Homework # 3
due Wednesday Oct 13 at 5PM

Reading: Hecht

Problems:

1. Light rays enter the plane surface of a glass hemisphere of radius 5 cm and refractive index 1.5. a) Using the system matrix representing the hemisphere, determine the exit elevation and angle of a ray that enters parallel to the optical axis and at an elevation of 1 cm. b) Enlarge the system to a distance *x* beyond the hemisphere and find the new system matrix as a function of *x*. c) Using the new system matrix, determine where the ray described above crossed the optical axis.

2. **Imaging with a thick lens.** Consider a glass lens of refractive index *n*, thickness *d*, and two spherical surfaces of equal radii R (see figure). Determine the system matrix between the two planes at distances d1 and d2 from the vertices of the lens. The lens is placed in air (n=1). Show that the system is an imaging system (*i.e.*, the input and output planes are conjugate) if

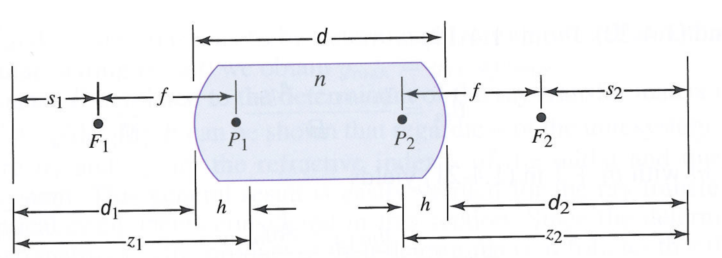
$$\frac{1}{z_1} + \frac{1}{z_2} = \frac{1}{f} \quad \text{or} \quad s_1 s_2 = f^2$$

where $z_1 = d_1 + h, s_1 = z_1 - f, z_2 = d_2 + h, s_2 = z_2 - f$

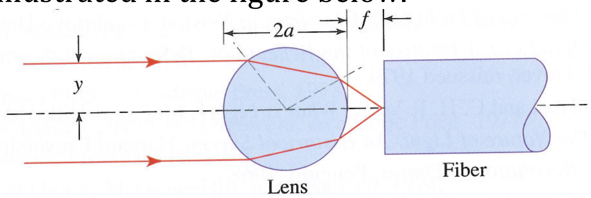
and $h = \dfrac{(n-1)fd}{nR}$

and $\dfrac{1}{f} = \dfrac{(n-1)}{R}[2 - \dfrac{n-1}{n}\dfrac{d}{R}]$.

The points $F_1$ and $F_2$ are known as the front and back focal points, respectively. The points $P_1$ and $P_2$ are known as the first and second principal points, respectively. Show the importance of these points by tracing the trajectories of rays that are incident parallel to the optical axis.



3. **Fiber Coupling Spheres:** Tiny glass balls are often used as lenses to couple light into and out of optical fibers. The fiber end is located at a distance *f* from the sphere. For a sphere of radius $a$ = 1mm and refractive index $n$ = 1.8, determine *f* such that a ray parallel to the optical axis at a distance $y$ = 0.7 mm is focused onto the fiber, as illustrated in the figure below.




4. **(Hecht 5.40)** Two positive lenses are to be used as a laserbeam expander. An axial 1.0-mm diameter beam enters a short focal length positive lens, which is followed by a somewhat longer focal length positive lens from which it emerges with a diameter of 8.0 mm. Given that the first lens has a 50.0 mm focal length, determine the focal length of the second lens and the separation between lenses. Draw a diagram.

5. **(Hecht 5.72)** Given a fused silica fiber with an attenuation of 0.2 dB/km, how far can a signal travel along it before the power level drops by half?

6. **(Hecht 5.73)** The number of modes in a stepped-index fiber is provided by the expression $N_m \approx 0.5(\pi D\, NA/\lambda_0)^2$. Given a fiber with a core diameter of 50 μm and $n_c$=1.482 and $n_f$=1.500, determine $N_m$ when the fiber is illuminated by an LED emitting at a central wavelength of 0.85 μm. What if the LED emits at 1.55 μm?

7. **(Hecht 6.16)** A positive meniscus lens with an index of refraction of 2.4 is immersed in a medium of index 1.9. The lens has an axial thickness of 9.6 mm and radii of curvature of 50.0 mm and 100 mm. Compute the system matrix when light is incident on the convex face and show that its determinant is equal to 1.

8. Any eyepiece is made of two thin lenses each of +20-mm focal length, separated by a distance of 16mm. Where must a small object be positioned so that light from the object is rendered parallel by the combination?

9. **(Hecht 6.24)**

6.24* Figure P.6.24 shows two identical concave spherical mirrors forming a so-called confocal cavity. Show, without first specifying the value of $d$, that after traversing the cavity two times the system matrix is

$$\begin{bmatrix} \left(\frac{2d}{r}-1\right)^2-\frac{2d}{r} & \frac{4}{r}\left(\frac{d}{r}-1\right) \\ 2d\left(1-\frac{d}{r}\right) & 1-2\frac{d}{r} \end{bmatrix}.$$

Then for the specific case of $d = r$ show that after four reflections the system is back where it started and the light will retrace its original path.

Figure P.6.24