University of California Santa Barbara

Johnson Noise

And the Derivation of Absolute Temperatures and Boltzmann's Constant

Johnson Noise Lab

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Abstract

Johnson Noise is the thermodynamic property inherent to the resistance of a circuit, reliant wholly on the temperature and conductivity of the system and independent of any geometry or materials used in the individual components. The phenomena of Johnson-Nyquist Noise was discovered by J.B. Johnson at Bell Labs in 1926 and theoretically derived by Harry Nyquist later that year and published in the Physical Review [1]. The following lab will utilize a series of shorted resistors and a filtered amplifier to create a readable signal from which the fundamental thermodynamic constant and the absolute temperature of the resistor may be derived.

1 Equipment

The spectral analyzer used for this lab is a precision digital instrument which converts electric signals into the frequency domain via Fast Fourier Transformation. The analyzer will be used to both characterize the gain of the amplifier-filter chain, and to calculate the Root Mean Square (RMS) band voltage of the noise of a resistor within the passed frequency band. Similarly, the low noise filter and pre-amplifier are sensitive electronic devices that respectively restrict output to a band of frequencies with high rolloff, and amplifies the applied signal with a variable low-noise gain. Both the pre-amp and the spectral analyzer can be seriously damaged by improper usage; read the instructions on their handling carefully and note the possible sources of damage. A White Noise Generator is also included to allow an easy characterization of the system's gain.

1.1 Spectral Analyzer

The SR760 spectral analyzer provided comes with a floppy disk RW drive to allow the storage and transfer of experimental data to any system with a compatible drive. An external usb compatible floppy drive is supplied to allow data to be transfered to any computer.

To turn the analyzer on, use the button on the bottom left of the screen and allow for a short boot period. The analyzer will always begin with its auto-offset feature on. This feature can not only slow data taking, but change the state of the system enough to affect the data. For this reason the feature should only be used inbetween experiments. To turn off the auto-offset, press the **INPUT** hard button located in the *Menu* section of the analyzer's control panel and use the lowest soft button on the panel located next to the screen to turn off the auto-offset.

In this menu the signal source, grounding, and coupling can all be changed, as well as the amplification and attenuation of the signal under Input Range. Make sure the source is set to A, and that the grounding is not on Float. Coupling should be set to AC for this experiment.

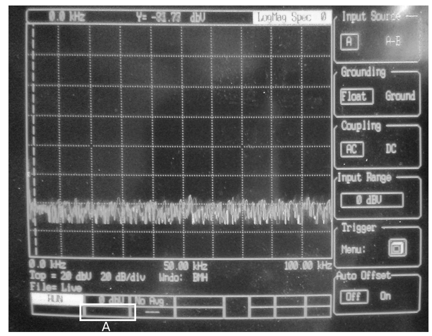
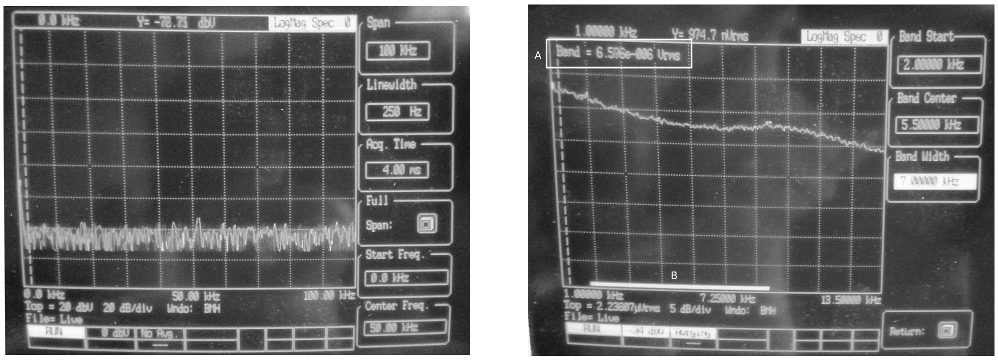


Figure 1: Photo of the Input menu. Soft-button options are on right of the screen, and a box (A) showing the location of the Overload warning system

The primary concern for the spectral analyzer is a large signal being further amplified by the analyzer's own circuits and causing an overload, burning the components out. Should the device be in danger of this the Overload feature at the bottom of the screen, found under the dbV reading, will flash on. To prevent this, hit the **AUTO RANGE** hard button in the *Entry* group. This will automatically set the input attenuation to an appropriate level. The input range should not be manually adjusted until the analyzer and its safe operation is sufficiently understood. Pressing the auto-range button will allow the feature to be turned on and off as needed.

By pressing the **Freq** button one can change the range of the analyzer's screen and center the graph on any frequency between 0 and 100 kHz. Span will tell you the width of the window, and can be reduced from the 100 kHz maximum in one-half-width decrements. Linewidth sets the unit divisions that the screen marker moves through, which can be accessed by hitting the **MARKER** button found under *Control*. Be sure to hit the marker button whenever interacting with the screen to prevent accidental changing of the active submenu. The final option in the frequency menu is the Start Freq soft button, which allows the window to begin at a specific allowed frequency. All options in the Range or Start groups are automatically modified when any feature within that group is changed.

The **MEAS** button will bring up a list of submenus. Of primary interest to this lab is the Units tab, which sets the units the analyzer outputs to. Make



(a) Frequency Menu (b) Analysis Menu

Figure 2: Photos of the Frequency and Analysis menus. In the Analysis window, the band voltage (A) can be found in the upper left, and a bar (B) showing the width of the analysis band

sure the option for Volts RMS is active. Other than Measure, which should be set to Spectrum, the options available are not necessary. Consult the user manual should they interest you.

The **ANALYSIS** button brings up the possible real-time analysis features that can be performed. You can turn on and off the Band analysis feature which will be used to record the total voltage within a specific bandwidth here. When analysis is turned on, the analyzer will transfer you to a menu that allows you to set the width, midpoint, and starting point of the band. The calculated value of the band is recorded in the upper left of the screen while analysis is active.

The **AVERAGE** button brings up a number of options useful for both troubleshooting your experiment and for data acquisition. Turning Averaging on and off allows you to view the input signal in real-time or as an averaged frequency-dependent magnitude. The **START** and **PAUSE/CONT** hard buttons under *Control* will be used with this feature for data taking. The Number Averages option sets how many samples will be taken in each average, which should be changed depending on whether you are using the Linear or Exponential averaging found under Averaging Mode. Linear mode is a single data set taken over the entirety of the sample size, while Exponential is a running average that updates continuously. Exponential is best used for setup debugging and with a sample size in the hundreds, while Linear is used for official data acquisition and using sample sizes of the thousands of

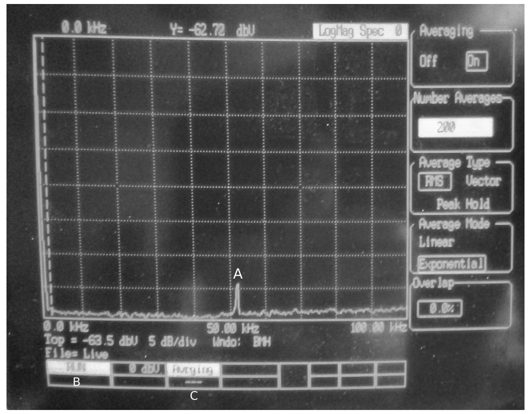


Figure 3: Photo of the Average menu, showing the CRT self-interference (A), the Run/Stop setting (B), and the averaging on/off setting (C)

samples. The number pad on the front panel can be used to set the number of samples.

The final button of interest on the analyzer is the **STORE/RECALL** option. This allows the saving of ASCII and graphical data, and the recall of graphs to and from the supplied 3 inch floppy. When saving use the Save: Data option then press the File Name soft button. Hit **ALT** in the lower left corner of the *Entry* section of the panel and input a filename. The system of the analyzer allows a 12 character filename for any save data, 4 characters of which will be the .csv filetype, which you must append personally. This allows a maximum of 8 characters for your filename. Be careful to always press the alt feature, as forgetting will cause you to use the features of the hard buttons instead of their alpha characters, which may cause you to lose your data run. After entering your filename, press Enter, then Save: ASCII and wait for the analyzer to finish writing to disk. The drive will make noise during this procedure, which is normal. Files can be deleted by using the external drive and any computer, or by using the Disk: Utilities function and scrolling down to the desired file with the panel knob and hitting Erase File.

1.2 Filter

Take some time to familiarize yourself with the low noise filter supplied, where its inputs and outputs are located, and how the dials function. The filter is turned on with the toggle switch located on the front of the device, and can be moved to Low Pass, High Pass, Band Pass, and Band Reject modes. This experiment will use the Band Pass feature.

1.3 Pre-Amplifier

The pre-amp is the easiest device in this lab to damage, so please exercise care. Unplug the device at the back if it is plugged in (gripping the plug and not the cord, please) and turn the amp on. The amplifier will be running off battery power for this experiment to further reduce system noise. Be sure to plug the amplifier back in after you are finished with the lab so the next group has a charged battery. Check that the amplifier is set to ground at the lower left of the front panel. If not, hit the **COUPLING** button until **GND** is lit. At the upper right of the amplifier is the **RESET** button and the **OVLD** light. This is the warning system of the amplifier. Overloading the amplifier will damage its internals. To prevent this, at any time that the light remains solid, move the coupling back to ground and reduce the voltage of whatever input your are passing. If the light persists, hit reset and see if this fixes the problem. If not, power down and inform your lab supervisor. Whenever anything is plugged into or unplugged from the connectors on the amplifier it should be returned to ground to prevent transients from overloading the system.

Check that the Source of the amplifier is set to A, and then examine the Filter Cutoffs. For this lab the bandwidth of the system should be set to 7 kHz starting from a frequency of 3 kHz. Therefore, the 3 kHz and 10 kHz lights should be active and the low noise filter should match this. While the amp is on ground, play with the **HIGH-PASS**, **LOW-PASS**, and **ROLLOFF** buttons to understand how they change the bandpass of the filter. Return them to 3 and 10 kHz and then look at the Gain Mode and Gain options. For this lab, the Low Noise mode should be used, and the gain should be reduced from whatever level it is currently at to the lowest level possible (1x).

1.4 White Noise Generator

The White Noise Generator that the University has supplied outputs a "white" electric signal, covering a broad spectrum with a nearly flat, constant signal. The internals can be seen through the cover, and include quite large capacitors, a massive inductor, and a gas-discharge tube that is the primary noise source. The generator is turned on with the toggle on the front, and requires a thirty second warm-up period before it becomes usable. The generator should be set to the 500 kHz range and Low output. The voltage can be increased by turning the left-hand knob clockwise, and the output should equal the level visible on the attached voltmeter multiplied by the factor set by the knob on the right. Set both these to their lowest level to start.

1.5 Liquid Nitrogen

This lab utilizes liquid nitrogen, a dangerous cryogenic liquid that can cause burns upon direct contact. As such, in addition to the supplied dewers are a face guard and gloves, which must be worn whenever the liquid is transferred between vessels. Gloves should also be worn whenever directly interacting with the liquid, such as inserting or removing the resistor elements.

1.6 Miscellaneous Equipment

A Digital Multimeter with a BNC adapter, and several high quality BNC cables are supplied for this lab, as well as an ambient thermometer.

2 Experiment 1: Gain

2.1 Theory

The Johnson noise that is to be measured is on the scale of nanovolts, far below the ability of most conventional electronics to detect. To remedy this issue, a low noise pre-amplifier and filter will be used to amplify the signal enough within a specific frequency window to be characterized. The addition of this amplifier chain modifies the derivation of the voltage of the Johnson Noise (as will be shown later.)
The effective gain of a system of electronics as a function of frequency is given as

$$g(f) = \frac{V_o(f)}{V_i(f)} \tag{1}$$

which is the RMS voltage of the output of the amplifier/filter chain when fed a signal by the white noise generator, over the RMS voltage of the white noise generator itself. This equation for gain assumes a pure-sinusoidal signal of frequency f, which the white noise generator produces effectively for all frequencies simultaneously.

2.2 Methodology

Turn the spectral analyzer on with nothing attached to the input, and remove the auto-offset, as well as band analysis if it is on (it slows down the analyzer quite a bit.) Turn on exponential averaging and auto-ranging, and be sure that the frequency range covers the full 100 kHz possible. Now, observe the analyzer's window. A non-zero level of background noise should be seen with many peaks, most of which are not of interest to us. Going to the Input screen, move to the Input Range option and **slowly** increase the attenuation, observing the changes to the background levels that occur. Once the attenuation is around -4 to 0 dBV, you should only see two peaks. The peak at the far left is the $\frac{1}{f}$ interference (or "pink noise") found in every form of electronics. To remove this noise the low frequency range is ignored with a high-pass filter. The spike found around 50 kHz is an unfortunate side effect of the CRT monitor, and the magnets that make it work, that the analyzer uses.

Question

What is the refresh rate of the monitor?

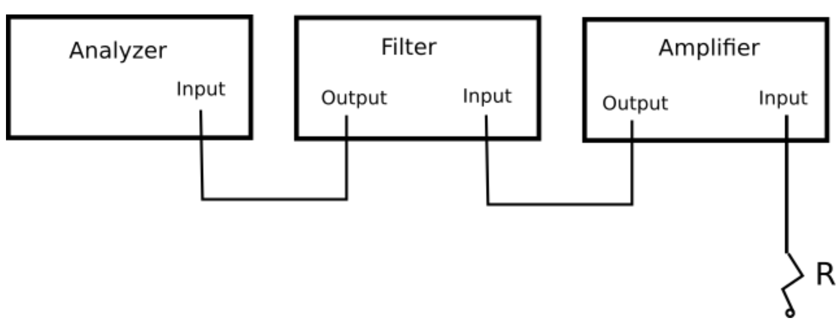


Figure 4: Representation of the setup of the equipment chain, with the amplified resistor feeding the filter and spectral analyzer

In order to prevent this spike interfering with the experiment, a low-pass filter is used below its range to exclude this region. However, it is still a good idea to keep the analyzer as far from the pre-amplifier as possible to be sure

that any noise it produces is not picked up by the amp and boosted.
To continue, turn on the white noise generator, and reduce its output to the minimum. Once the generator is active, return the analyzer to auto-range, and plug the white noise generator into the analyzer. Increase the signal of the generator until you see the signal begin to rise above the background level, then remove the generator from the analyzer. Turn on the filter and pre-amp and make sure the amp is at ground. Connect all the cables such that the pre-amp is fed by the generator and outputs to the filter, which itself outputs to the analyzer. Move the amp to AC coupling and increase its gain setting until it overloads, then reduce to the gain setting directly below this point. Once you have recorded this max gain setting, increase the output of the generator until the overload light begins to flash, then reduce the output until it stops. Record the settings of your equipment and calculate what the output of the generator should be. Move the generator back to the analyzer and check that the output is above the background, and that the signal is white. Using the marker, check along the signal to see that the output equals the expected voltage the generator should be producing. Change the frequency range of the analyzer to a window size that captures the entire bandpass and enough of the region on either side without running into the problem regions seen before, then set averaging to Linear with a sample count in the area of 1500 samples or higher and hit run. Save enough of these averaged data sets to get a gaussian distribution, then plug the generator back into the amp-filter chain and do the same for the amplified output.

2.3 Analysis

Move the saved data to an external computer and average the RMS voltage data you took for $V_o(f)$ and $V_i(f)$ and calculate eq. 1, as well as the uncertainties. Graph $g(f)$ against frequency and apply a fit to get an equation for $g(f)$.

3 Experiment 2: Johnson Noise, Part 1

3.1 Theory

According to Nyquist's theorem the RMS voltage V_j across any resistor R at a temperature T is

$$dV_j^2 = 4Rk_BTdf$$

over the frequency interval df. The derivation of this equation was performed purely via the treatment of simple circuits and thermodynamical principles, and at only four pages long is worth a read.

Measurement of this voltage by any system creates a modified, frequency dependent resistance R_f due to the capacitance in the system C forming a low-pass filter (essentially a frequency dependent *short*) as expressed by

$$R_f = \frac{R}{1 + (2\pi fCR)^2}$$

Additionally, the inclusion of the amplifier to the measurement system creates an amplified voltage dependent on the Johnson voltage

$$dV^2 = [g(f)]^2 dV_j^2$$

Substituting the Johnson voltage and integrating this equation, the final expected RMS voltage as shown by the spectral analyzer is

$$V^2 = 4Rk_BTG \tag{2}$$

where G is the Band Gain of the system

$$G = \int_0^\infty \frac{[g(f)]^2 df}{1 + (2\pi fCR)^2} \tag{3}$$

Every value of resistor will require its own numerical value for G.

The square of the RMS voltage recorded by the spectral analyzer is not just the amplified Johnson noise, however, but rather the Johnson voltage and the ambient noise of the system added in quadrature

$$V_{meas}^2 = V^2 + V_{system}^2 \tag{4}$$

3.2 Methodology

Using the supplied DMM, measure the actual resistance of each resistor tail and quickly estimate the uncertainty. To calculate G you will need the capacitance C as seen by the resistor under investigation. Remembering your analog electronics, or referencing an available source, determine how much of the system's capacitance is "seen" by the resistor (the amplifier can be seen as a massive, large-gain *transistor*.)

> **Question**
>
> Are you looking into the "base" or "emitter" of the amplifier as the resistor? Is the impedance larger or smaller on the other side? Which side of the amplifier then is more important for C as a result?

Once you have determined how much of the impedance is a concern, look up the values needed and calculate the capacitance of the BNC cables as necessary, and then sum the series capacitance (the connectors for the input and output of all the electronics have a capacitance, which can be found in their user manuals.

Record the room temperature, then turn on the spectral analyzer and remove the auto-offset as before, and reduce the frequency range to the 3-10 kHz range. Turn on Band Analysis and set it to cover the bandpass. With the amplifier calibrated to the gain settings you found earlier, attach the resistor tails and run Linear averaging as before. Record the band voltage, located in the upper left of the analyzer monitor, and continue averaging until you have a data set that is sufficiently large. Repeat this for each resistor, including the tail labeled "short" which will give the value of V_{system}.

> **Question**
>
> Is there any difference between the band voltage of the Short tail and the amplifier at ground coupling? Do you expect there to be?

Keep an eye on the temperature of the room throughout the test. Any changes in the temperature should factor into your uncertainty.

3.3 Analysis

Average the voltages you measured, and subtract the short from each resistor value in quadrature as implied by eq. 4. Calculate a numerical value for G

using the values you measured and the equation for $g(f)$ you found and integrating between 3 and 10 kHz.

Question

Does G converge? How fast? Is it possible to integrate over the whole of the zero to infinity range? Is it sensible to?

Solving eq. 2 for resistance, the temperature of the resistor can be found by graphing

$$\frac{V^2}{4k_B G} = R$$

and applying a line fit. Look up the accepted value of Boltzmann's constant and do so. The slope of the line should be the temperature of the resistor in absolute scale. From this value and the temperature of the room in Celsius, a value for Absolute Zero in Celsius can be calculated.

Question

How close is your value of absolute zero? Given your result, is Johnson Noise useful for measuring temperature?

4 Experiment 3: Johnson Noise, Part 2

4.1 Theory

Liquid Nitrogen (LN_2) in an environment of Standard Temperature and Pressure (STP) is a boiling liquid, i.e. in a constant state of phase change, and a liquid in the process of phase transition under constant conditions maintains a reliable temperature. By submerging the resistors in LN_2 the Johnson Noise at a specific temperature can be found. Additionally, with two points of temperature data, the thermodynamic constant k_B can be calculated.

4.2 Methodology

Empty the dewer attached to the table and, observing standard safety precautions, fill the vessel halfway with liquid nitrogen. Cover the dewer and, using gloves, drop the resistor end of a tail into the dewer. Wait for the liquid to stop boiling and reduce to a light simmer with small bubbles coming off the upper end of the tail (this takes around thirty seconds.) Plug the tail into the amplifier and record the band voltage as before. At this point, please remember to plug the analyzer back in after you have powered it down.

Question

Is it necessary to chill the Short tail? Do you expect its band voltage to change?

4.3 Analysis

Calculate the temperatue of liquid nitrogen as before, and compare to the accepted value of 77^o Kelvin. Solving eq 2 again, this time as a function of temperature, the fundamental constant of thermodynamics can be found by graphing

$$\frac{V^2}{4RG} = T$$

and applying a line fit. The slope is your value of k_B.

Question

What could be done to improve your findings? What temperatures would you prefer to work in?

References

[1] Harry Nyquist, *Thermal Agitation of Electric Charge in Conductors* Physical Review, Vol 32 1928

[2] *Johnson Noise and Shot Noise* MIT Department of Physics, 2013