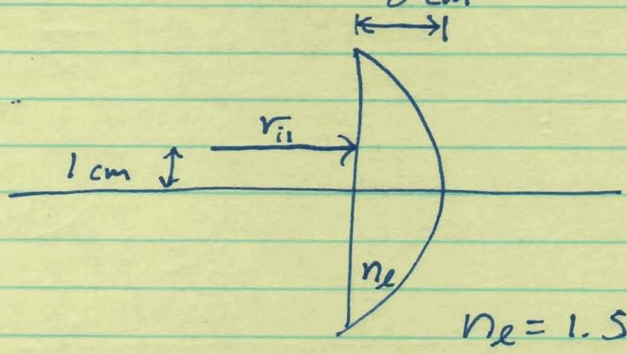
1) a)



$n_e = 1.5$

$$r_{i1} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} \alpha_{i1} \\ y_{i1} \end{pmatrix}$$

$$R_1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad T_1 = \begin{pmatrix} 1 & 0 \\ 10/3 & 1 \end{pmatrix} \quad R_2 = \begin{pmatrix} 1 & -1/10 \\ 0 & 1 \end{pmatrix}$$

$$R_2 T_1 R_1 r_{i1} = r_{t2} = \begin{pmatrix} -1/10 \\ 1 \end{pmatrix}$$

b) now add a translation $T_2 = \begin{pmatrix} 1 & 0 \\ x & 1 \end{pmatrix}$

$$T_2 R_2 T_1 R_1 = \begin{pmatrix} 2/3 & -1/10 \\ 2x/3 + 10/3 & -x/10 + 1 \end{pmatrix}$$

c) $$T_2 R_2 T_1 R_1 r_{i1} = \begin{pmatrix} -1/10 \\ -x/10 + 1 \end{pmatrix} = \begin{pmatrix} \alpha_{t2} \\ y_x \end{pmatrix}$$

to find where r_{t2} crosses the optical axis set $y_x = 0$

$\Rightarrow \ x/10 = 1, \quad \boxed{x = 10 \text{ cm}}$

Problem #2

Computing the system matrix
and plugging in the constrains
we get that the element c, or a21 is zero,
which means that a ray
emitted from a given point on the
object plane will be focused to
a corresponding location on
the image plane no matter what the emitted angle.

$$\text{T1} = \begin{pmatrix} 1 & 0 \\ \text{d1} & 1 \end{pmatrix};\ \text{R1} = \begin{pmatrix} 1 & \frac{(1-n)}{R} \\ 0 & 1 \end{pmatrix};\ \text{T2} = \begin{pmatrix} 1 & 0 \\ \frac{d}{n} & 1 \end{pmatrix};$$

$$\text{R2} = \begin{pmatrix} 1 & \frac{(1-n)}{R} \\ 0 & 1 \end{pmatrix};\ \text{T3} = \begin{pmatrix} 1 & 0 \\ \text{d2} & 1 \end{pmatrix};\quad \text{d1} = \text{z1} - h;$$

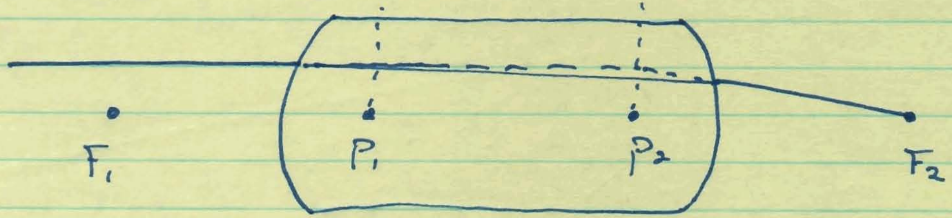
$$\text{d2} = \text{z2} - h;\ \text{z1} = \text{s1} + f;\ \text{z2} = \text{s2} + f;$$

$$\text{s1} = \frac{f^2}{\text{s2}};\ h = \frac{(n-1)\ f * d}{n * R};\ f = \frac{1}{\frac{(n-1)}{R}\left(2 - \frac{(n-1)}{n}\ \frac{d}{R}\right)};$$

MatrixForm[FullSimplify[T3.R2.T2.R1.T1]]

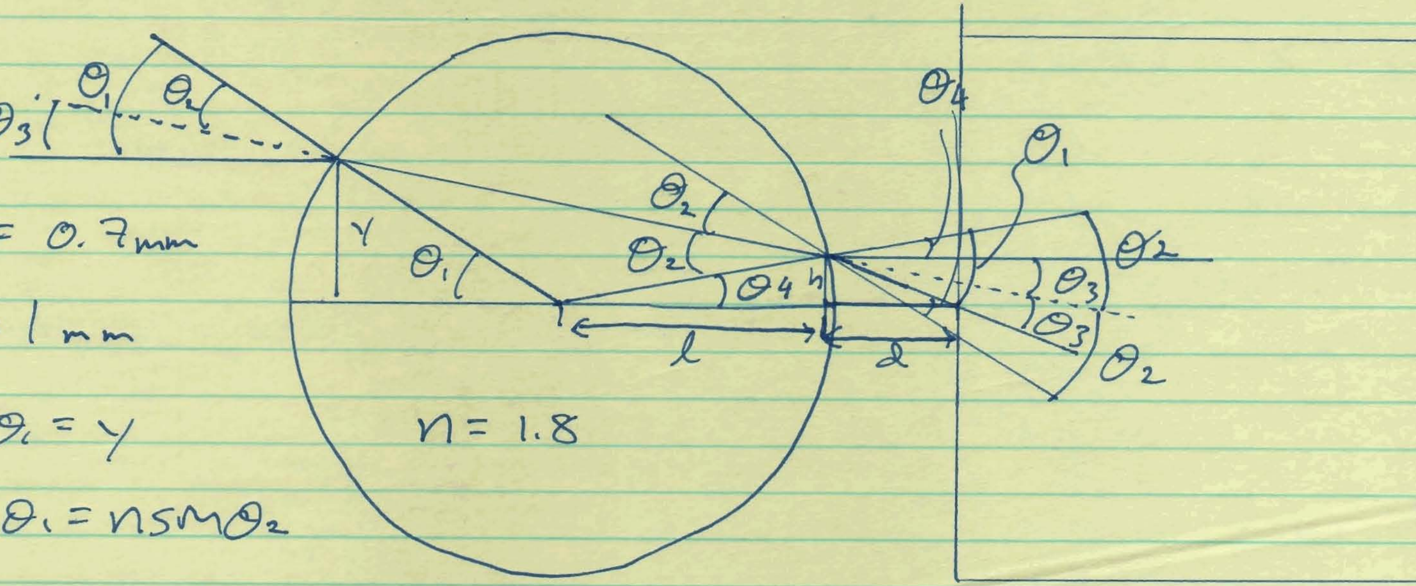
$$\begin{pmatrix} \frac{n\,R^2}{(-1+n)\ (d\ (-1+n)-2\ n\ R)\ \text{s2}} & \frac{(-1+n)\ (d\ (-1+n)-2\ n\ R)}{n\,R^2} \\ 0 & \frac{(-1+n)\ (d\ (-1+n)-2\ n\ R)\ \text{s2}}{n\,R^2} \end{pmatrix}$$

2)



see fig 6.1
in Hecht
p. 243

3)



$y = 0.7\,mm$

$R = 1\,mm$

$\sin\theta_1 = y$

$\sin\theta_1 = n\sin\theta_2$

$\theta_3 = \theta_1 - \theta_2$

$\theta_4 = 2\theta_2 - \theta_1$

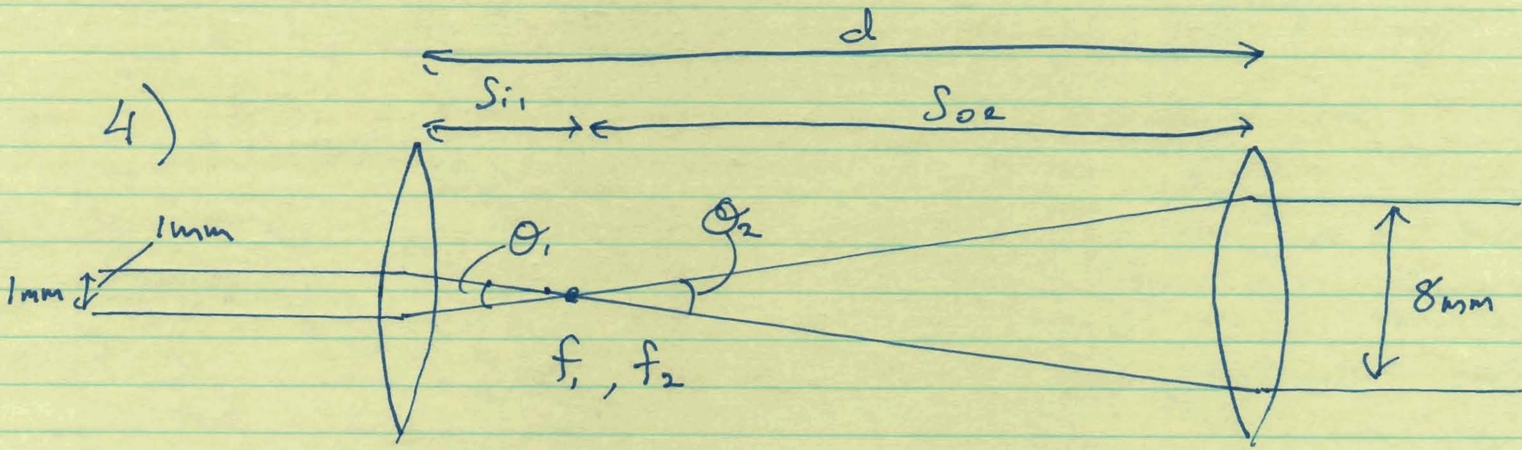
$\sin\theta_4 = h$

$\tan\theta_4 = \frac{h}{l}$

$\tan 2\theta_3 = \frac{h}{d}$

$f = d + l - 1 = 0.0248$

4)



$$\frac{1}{f_1} = \frac{1}{S_{i1}} + \frac{1}{\infty} = \frac{1}{50\,mm} \Rightarrow S_{i1} = 50\,mm$$

$$1/f_2 = \frac{1}{\infty} + \frac{1}{S_{o2}} = \frac{1}{d - S_{i1}}$$ from $d = S_{i1} + S_{o2}$

but since $\theta_1 = \theta_2$ (similar triangles)

we get $S_{o2} = 8 S_{i1}$

so $\underline{d} = 9 S_{i1} = \boxed{450\,mm}$ $\underline{f_2} = 8 S_{i1} = \boxed{400\,mm}$

5) $10 \log \left(\frac{1}{2}\right) = 3\,dB$

$$\frac{3\,dB}{0.2\,dB/km} = 15\,km$$

6) $NA = (n_f^2 - n_c^2)^{1/2}$

$n_f = 1.5 \quad n_c = 1.482 \quad , \quad NA = 0.2317$

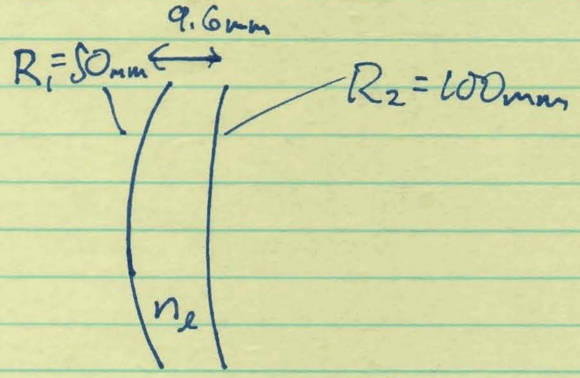
$N_m \approx 0.5\,(\pi D\, NA / \lambda_0)^2$

$D = 50\,\mu m$

$\lambda_0 = 0.85\,\mu m \quad , \quad N_m = 916$

$\lambda_0 = 1.55\,\mu m \quad , \quad N_m = 276$

7)



$n_m =$

$n_l = 2.4$

Problem #7

Here the lense is positive,

which means it's thicker in the middle.

```
Refr1 = {{1, (nl-nm)/R1}, {0, 1}}; T = {{1, 0}, {d/nl, 1}};
Refr2 = {{1, (nm-nl)/R2}, {0, 1}}; nl = 2.4;
nm = 1.9; d = 9.6; R1 = 50; R2 = 100;
MatrixForm[Refr2.T.Refr1]
```

$$\mathrm{Refr1} = \begin{pmatrix} 1 & \frac{(nl-nm)}{R1} \\ 0 & 1 \end{pmatrix};\ T = \begin{pmatrix} 1 & 0 \\ \frac{d}{nl} & 1 \end{pmatrix};$$

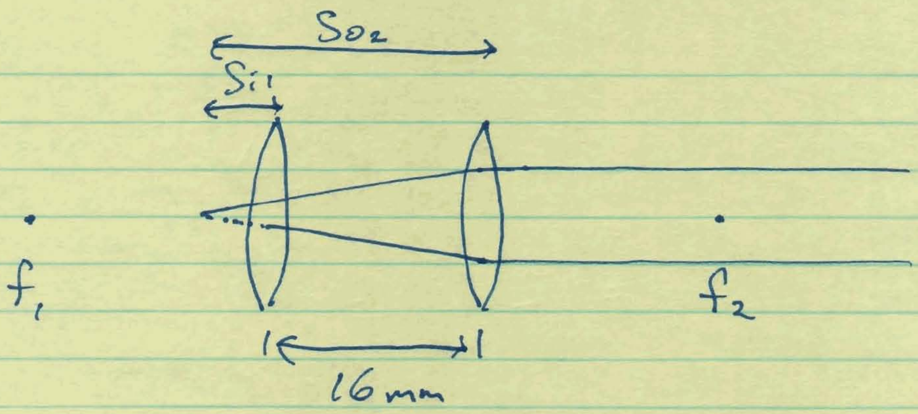
$$\mathrm{Refr2} = \begin{pmatrix} 1 & \frac{(nm-nl)}{R2} \\ 0 & 1 \end{pmatrix};\ nl = 2.4;$$

$$\begin{pmatrix} 0.98 & 0.0048 \\ 4. & 1.04 \end{pmatrix}$$

```
Det[Refr2.T.Refr1]
```

1.

8)



$f_1 = f_2 = 20\,mm$

$$\frac{1}{f_1} = \frac{1}{S_{i1}} + \frac{1}{S_{o1}} \qquad \frac{1}{f_2} = \frac{1}{\infty} + \frac{1}{S_{o2}} \Rightarrow S_{o2} = 20\,mm$$

$$S_{o2} + S_{i1} = d = 16\,mm \Rightarrow S_{i1} = -4\,mm$$

$$S_{o1} = \frac{1}{\frac{1}{20} - \frac{1}{-4}} = \frac{1}{\frac{1}{20} + \frac{5}{20}} = \frac{20}{6} = \boxed{\frac{10}{3}\,mm}$$

Problem #9

Look out for signs.

```
Trans1 = {{1, 0}, {d, 1}}; Trans2 = {{1, 0}, {-d, 1}};
```

$$\mathbf{Trans1} = \begin{pmatrix} 1 & 0 \\ d & 1 \end{pmatrix}; \ \mathbf{Trans2} = \begin{pmatrix} 1 & 0 \\ -d & 1 \end{pmatrix};$$

$$\mathbf{Refl1} = \begin{pmatrix} -1 & \frac{-2}{-R} \\ 0 & 1 \end{pmatrix}; \ \mathbf{Refl2} = \begin{pmatrix} -1 & \frac{-2}{R} \\ 0 & 1 \end{pmatrix};$$

MatrixForm[Simplify[Refl2.Trans2.Refl1.Trans1]]

$$\begin{pmatrix} \frac{4\,d^2-6\,d\,R+R^2}{R^2} & \frac{4\,(d-R)}{R^2} \\ \frac{2\,d\,(-d+R)}{R} & 1-\frac{2\,d}{R} \end{pmatrix}$$

$$\mathbf{Trans1} = \begin{pmatrix} 1 & 0 \\ R & 1 \end{pmatrix}; \ \mathbf{Trans2} = \begin{pmatrix} 1 & 0 \\ -R & 1 \end{pmatrix};$$

$$\mathbf{Refl1} = \begin{pmatrix} -1 & \frac{-2}{-R} \\ 0 & 1 \end{pmatrix}; \ \mathbf{Refl2} = \begin{pmatrix} -1 & \frac{-2}{R} \\ 0 & 1 \end{pmatrix};$$

MatrixForm[Simplify[Refl2.Trans2.
Refl1.Trans1.Refl2.Trans2.Refl1.Trans1]]

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

This of course is the identity matrix,
which when acting on any
ray gives the same ray back again.