Measuring Fermi surfaces

- Traditional and still most precise technique: *quantum oscillations*
  - electrons in applied B field make constant energy orbits on Fermi surface. Interference between multiple circuits creates quantum oscillations in magnetization (de Haas van Alphen), resistivity (Shubnikov de Haas), ...
- Angle resolved photoemission spectroscopy = ARPES
- Many other less direct methods

Quantum Oscillations

- Dynamics of a Bloch electron in a field

$$\hbar \frac{d\mathbf{k}}{dt} = -e\mathbf{v}_n(\mathbf{k}) \times \mathbf{B}$$

- This implies

$$\mathbf{k} \cdot \mathbf{B} = \text{const}$$

$$\epsilon_n(\mathbf{k}) = \text{const}$$

- In reciprocal space, electrons move in orbits of constant energy in planes normal to the magnetic field

Phase of the orbit

$$\phi = \phi_1 + \phi_2 \qquad \phi_1 = \oint \mathbf{k} \cdot \mathbf{dr} \qquad \phi_2 = \frac{e}{\hbar} \oint \mathbf{A} \cdot \mathbf{dr}$$

$$\hbar \frac{d\mathbf{k}}{dt} = -e \frac{d\mathbf{r}}{dt} \times \mathbf{B}$$

$$\hbar \mathbf{k} \times d\mathbf{k} = -e\mathbf{k} \times d\mathbf{r} \times \mathbf{B}$$

$$= -e(\mathbf{k} \cdot \mathbf{B})\, d\mathbf{r} + e(\mathbf{k} \cdot d\mathbf{r})\, \mathbf{B}$$

$$\hbar \mathbf{r} \cdot d\mathbf{k} = -e\mathbf{r} \cdot \mathbf{dr} \times \mathbf{B} = -e\mathbf{B} \cdot \mathbf{r} \times \mathbf{dr}$$

$$\hbar \hat{z} \cdot \oint \mathbf{k} \times d\mathbf{k} = eB \oint \mathbf{k} \cdot d\mathbf{r} = eB\phi_1 \qquad \hbar \oint \mathbf{r} \cdot d\mathbf{k} = -eB\hat{z} \cdot \oint \mathbf{r} \times \mathbf{dr}$$

$$\phi_1 = \frac{2eB}{\hbar} S_k \qquad \phi_2 = -\frac{\phi_1}{2}$$

Quantum oscillations

$$\phi = \frac{\hbar}{eB} S_k$$

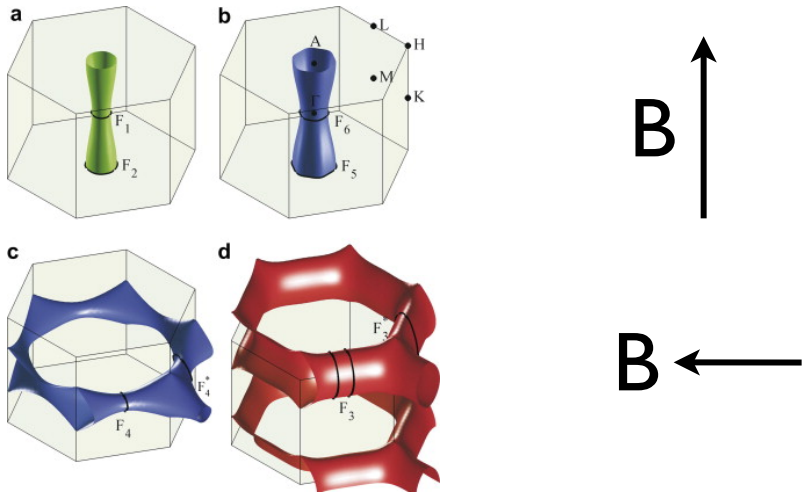
- This phase governs the interference of trajectories in which the electron circles the Fermi surface different numbers of times
- If the field is changed in such a way that ϕ changes by a multiple of 2π, the interference is identical

$$\Delta\left(\frac{1}{B}\right) = \frac{2\pi e}{\hbar S_k}$$ gives period of oscillations in 1/B

Extremal orbits

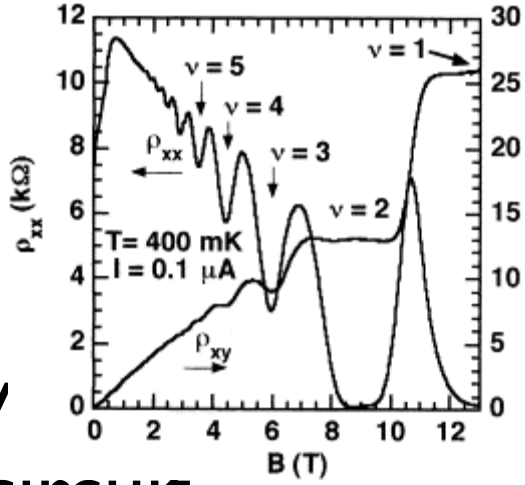
- For a 3d Fermi surface, the area depends upon the component of k along B - which ones contribute?
- Turns out that *extremal orbits* (max/min of S_k) dominate

e.g. MgB_2



By varying field orientation, Fermi surfaces can often be mapped out

Quantum picture



- This is a semi-classical description
- It applies if there are not too many interferences, i.e. electron cannot circuit the Fermi surface very many times before dephasing
- The quantum limit is *Landau levels* and the *quantum Hall effect*

Landau levels

- Simplest case: "free" 2d electrons in a magnetic field (applies to electrons in a semiconductor 2DEG)
- Hamiltonian

$$H = \frac{1}{2m}\left(\mathbf{p} + e\mathbf{A}\right)^2 \qquad \mathbf{A} = By\hat{x}$$

- Choose k_x eigenstate

$$\psi(x, y) = e^{ik_x x} Y(y)$$

Landau levels

- One obtains

$$\frac{1}{2m}\left(-\hbar^2\frac{d^2}{dy^2}+(eB)^2\left(y-\frac{\hbar k_x}{eB}\right)^2\right)Y=\epsilon Y$$

- This is a 1d simple harmonic oscillator with a frequency and center

$$\omega_c=\frac{eB}{c}$$

cyclotron frequency

$$y_0=\frac{\hbar k_x}{eB}=k_x\ell^2$$

$$\ell=\sqrt{\frac{\hbar}{eB}}$$

magnetic length

Landau levels

- Energy levels = Landau levels are

$$\epsilon_n = \hbar\omega_c(n + \tfrac{1}{2})$$

- Each is highly degenerate due to independence of energy on k_x
- How many?

$$k_x = \frac{2\pi}{L_x} i, \quad i = 0, 1, 2, \cdots$$

$$0 < y_0 = k_x \ell^2 < L_y$$

$$0 < i < \frac{L_x L_y}{2\pi\ell^2} \qquad N = \frac{A}{2\pi\ell^2}$$

Landau levels

- Degeneracy

$$N = \frac{A}{2\pi\ell^2} = AB\frac{e}{h} = \frac{\Phi}{\varphi}$$

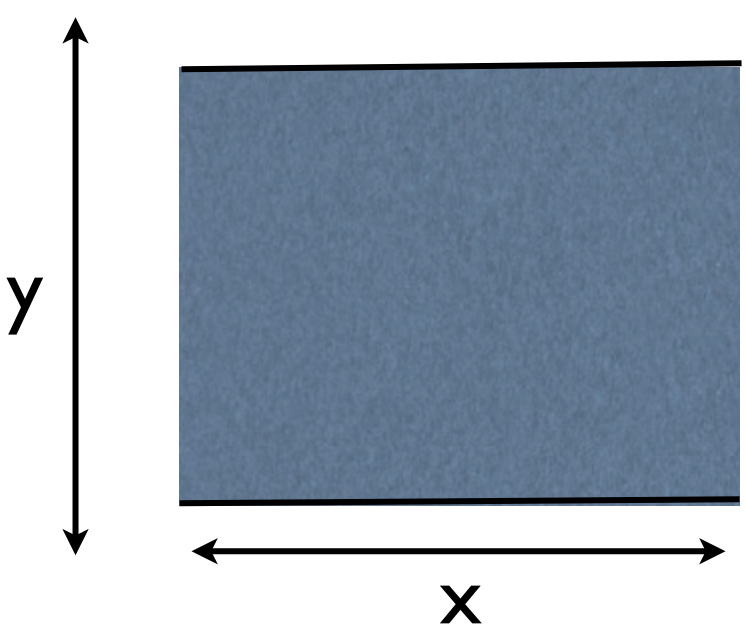
- Flux quantum

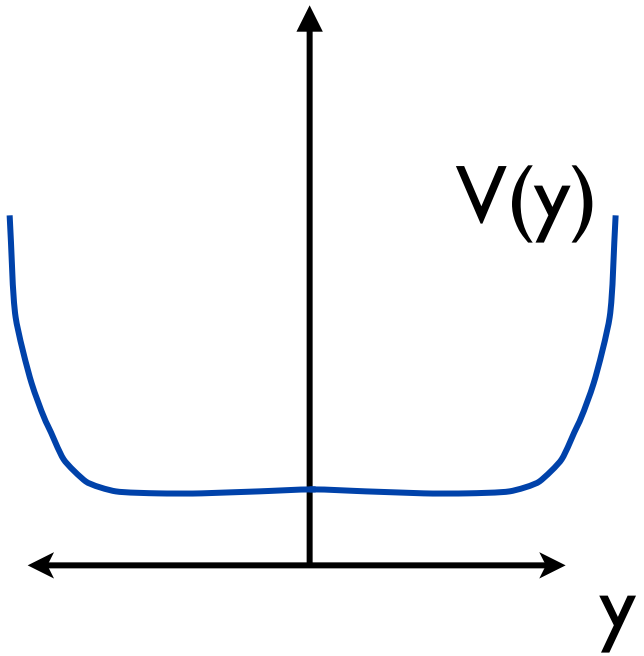
$$\varphi = h/e \approx 4 \times 10^{-15} T \cdot m^2$$

- This is basically the number of minimal quantized cyclotron orbits which fit into the sample area

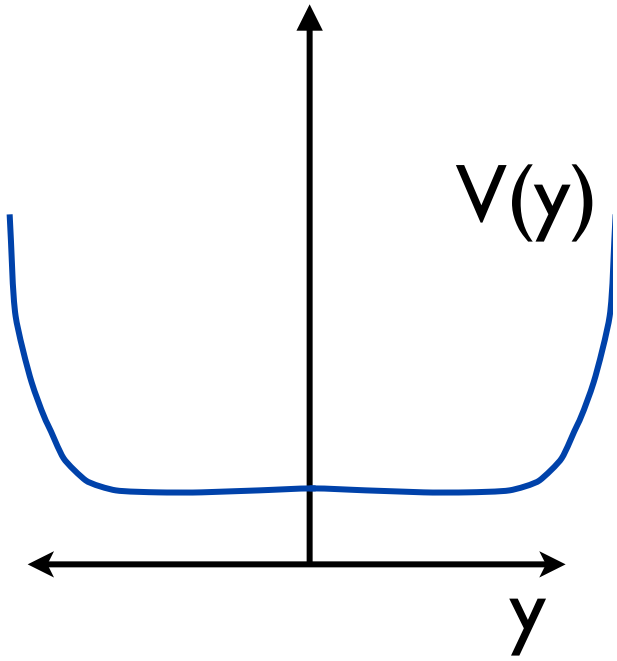
Edge states

- A simple way to understand the quantization of Hall effect, realized by Halperin
- Consider Hall bar





Edge states



$$\left[-\frac{\hbar^2}{2m}\frac{d^2}{dy^2}+\frac{1}{2}m\omega_c^2\left(y-k_x\ell^2\right)^2+V(y)\right]Y=\epsilon Y$$

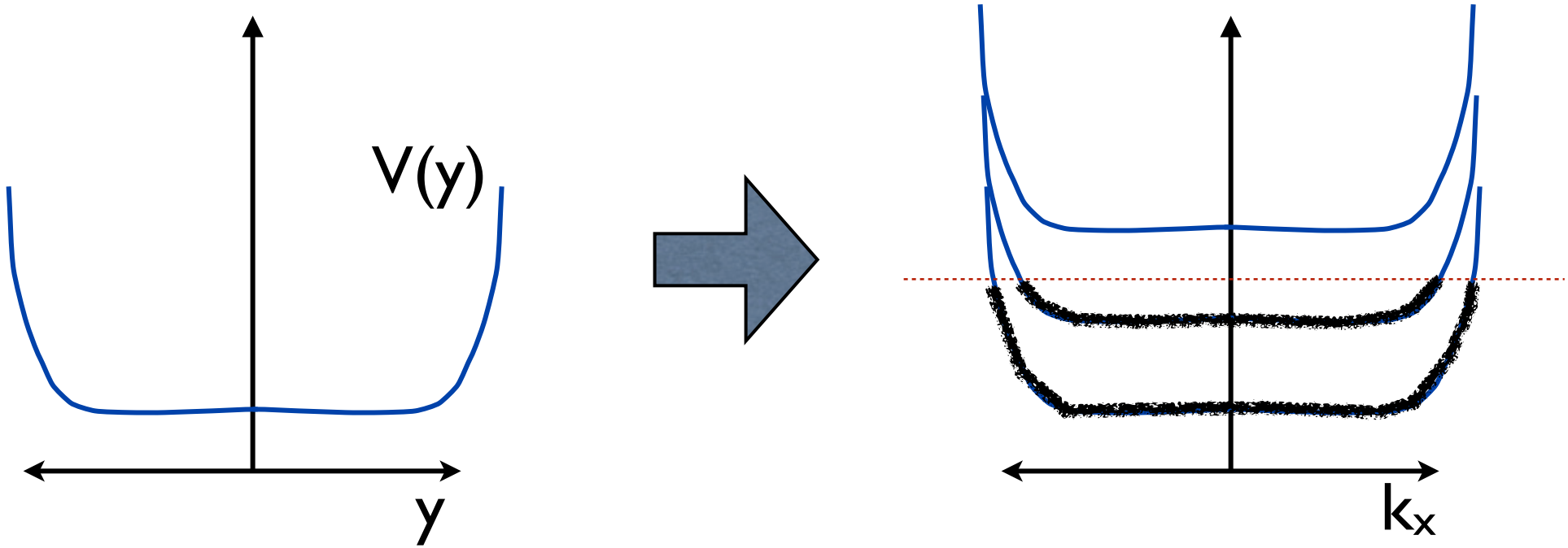
If V(y) is slowly varying, then we can approximate

$$V(y)\approx V(k_x\ell^2)$$

$$\epsilon_n\approx\hbar\omega_c(n+\tfrac{1}{2})+V(k_x\ell^2)$$

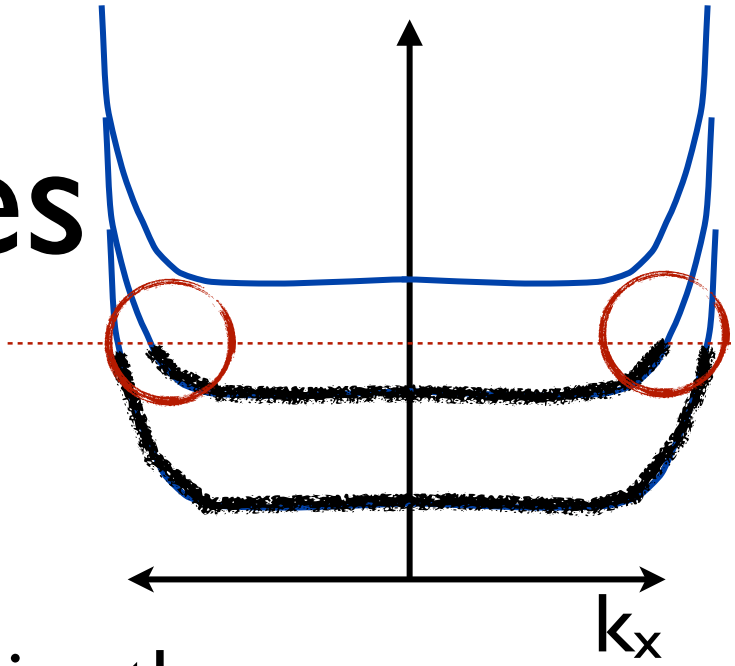
Edge states

$$\epsilon_n \approx \hbar\omega_c(n + \tfrac{1}{2}) + V(k_x \ell^2)$$



Low energy states at the edges of the system

Edge states

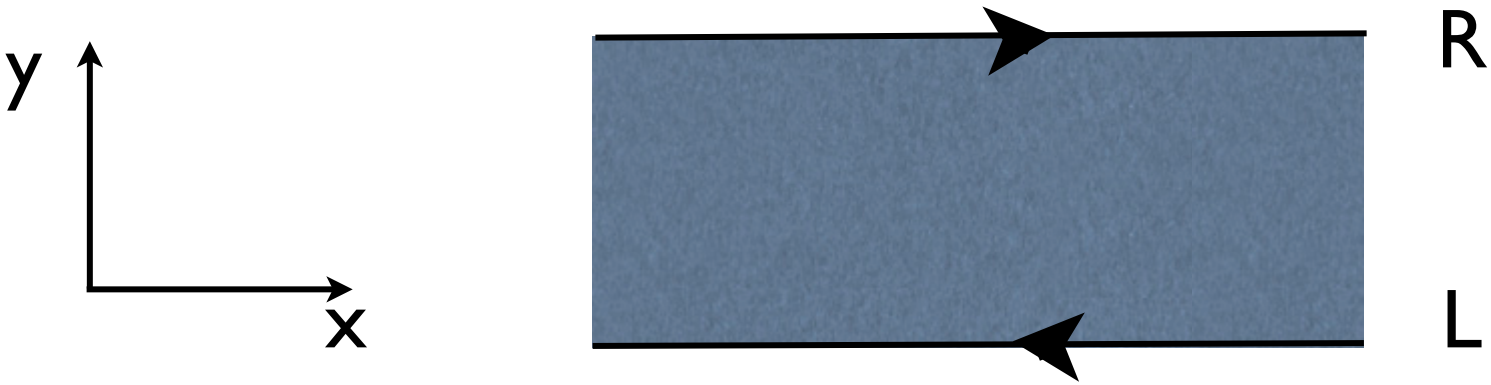


$$\epsilon_n \approx \hbar\omega_c(n+\tfrac{1}{2}) + V(k_x\ell^2)$$

- Near the edge, we can linearize the energy

$$k_x = \pm K_n + q_x \qquad\qquad \epsilon_n \approx \epsilon_F \pm v_n q_x$$

- This describes "right and left-moving chiral fermions" = edge states



Edge states

- Corresponds to semi-classical "skipping orbits"