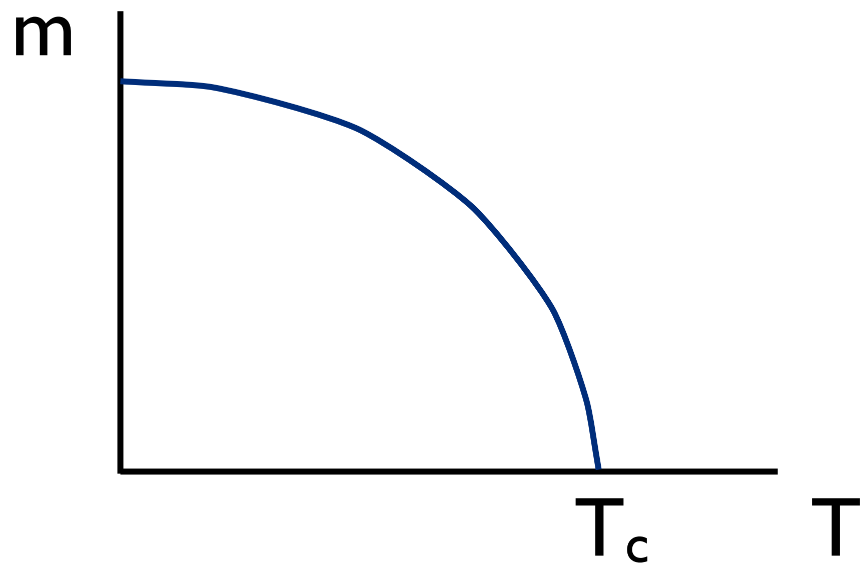
Mean field theory

- *Zero field* magnetization:



- T_c is called the "Curie point" or critical temperature

Susceptibility

- We may guess that the susceptibility gets large on approaching the Curie point, since the material almost forms a magnetization with no field at all.
- This is indeed true.
- Within MFT, just shift $h \rightarrow h + g\,\mu_B\,H$

Susceptibility

$$m = \frac{1}{2}\tanh\left[\frac{zJm + g\mu_B H}{2kT}\right] \approx \frac{zJm + g\mu_B H}{4kT}$$

$$M/N = g\mu_B m \approx \frac{g\mu_B}{1 - \frac{zJ}{4kT}} \frac{g\mu_B H}{4kT} = \frac{(g\mu_B)^2 H}{4kT - zJ}$$

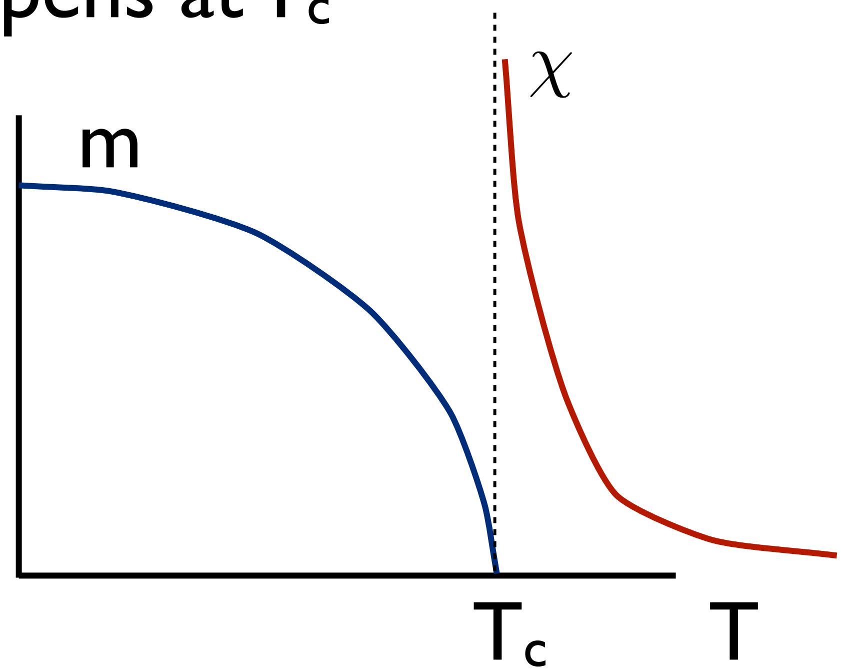
$$\boxed{\chi = \frac{1}{N}\frac{\partial M}{\partial H} = \frac{A}{T - T_c}}$$

"Curie-Weiss law"

Curie law is modified by shift of T by mean field T_c

Phase transition

- A lot happens at T_c



- Both m(T) and χ(T) are *non-analytic* at T_c
- This is actually a sign of a *phase transition*

Quantum treatment

- So far, we treated the ferromagnet in mean field theory
  - This is approximate. Usually qualitatively correct but not even always that.
  - We can do better for the Heisenberg ferromagnet
- Goal: find the actual ground state and excitations

Quantum treatment

- Hamiltonian $$H = -J \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j$$

$$= -J \sum_{\langle ij \rangle} \left(S_i^z S_j^z + \frac{1}{2} S_i^+ S_j^- + \frac{1}{2} S_i^- S_j^+ \right)$$

- Try the "obvious" ground state $|0\rangle = \prod_i |S_i^z = +S\rangle$

$$H|0\rangle = -J \sum_{\langle ij \rangle} S^2 |0\rangle = -NJS^2 \frac{z}{2} |0\rangle$$

Quantum treatment

- Is it the ground state?

$$H = -J \sum_{\langle ij \rangle} \left[\left(\frac{\mathbf{S}_i + \mathbf{S}_j}{2} \right)^2 - S(S+1) \right]$$

- Since $0 \le |S_i + S_j| \le 2S$ for two spins

$$E \ge -J \sum_{\langle ij \rangle} \left[\frac{2S(2S+1)}{2} - S(S+1) \right] = -JS^2 \frac{Nz}{2}$$

- Thus this is indeed a ground state!