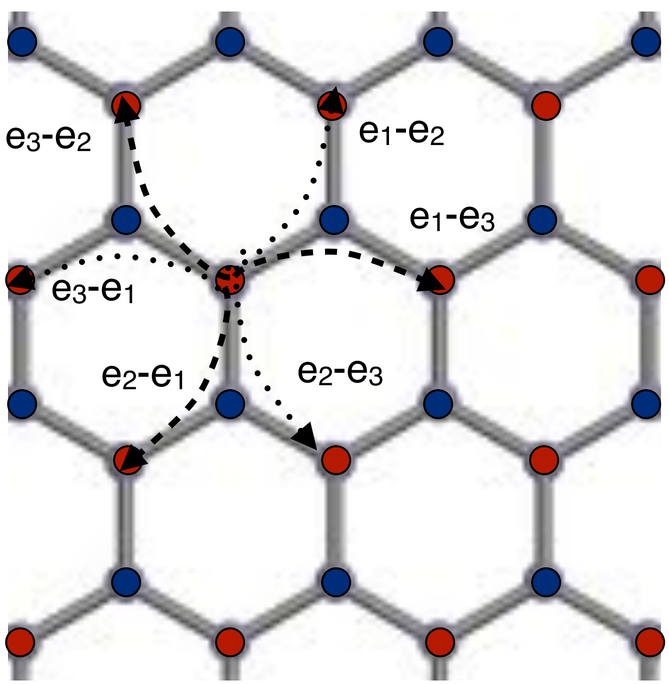
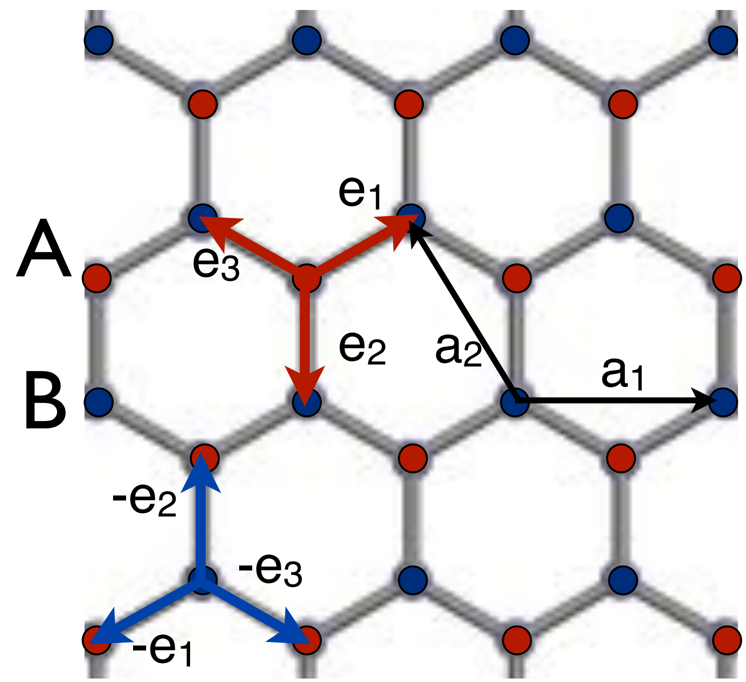
QSHE in graphene

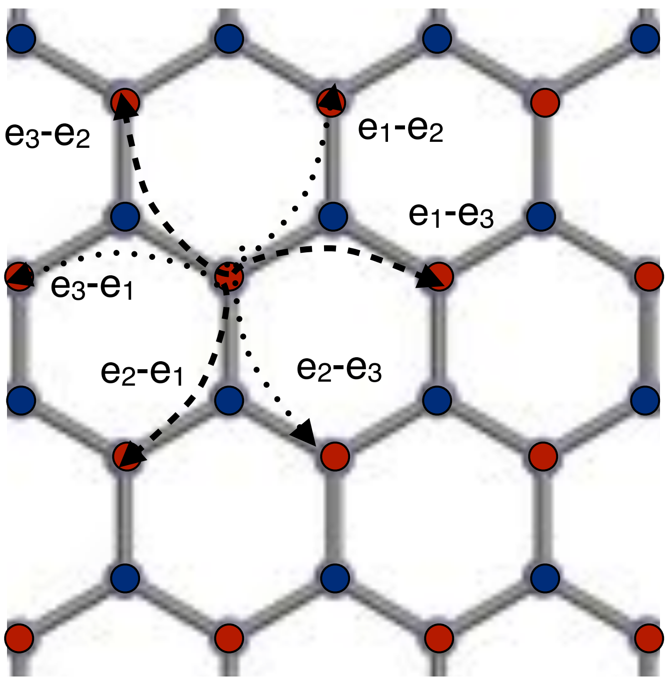
- We can go back to the tight-binding model and consider the coupling of spin and orbital motion





QSHE in graphene

- We can go back to the tight-binding model and consider the coupling of spin and orbital motion



$$\hat{H}\psi_R^{\uparrow} = \cdots \text{ (old terms)}$$
$$+i\lambda \sum_{i=1}^{3} \left[\psi^{\uparrow}_{R+e_i-e_{i+1}} - \psi^{\uparrow}_{R+e_i-e_{i-1}} \right]$$
$$R \in A$$

opposite sign for B sublattice

QSHE in graphene

$$\hat{H}\psi_R^\uparrow = \cdots + i\lambda \sum_{i=1}^{3} \left[\psi^\uparrow_{R+e_i-e_{i+1}} - \psi^\uparrow_{R+e_i-e_{i-1}}\right] \qquad R \in A$$

$$\psi_R = \psi_{A/B} e^{i\mathbf{k}\cdot\mathbf{R}}$$

$$\begin{pmatrix} \epsilon_0 + \Delta(k) & f(k) \\ f^*(k) & \epsilon_0 - \Delta(k) \end{pmatrix} \begin{pmatrix} \psi_A^\uparrow \\ \psi_B^\uparrow \end{pmatrix} = \epsilon \begin{pmatrix} \psi_A^\uparrow \\ \psi_B^\uparrow \end{pmatrix}$$

$$-\Delta^\downarrow = \Delta^\uparrow = \Delta(k) = -2\lambda \sum_{i=1}^{3} \sin\left[k \cdot (e_i - e_{i+1})\right]$$

spectrum $$\epsilon_\pm(k) = \epsilon_0 \pm \sqrt{|f(k)|^2 + (\Delta(k))^2}$$

QSHE in graphene

$$\epsilon_\pm(k) = \epsilon_0 \pm \sqrt{|f(k)|^2 + (\Delta(k))^2}$$

- We still have separated conduction and valence bands, but now it is harder for them to touch: need f=Δ=0!
- Can calculate

$$\Delta(K) = -3\sqrt{3}\lambda \qquad\qquad \Delta(K') = +3\sqrt{3}\lambda$$

- So there is no touching - i.e. a full gap

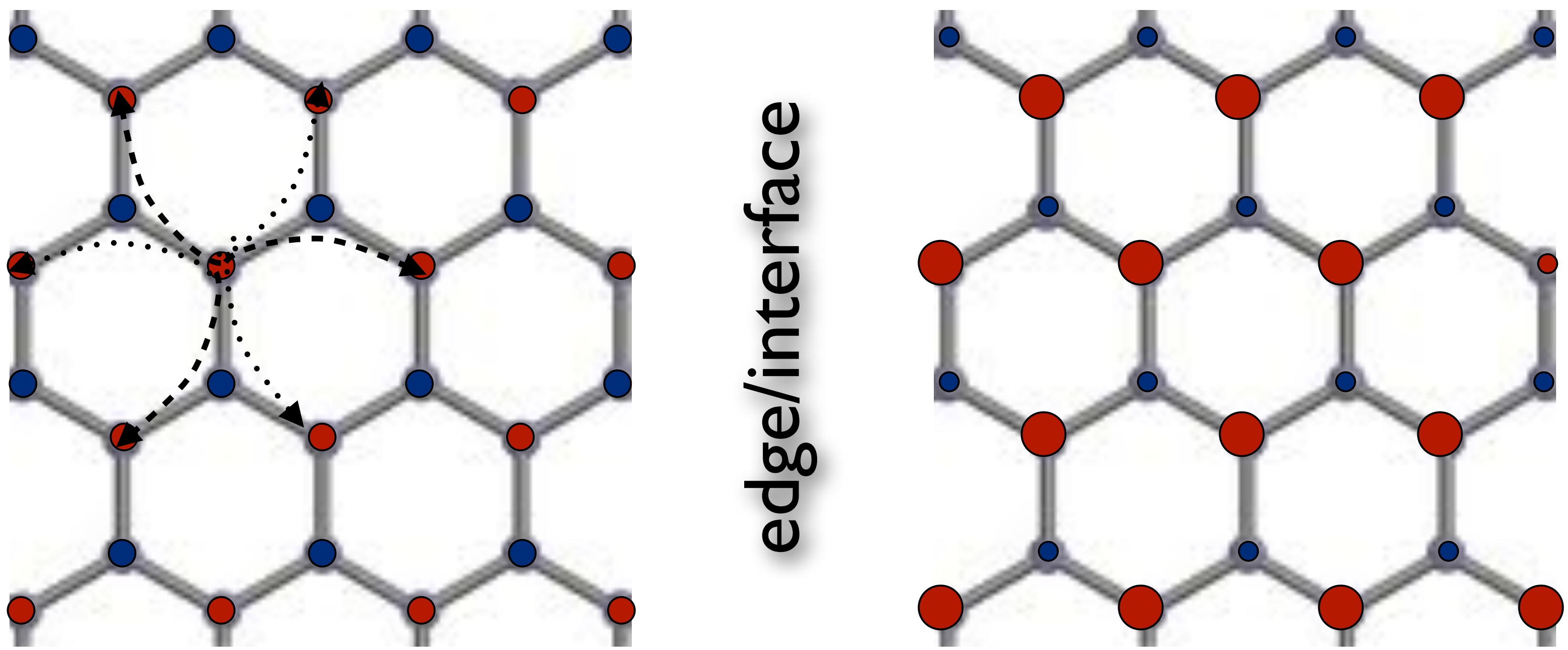
QSHE in graphene

- From just the energy bands, it might appear like an ordinary semiconductor or band insulator
- BUT...it has unusual edge states, similar to IQHE
- To see them, we need to model an edge - complicated!

QSHE in graphene

- Instead of an edge = interface to vacuum, we consider an interface to a "trivial" insulator
  - n.b. "trivial" means *topologically trivial.* This means that it can be deformed into just a bunch of localized electrons bound to individual atoms
  - It is expected that edge states to a trivial insulator are the same as those to vacuum, which is also "trivial"
  - This can be verified explicitly, but it is more involved

QSHE in graphene

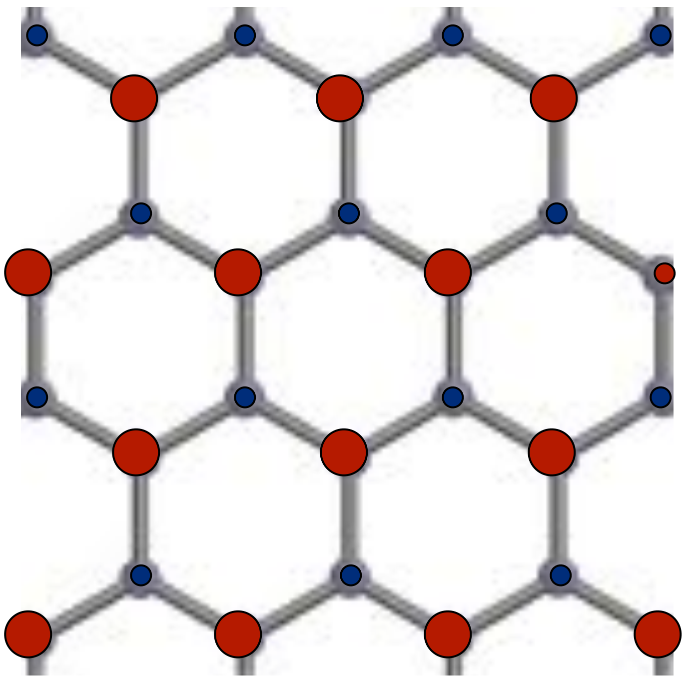


SOC $\lambda \neq 0$

Staggered potential
$V \neq 0$

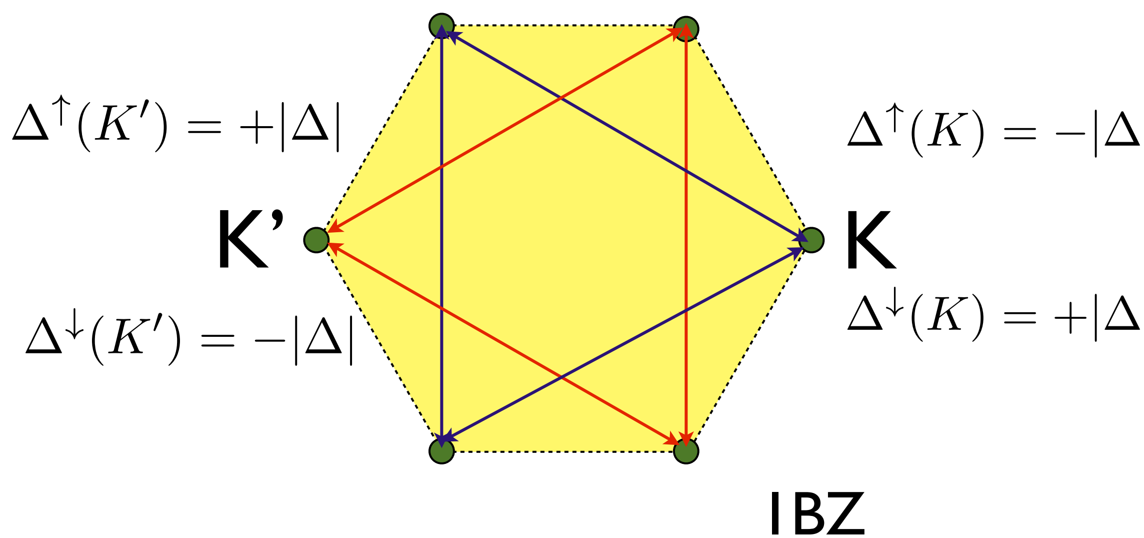
QSHE in graphene

- Back to tight-binding model

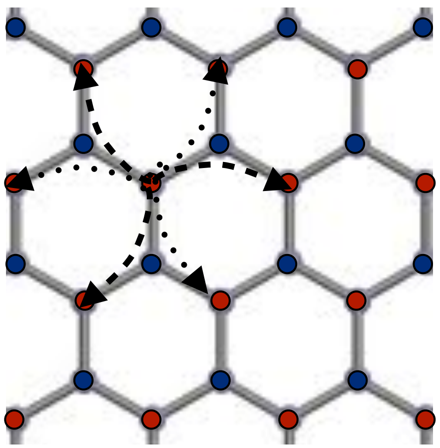


Staggered potential
V≠0

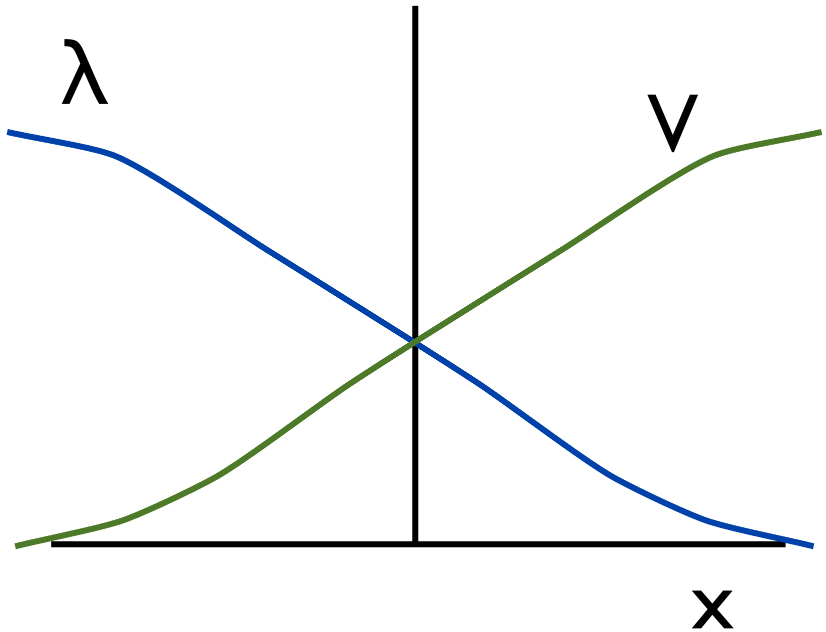
$$\begin{pmatrix} \epsilon_0 + \Delta^\sigma(k) + V & f(k) \\ f^*(k) & \epsilon_0 - \Delta^\sigma(k) - V \end{pmatrix} \begin{pmatrix} \psi_A^\sigma \\ \psi_B^\sigma \end{pmatrix} = \epsilon \begin{pmatrix} \psi_A^\sigma \\ \psi_B^\sigma \end{pmatrix}$$

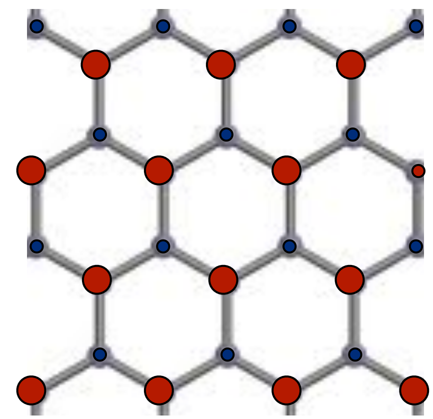


QSHE in graphene



SOC $\lambda \neq 0$





Staggered potential $V \neq 0$

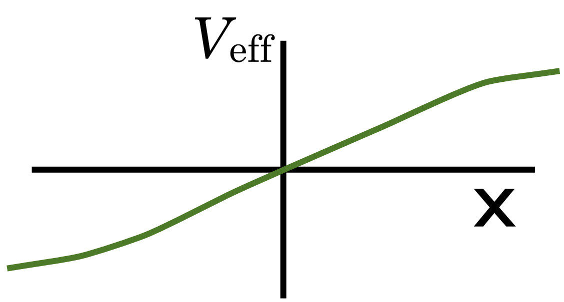
k	spin	gap
K	⇑	V-|Δ|
K	⇓	V+|Δ|
K’	⇑	V+|Δ|
K’	⇓	V-|Δ|

gap vanishes

QSHE in graphene

- Turns out the vanishing gap supports edge states!
  - We can see this using the Dirac equation!
  - Near K point, for up electrons

$$\begin{pmatrix} \epsilon_0 + V_{\text{eff}} & v(q_x + iq_y) \\ v(q_x - iq_y) & \epsilon_0 - V_{\text{eff}} \end{pmatrix} \begin{pmatrix} \psi_A \\ \psi_B \end{pmatrix} = \epsilon \begin{pmatrix} \psi_A \\ \psi_B \end{pmatrix}$$



$$q_x \rightarrow -i\partial_x$$

QSHE in graphene

- Dirac equation for K, up spin

$$\begin{pmatrix} \epsilon_0 + V_{\rm eff}(x) & -iv(\partial_x - q_y) \\ -iv(\partial_x + q_y) & \epsilon_0 - V_{\rm eff}(x) \end{pmatrix} \begin{pmatrix} \psi_A \\ \psi_B \end{pmatrix} = \epsilon \begin{pmatrix} \psi_A \\ \psi_B \end{pmatrix}$$

- Look for a special solution

$$\begin{pmatrix} \psi_A \\ \psi_B \end{pmatrix} = \begin{pmatrix} 1 \\ i \end{pmatrix} g(x) \qquad \epsilon = \epsilon_0 - v q_y$$

$$g(x) = g_0 e^{-\frac{1}{v}\int_0^x dx'\, V_{\rm eff}(x')}$$

converges for positive and negative x because $V_{\rm eff}(x)$ changes sign

QSHE in graphene

- Dirac equation for K, up spin

$$\left(\begin{array}{cc} \epsilon_0 + V_{\rm eff}(x) & -iv(\partial_x - q_y) \\ -iv(\partial_x + q_y) & \epsilon_0 - V_{\rm eff}(x) \end{array}\right)\left(\begin{array}{c} \psi_A \\ \psi_B \end{array}\right) = \epsilon \left(\begin{array}{c} \psi_A \\ \psi_B \end{array}\right)$$

- Look for a special solution

$$\left(\begin{array}{c} \psi_A \\ \psi_B \end{array}\right) = \left(\begin{array}{c} 1 \\ i \end{array}\right) g(x)$$

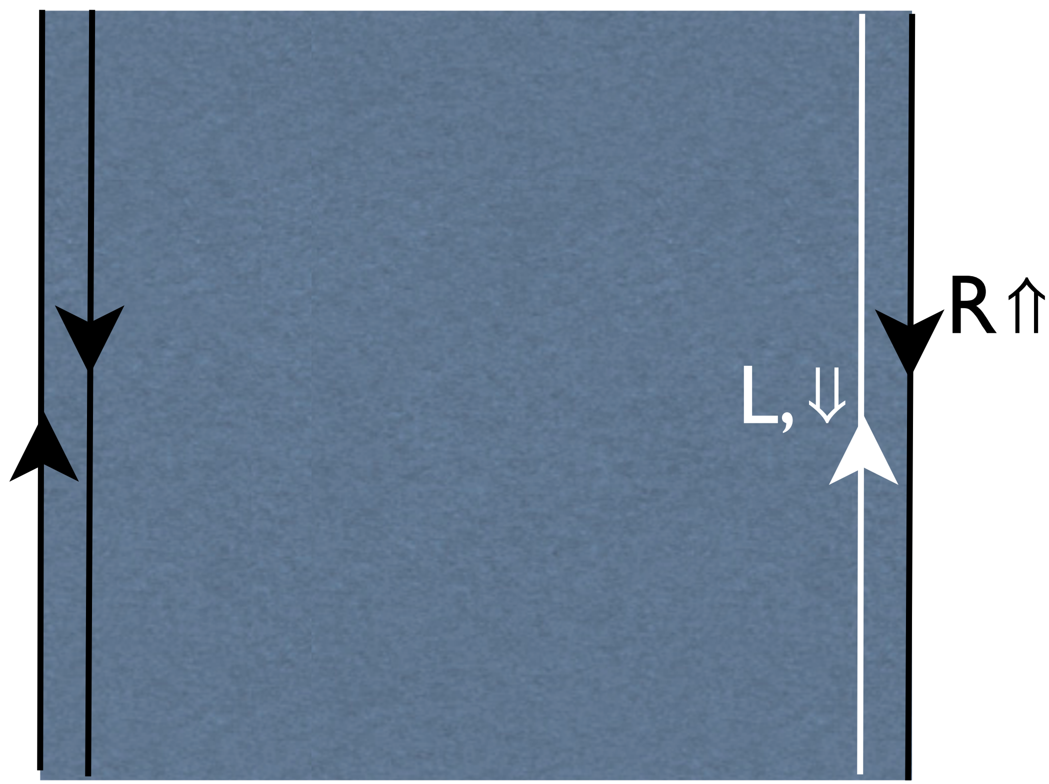
$$\epsilon = \epsilon_0 - v q_y$$

$$g(x) = g_0 e^{-\frac{1}{v}\int_0^x dx' \, V_{\rm eff}(x')}$$

describes a chiral edge state like in IQHE

QSHE in graphene

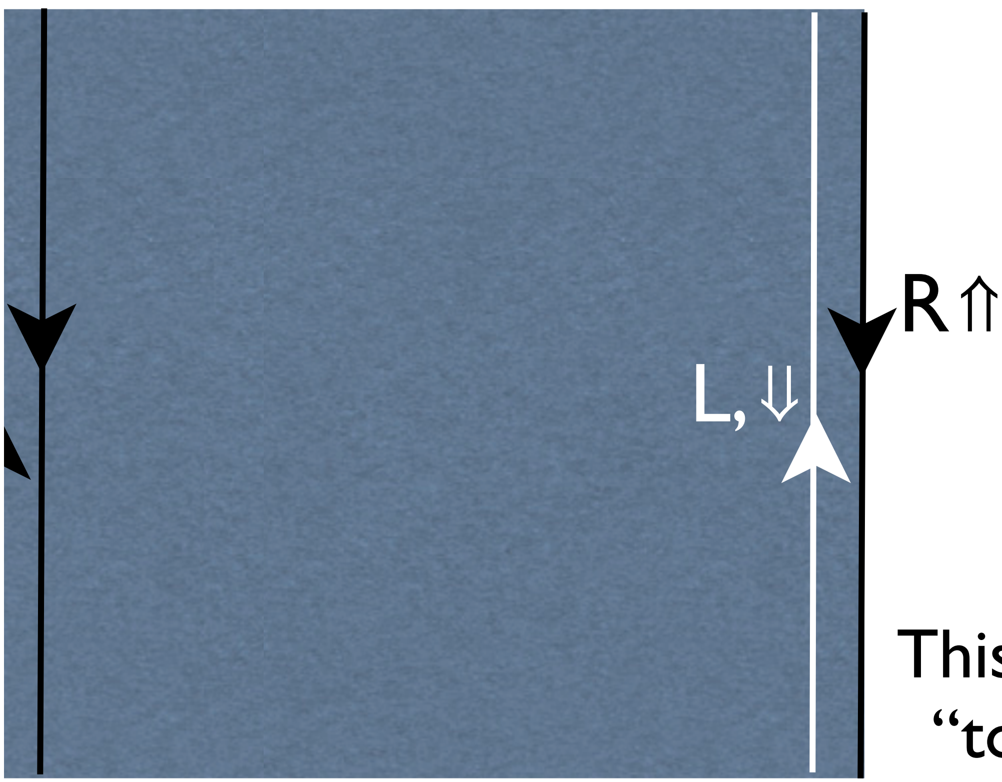
- Edge states!



like IQHE but with counter-propagating edge states for opposite spin

QSHE in graphene

- Edge states!



Lots of cool consequences

related physics in 3d

Verified in many experiments

This is now considered just one of many "topological" phases that can occur in quantum systems!