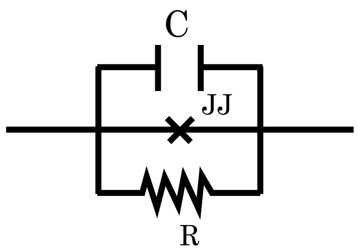


RCSJ model

- Same equation as a *pendulum* or particle in a *tilted washboard* potential

$$\frac{d^2\gamma}{dt^2} + \frac{1}{RC}\frac{d\gamma}{dt} + \frac{2eI_c}{\hbar C}\sin\gamma = \frac{2eI}{\hbar C}$$

- $I<I_c$: constant phase: V=0
- $I>I_c$: pendulum spins: non-zero average voltage



RCSJ model

- Consider over-damped limit $\frac{1}{RC} \gg \sqrt{\frac{2eI_c}{\hbar C}}$

$$\cancel{\frac{d^2\gamma}{dt^2}} + \frac{1}{RC}\frac{d\gamma}{dt} + \frac{2eI_c}{\hbar C}\sin\gamma = \frac{2eI}{\hbar C} \quad \Rightarrow \quad \frac{d\gamma}{dt} = \frac{2eI_cR}{\hbar}\left(\frac{I}{I_c} - \sin\gamma\right) > 0$$

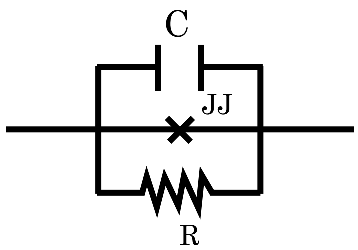
$$\Rightarrow \quad \int \frac{d\gamma}{I/I_c - \sin\gamma} = \int \frac{2eI_cR}{\hbar}dt = \frac{2eI_cR}{\hbar}t$$

time for one cycle

$$t_{2\pi} = \frac{\hbar}{2eI_cR}\int_0^{2\pi} \frac{d\gamma}{I/I_c - \sin\gamma} = \frac{\hbar}{2eI_cR}\frac{2\pi}{\sqrt{(I/I_c)^2 - 1}}$$

DC voltage

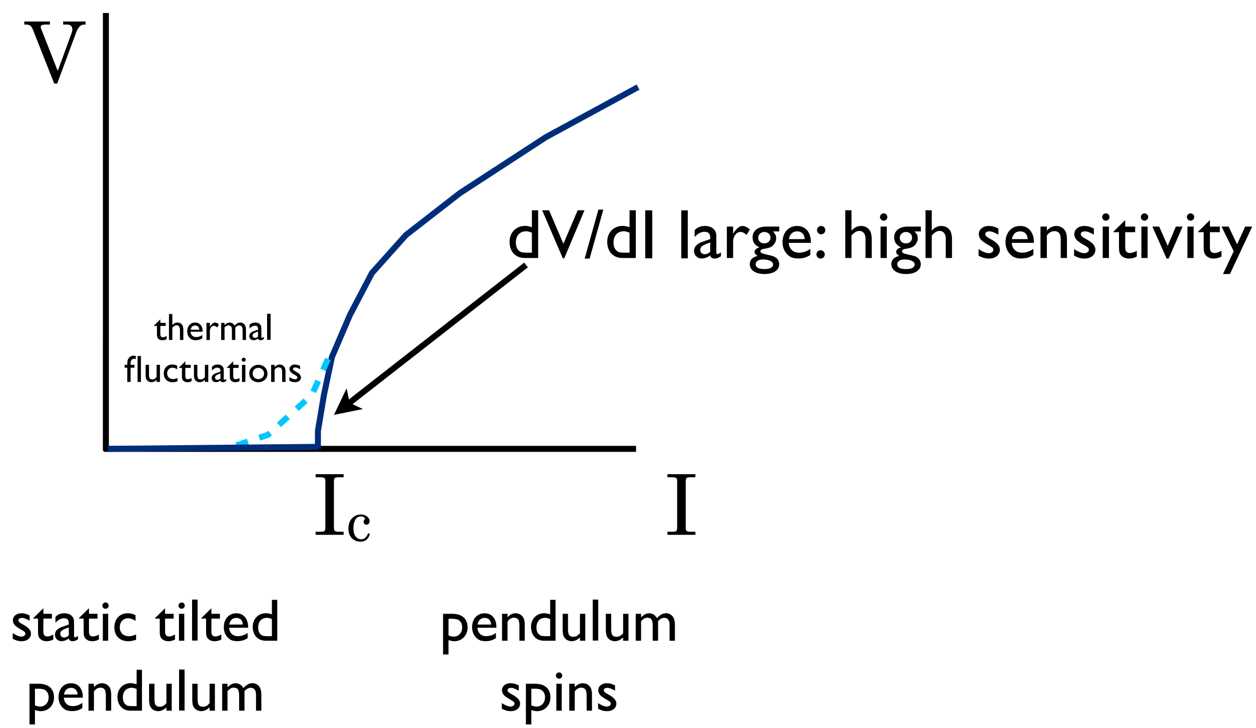
$$V = \frac{\hbar}{2e}\left\langle \frac{d\gamma}{dt} \right\rangle_{av} = \frac{\hbar}{2e}\frac{2\pi}{t_{2\pi}} = R\sqrt{I^2 - I_c^2}$$



RCSJ model

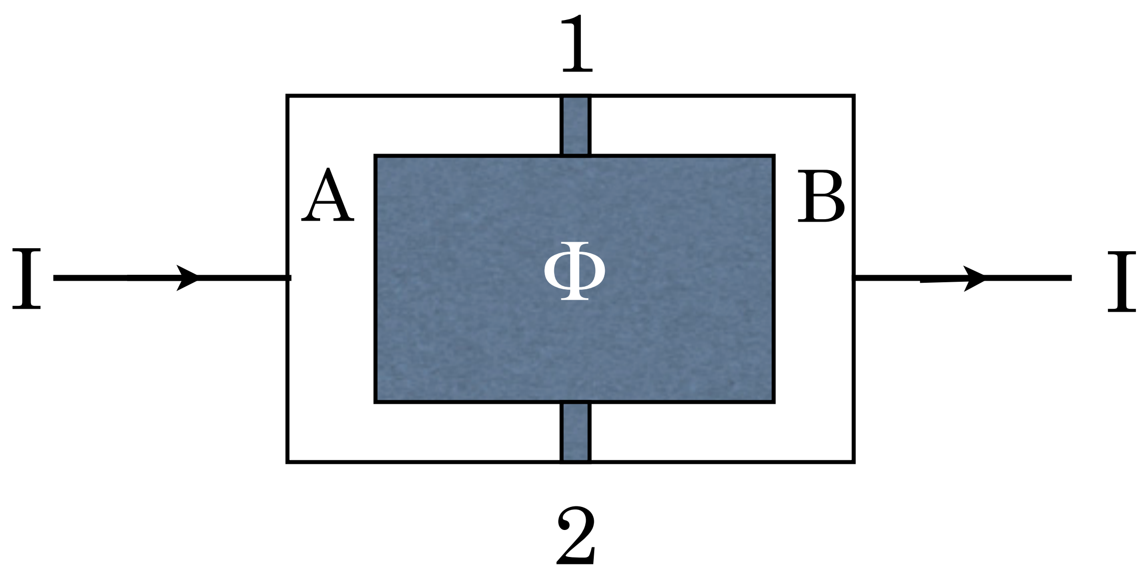
- Consider over-damped limit $\frac{1}{RC} \gg \sqrt{\frac{2eI_c}{\hbar C}}$

$$V = R\sqrt{I^2 - I_c^2}$$

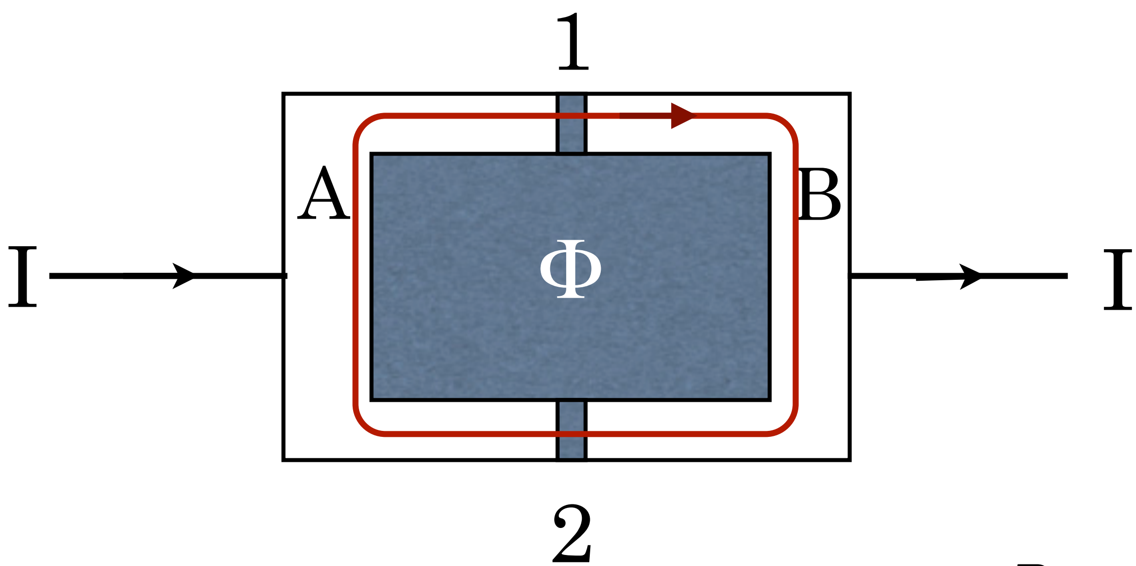


SQUIDs

- SQUID = Superconducting QUantum Interference Device.
- Many kinds of SQUIDs. Here just consider DC-SQUID, a sensitive magnetometer



dc SQUID



in SC, current is negligible, since only small currents can cross barriers.

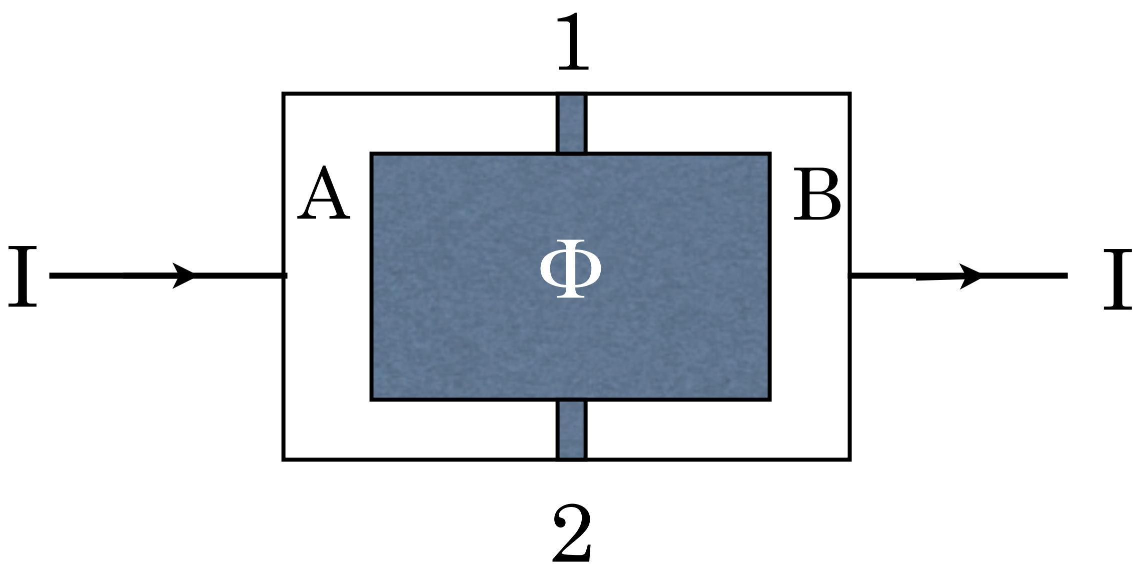
$$\mathbf{J} \propto \nabla\theta - \frac{2e}{\hbar}\mathbf{A} \approx 0$$

$$\oint \mathbf{A}\cdot d\ell = \oint_{\text{inside SC}} \mathbf{A}\cdot d\ell + \int_{A_1}^{B_1} \mathbf{A}\cdot d\ell + \int_{B_2}^{A_2} \mathbf{A}\cdot d\ell$$

$$\Phi = \frac{\hbar}{2e}\oint_{\text{inside SC}} \nabla\theta\cdot d\ell + \int_{A_1}^{B_1} \mathbf{A}\cdot d\ell + \int_{B_2}^{A_2} \mathbf{A}\cdot d\ell$$

$$= \frac{\hbar}{2e}\left(\theta_{A_1} - \theta_{A_2} + \theta_{B_2} - \theta_{B_1}\right) + \int_{A_1}^{B_1} \mathbf{A}\cdot d\ell + \int_{B_2}^{A_2} \mathbf{A}\cdot d\ell \;= \frac{1}{2\pi}\frac{h}{2e}\left(\gamma_1 - \gamma_2\right)$$

dc SQUID

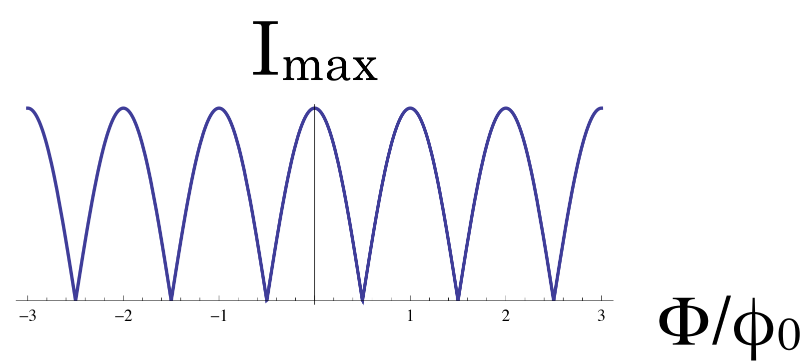


$$\gamma_1 - \gamma_2 = 2\pi \left(\frac{\Phi}{\varphi_0}\right)$$

$$I = I_c \sin\gamma_1 + I_c \sin\gamma_2 \ = 2I_c \cos\left(\frac{\gamma_1 - \gamma_2}{2}\right) \sin\left(\frac{\gamma_1 + \gamma_2}{2}\right)$$

$$= 2I_c \cos\left(\frac{\pi\Phi}{\varphi_0}\right) \sin\left(\frac{\gamma_1 + \gamma_2}{2}\right)$$

$$I_{\max} = 2I_c \left|\cos\left(\frac{\pi\Phi}{\varphi_0}\right)\right|$$



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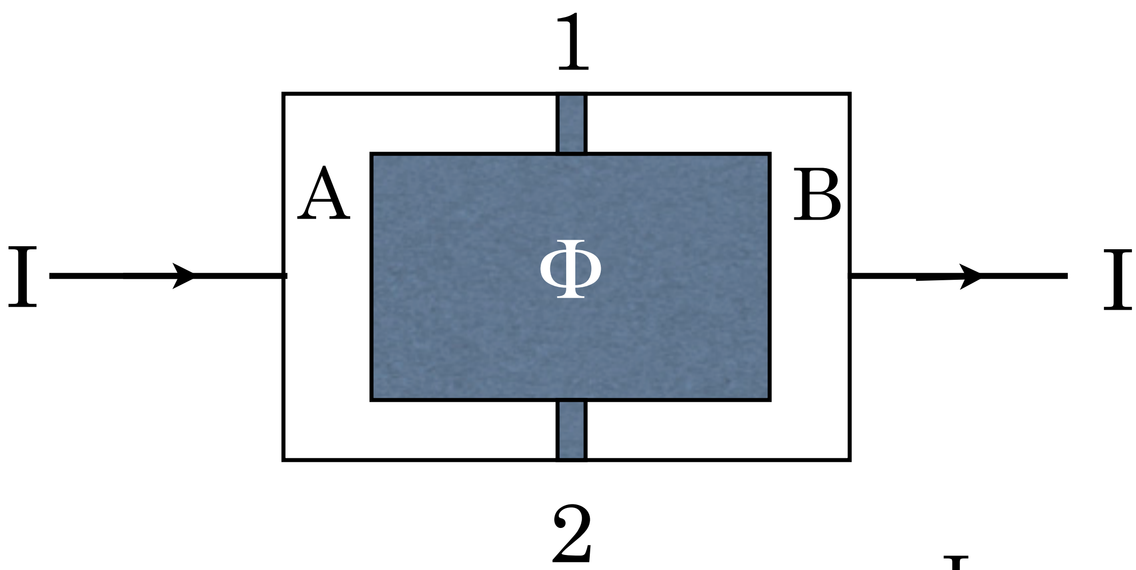
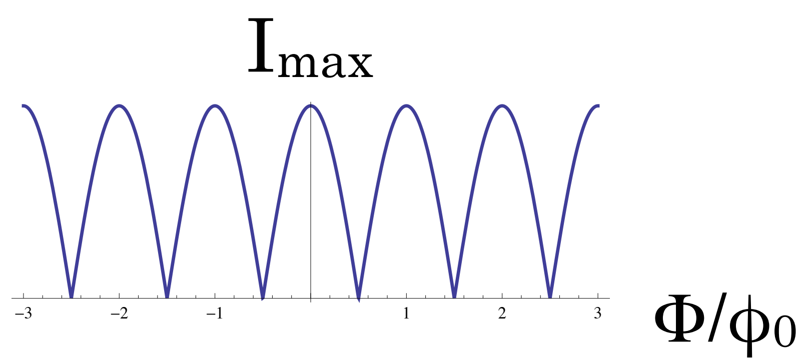


$$\gamma_1 - \gamma_2 = 2\pi \left(\frac{\Phi}{\varphi_0} \right)$$

$$I_{\max} = 2I_c \left| \cos\left(\frac{\pi\Phi}{\varphi_0} \right) \right|$$



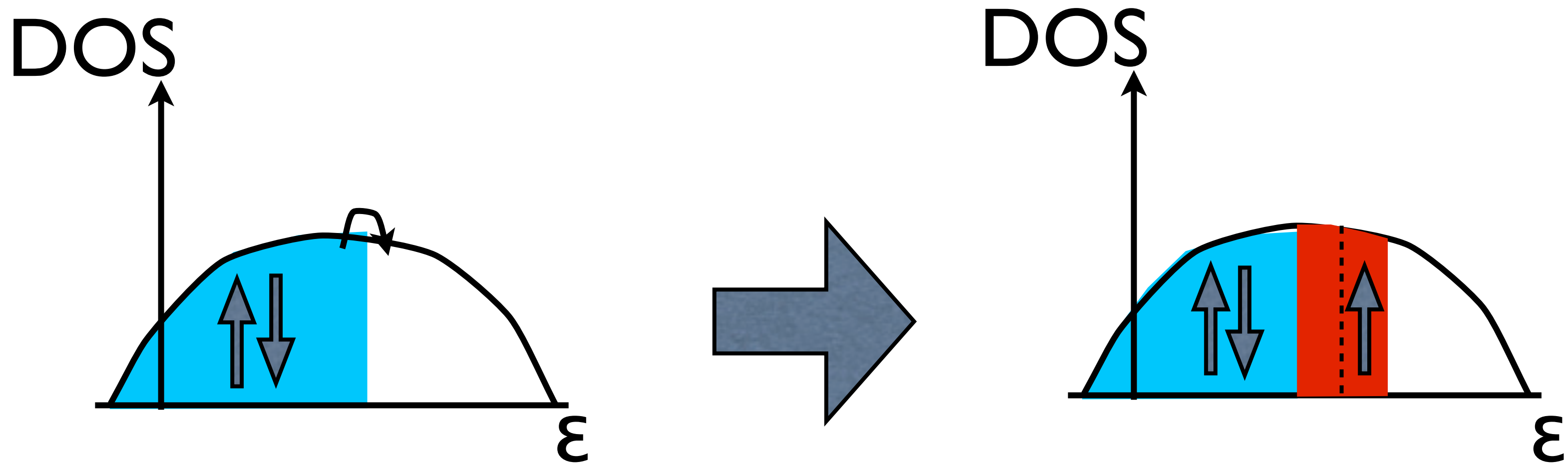
typically operate slightly above the max critical current, where voltage then varies rapidly with flux

Magnetism

- Ferromagnetism has been known since ancient times, at least to the Greeks and Chinese
- Basic physics: electrons have spin $S=1/2$. Sometimes they align
- Much more common is *antiferromagnetism*: electron spins orient but in a way that adds up to zero net moment

Magnetism

- Mechanism?
  - Beyond band theory: for non-interacting electrons, it always costs energy to have spin polarization



  - Magnetism is *always* due to e-e interactions

Magnetism

- So like superconductivity, magnetism is an effect of interactions between electrons
- But unlike superconductivity, magnetism requires *strong* interactions
  - Arbitrarily weak attraction leads to superconductivity, hence most metals become superconducting - but usually at quite low T
  - Few metals are magnetic. In fact, most antiferromagnets are insulating.
- Basic reason: interactions must overcome kinetic energy. Insulators have the least KE