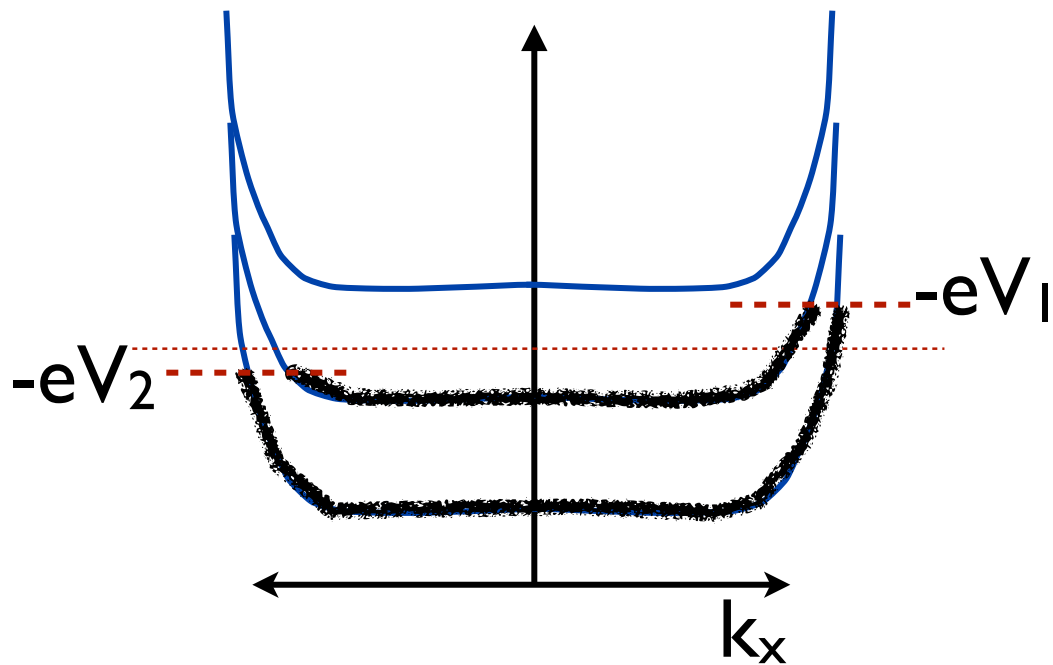
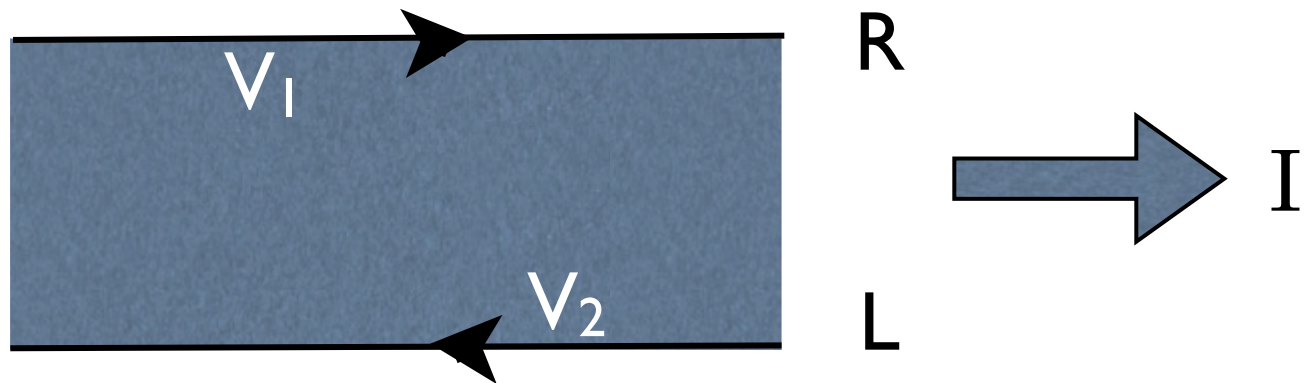


# Quantum Hall effect



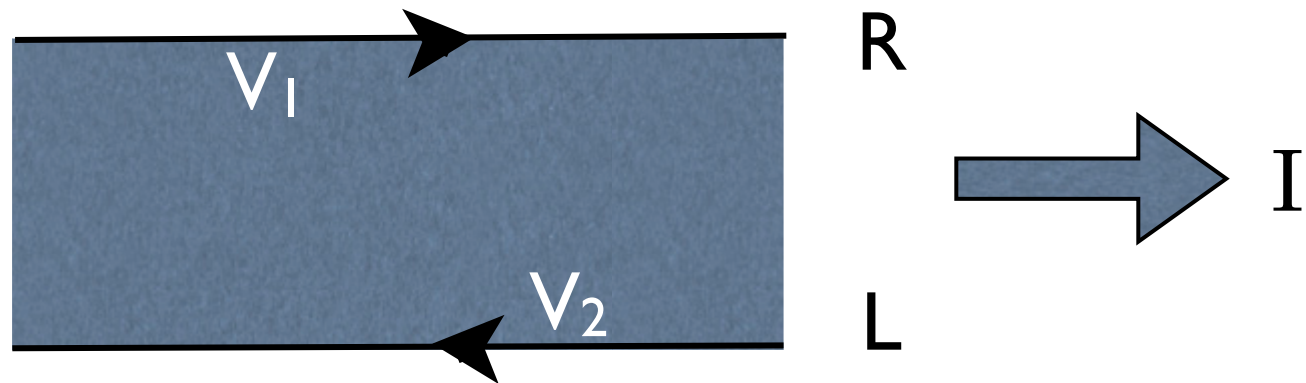
$$\delta n_R = \frac{-eV_1}{2\pi\hbar v} \quad \delta n_L = \frac{-eV_2}{2\pi\hbar v}$$

$$I = -e(n_R v - n_L v)$$

$$= \frac{e^2}{h} (V_1 - V_2)$$

(for each LL)

# Quantum Hall effect

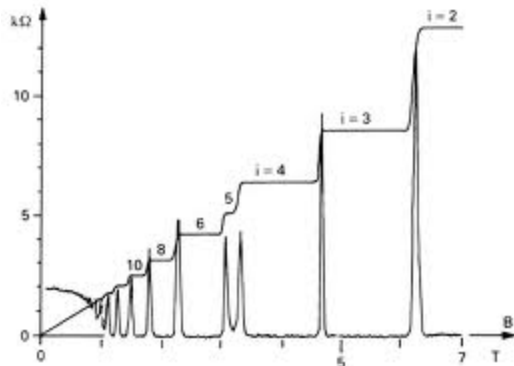


$$I_x = N \frac{e^2}{h} V_y \quad G_{xy} = N \frac{e^2}{h} \quad \text{n.b. } h/e^2 = 25 \text{ kOhms}$$

$$V_x = 0 \quad G_{xx} = 0$$

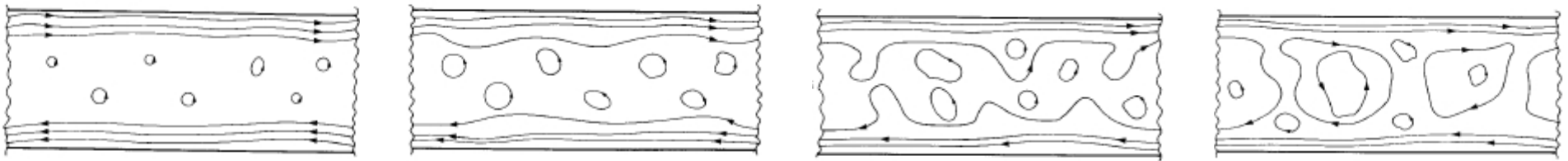
R+L movers are *separately* at equilibrium. No dissipation

$$\sigma_{xx} = \rho_{xx} = 0$$



# Robustness

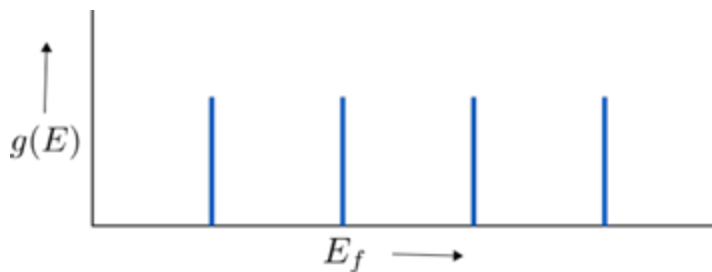
- In real samples, disorder is important, and splits the degeneracy of the bulk LL states
- BUT...it cannot “back-scatter” from one edge to another



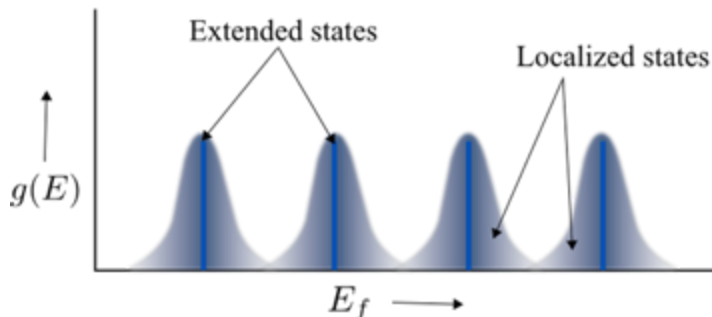
somewhere here an edge state  
“peels off” from boundary and  
crosses the bulk

# Robustness

- In real samples, disorder is important, and splits the degeneracy of the bulk LL states
- BUT...it cannot “back-scatter” from one edge to another



it turns out truly delocalized states occur only at one energy...this is NOT obvious



consequently, IQHE “steps” in Hall conductance are expected to be *infinitely sharp* at  $T=0$ , for a large sample

# IQHE phases

- Actually, the states with different integer quantum Hall conductivity are *different phases of matter* at  $T=0$ : they are sharply and qualitatively distinguished from one another by  $\sigma_{xy}$
- This means that to pass from one IQHE state to another requires a quantum *phase transition*: this corresponds to the point at which the edge state delocalizes and “percolates” through the bulk
- However, unlike most phases of matter, IQHE states *break no symmetry*
- They are distinguished not by symmetry but by “topology” (actually the Hall conductivity can be related to topology...a bit of a long story)

# Topological Insulators

- So the IQHE states are examples of what we now call “Topological Insulators”: states which are distinguished by “protected” edge states
- Until very recently, it was thought that this physics was restricted to high magnetic fields and 2 dimensions
- But it turns out there are other TIs...even in zero field and in both 2d and 3d!

# Topological Insulators

- The simplest TI (other than IQHE) is the 2d “Quantum Spin Hall” state
- It was actually invented by C. Kane and E. Mele in 2005 - they were thinking about the effect of spin-orbit coupling in graphene!
- It has been experimentally confirmed, but not in graphene. However, we can still use that example.

# QSH in graphene

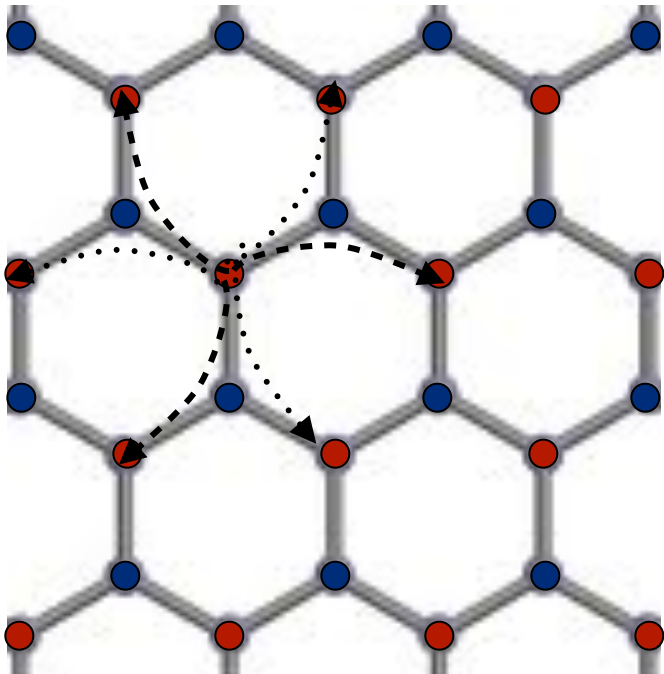
- We need to add SOC to graphene
- What is SOC? You usually learn about it in atoms

$$H_{SOC} = \lambda \mathbf{L} \cdot \mathbf{S}$$

- This gives rise to “fine structure” of the atom - energy is not spin-independent but instead depends upon  $J = L + S$
- It is also present in solids, but is more complicated because solids are not spherically symmetric

# QSH in graphene

- We can go back to the tight-binding model and consider the coupling of spin and orbital motion



Give spin up electrons  
different amplitude to “turn  
right” than to “turn left”

$$+-\lambda$$

And reverse the amplitudes  
for spin down electrons