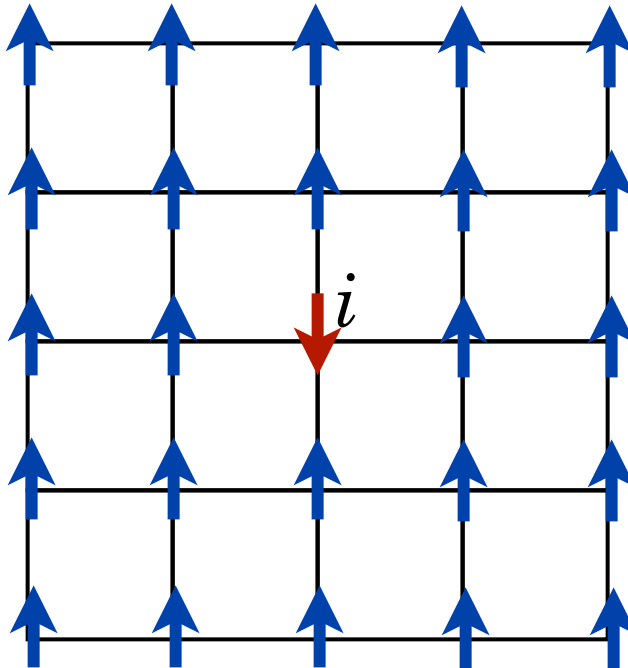


Quantum treatment

- Excitations - lower the spin once

$$|i\rangle = \frac{S_i^-}{\sqrt{2S}} \prod_j |S_j^z = S\rangle = |S_i^z = S - 1\rangle \prod_{j \neq i} |S_j^z = S\rangle$$

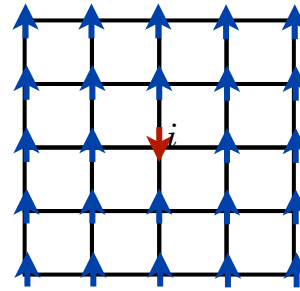
e.g.
 $S = 1/2$



Quantum treatment

- Hamiltonian

$$H = -J \sum_{\langle ij \rangle} \left(S_i^z S_j^z + \frac{1}{2} S_i^+ S_j^- + \frac{1}{2} S_i^- S_j^+ \right) \equiv H_z + H_{\pm}$$

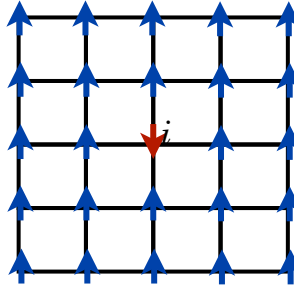


- zz terms

$$H_z |i\rangle = (E_0 - Jz[S(S-1) - S^2]) |i\rangle = (E_0 + JSz) |i\rangle$$

Quantum treatment

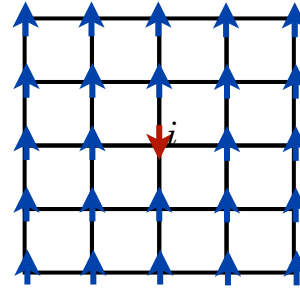
- +- terms
$$H_{\pm} = -\frac{J}{2} \sum_{\langle jk \rangle} (S_j^+ S_k^- + S_j^- S_k^+)$$



$$H_{\pm}|i\rangle = -\frac{J}{2} \sum_{\langle ji \rangle} S_j^- S_i^+ |i\rangle$$

$$= -\frac{J}{2} 2S \sum_{\langle ji \rangle} |j\rangle = -JS \sum_{\mu} |i + \mu\rangle$$

Quantum treatment



- All together

$$H|i\rangle = (E_0 + JSz)|i\rangle - JS \sum_{\mu} |i + \mu\rangle$$

- Looks like a hopping Hamiltonian

- Bloch:

$$|\mathbf{k}\rangle = \frac{1}{\sqrt{N}} \sum_i e^{i\mathbf{k}\cdot\mathbf{r}_i} |i\rangle$$

$$H|\mathbf{k}\rangle = \left(E_0 + JSz - JS \sum_{\mu} e^{i\mathbf{k}\cdot\mathbf{e}_{\mu}} \right) |\mathbf{k}\rangle$$

$$E_0 + \epsilon(\mathbf{k})$$

Quantum treatment

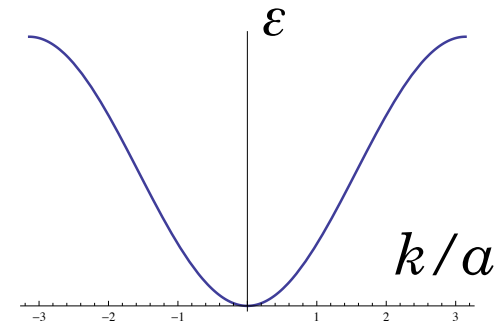
- These are “spin waves” or “magnons”

$$\begin{aligned}\epsilon(\mathbf{k}) &= JSz - JS \sum_{\mu} e^{i\mathbf{k}\cdot\mathbf{e}_{\mu}} \\ &= 2JS \left(d - \sum_{\alpha=1}^d \cos k_{\alpha} a \right) \\ &\approx JSa^2 k^2 \quad \text{quadratic for small } k\end{aligned}$$

- Magnons are gapless

Magnons

- Magnons are gapless
- Energy vanishes as $k \rightarrow 0$
- This is because

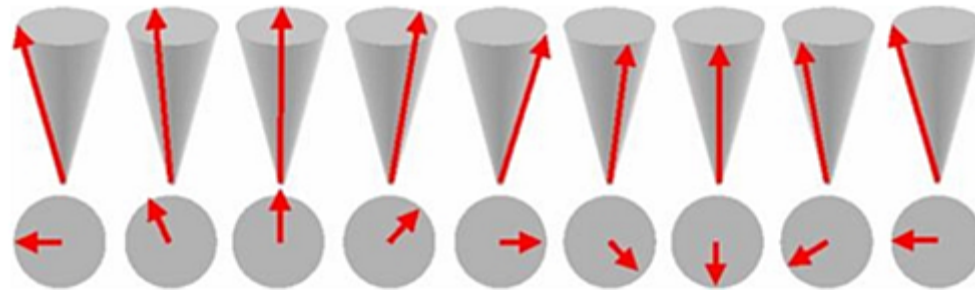


$$|\mathbf{k} = 0\rangle \propto S_{\text{TOT}}^-|0\rangle$$

- This is an example of a “Goldstone mode”
- “Goldstone’s theorem”: if H has a continuous symmetry that is “broken” by the ground state, there will be a gapless mode

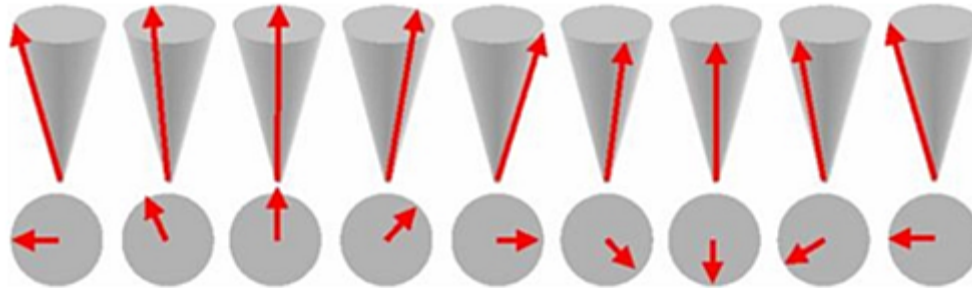
Magnons

- Spin wave/magnon can be regarded as a quantized precession wave of slightly tilted spins



- $\epsilon \sim k^2$ behavior, a general property for ferromagnets, can be understood this way

Magnons



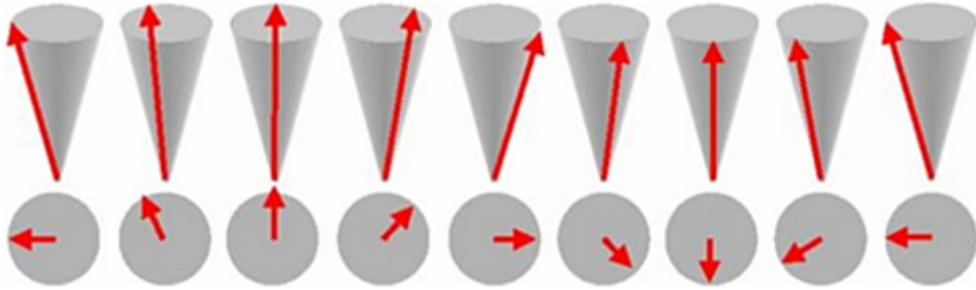
- Think of effective field due to other spins

$$\mathbf{h}(\mathbf{r}) \approx c_0 \mathbf{m} + c_1 \nabla^2 \mathbf{m} + \dots$$

- Local spins precess in this field

$$\partial_t \mathbf{m} = \mathbf{h} \times \mathbf{m} \approx c_1 \nabla^2 \mathbf{m} \times \mathbf{m}$$

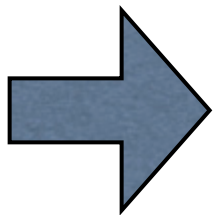
Magnons



$$\partial_t \mathbf{m} = \mathbf{h} \times \mathbf{m} \approx c_1 \nabla^2 \mathbf{m} \times \mathbf{m}$$

$$\mathbf{m} = (m_x, m_y, \sqrt{m_0^2 - m_x^2 - m_y^2}) \approx (m_x, m_y, m_0)$$

$$\partial_t \begin{pmatrix} m_x \\ m_y \end{pmatrix} = c_1 m_0 \begin{pmatrix} 0 & -k^2 \\ k^2 & 0 \end{pmatrix} \begin{pmatrix} m_x \\ m_y \end{pmatrix}$$



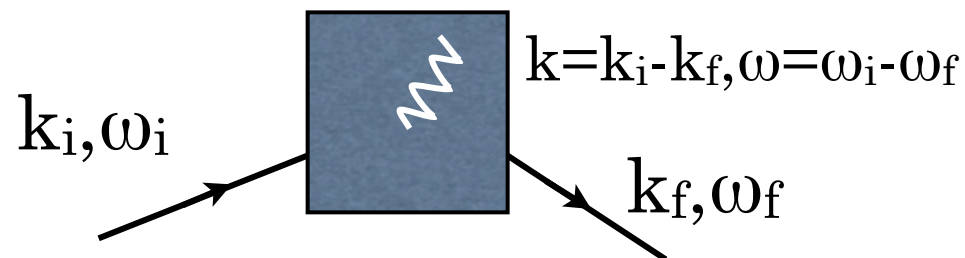
$$\omega = \pm c_1 m_0 k^2$$

Neutron scattering

- Neutron has a $S=1/2$ similar to an electron, and its own dipole moment, which interacts with magnetic dipoles in materials

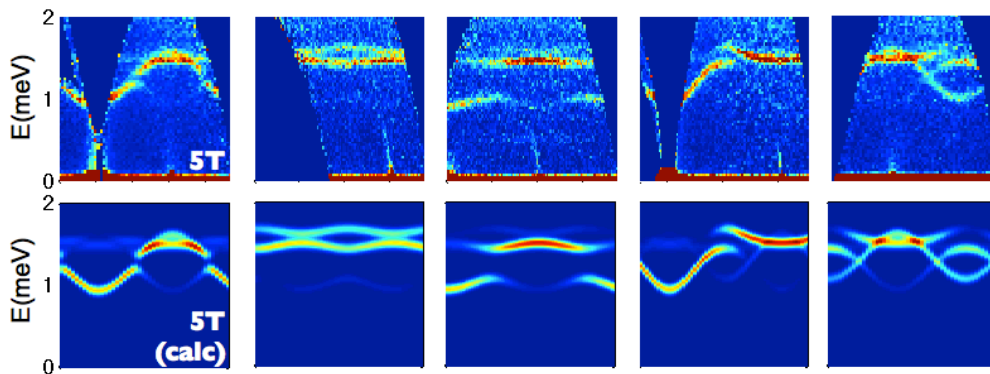
$$H_{d-d} = -\frac{\mu_0}{4\pi r^3} [3(\mathbf{m} \cdot \mathbf{r})(\mathbf{m}' \cdot \mathbf{r}) - \mathbf{m} \cdot \mathbf{m}']$$

- Consequently, a neutron can exchange energy and momentum with electronic spins

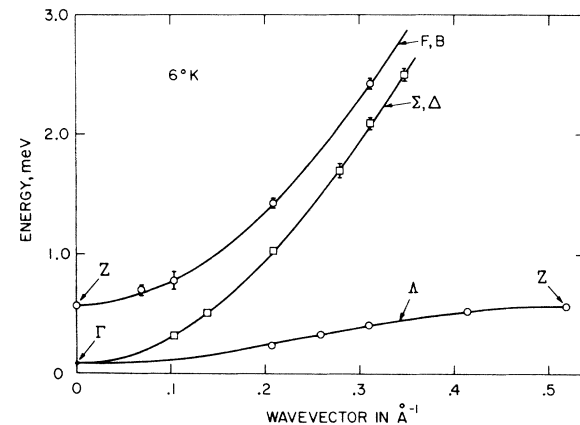


Neutron scattering

- Integrate over all energies: get total scattering
 - tool for detecting magnetic ordering in solids
- Resolve both momentum and energy of neutrons: measure spin waves



example of spin waves in $\text{Yb}_2\text{Ti}_2\text{O}_7$ (2012)



first measurement in an insulating FM: CrBr_3 , 1971

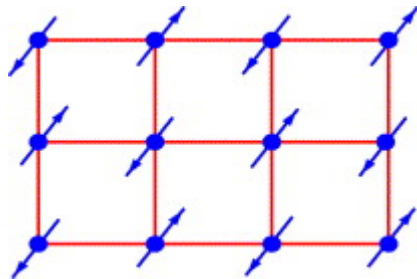
Antiferromagnets

- Actually it is much more common to have interactions that favor anti-aligned spins

$$H = +|J| \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j$$

- This is trickier, because even classically spins take on different orientations

Néel state



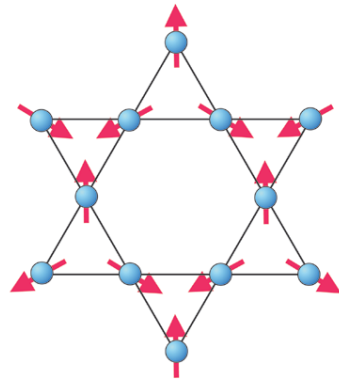
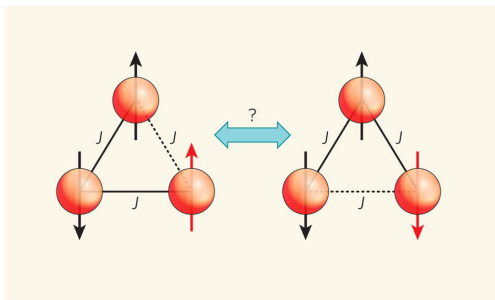
“unfrustrated” lattice:
can minimize all pairs of
interactions simultaneously

Antiferromagnets

- Actually it is much more common to have interactions that favor anti-aligned spins

$$H = +|J| \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j$$

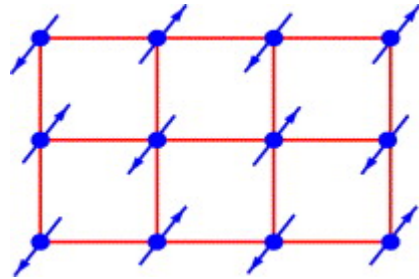
- This is trickier, because even classically spins take on different orientations



“frustrated” lattice:
spins must compromise

Unfrustrated case

- For example, square lattice

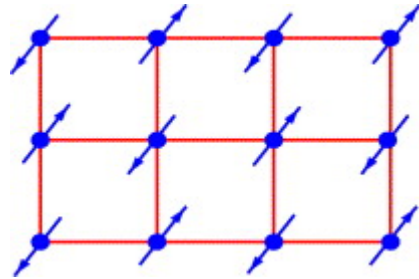


$$\mathbf{n} = \langle \mathbf{S}_i \rangle (-1)^{x_i + y_i}$$

- In MFT, find *staggered magnetization* \mathbf{n} behaves the same as m for a ferromagnet
- But quantum situation is different!

Unfrustrated case

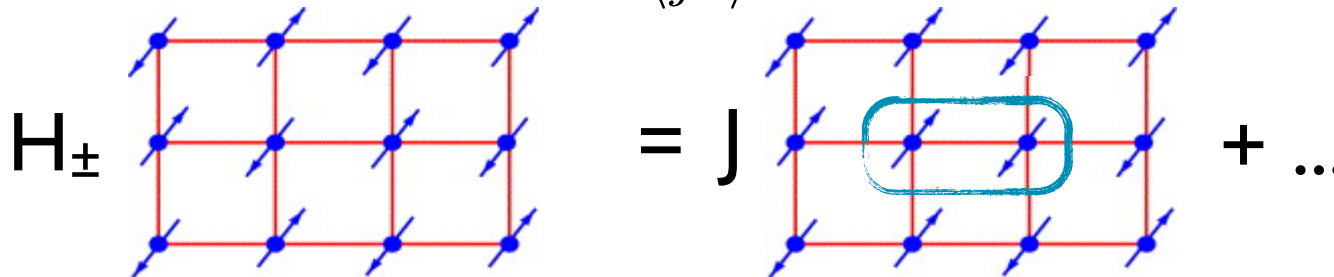
- Guess the ground state?



$$|\Psi\rangle = \prod_i |S_i^z = (-1)^{x_i+y_i} S\rangle$$

- Is it an eigenstate? NO!!

$$H_{\pm} = \frac{|J|}{2} \sum_{\langle jk \rangle} (S_j^+ S_k^- + S_j^- S_k^+)$$



Singlets

- For two spins, the ground state is actually a singlet

$$|s\rangle = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$

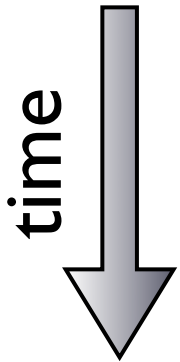
- This can be viewed as a *quantum superposition* of two classical Néel states
- Consequently, in a real antiferromagnet, the magnetic order will be reduced or even removed by quantum fluctuations below that of MFT
- However, it turns out that most antiferromagnets still manage to order

A brief history of magnetism

~500BC: Ferromagnetism
documented in
Greece, India, used in
China



sinan, ~200BC



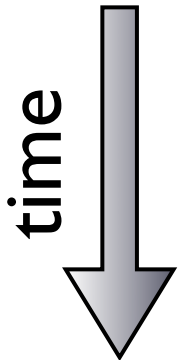
1949AD: Antiferromagnetism
proven experimentally

A brief history of magnetism

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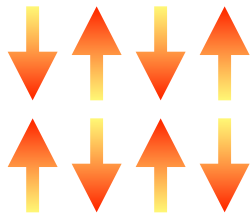
Why so long???

1949AD: Antiferromagnetism proven experimentally

A debate

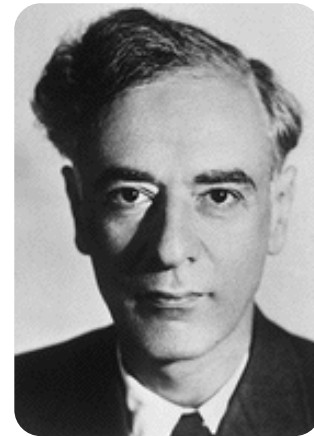


Néel

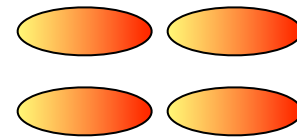


antiferromagnet

$$H = |J| \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j$$



Landau



$$\text{Oval} = \frac{1}{\sqrt{2}} (\uparrow\downarrow - \downarrow\uparrow)$$

singlets

Landau's dream

- People are still looking for the “quantum disordered” antiferromagnet, with a singlet ground state instead of a Néel one