HRK 24.6

24.6 (a)

First calculate the pressure in standard units

$$P = \frac{1.1 \times 10^{-7}\ cm\ Hg}{76\ cm\ Hg}(1.01 \times 10^5\ Pa) = 1.46 \times 10^{-4}\ Pa$$

The 'number density" ρ_n is given by

$$\begin{aligned} \rho_n &= \frac{N}{V} = \frac{P}{kT} \\ &= \frac{1.46 \times 10^{-4}}{(1.38 \times 10^{-23})(295)} \\ &= 3.59 \times 10^{16} \quad \text{molecules}/m^3 \end{aligned}$$

24.6 (b)

$$\begin{aligned} \lambda &= \frac{1}{\sqrt{2}\ \pi \rho_n d^2} \\ \lambda &= \frac{1}{\sqrt{2}\ \pi (3.59 \times 10^{16})(2.2 \times 10^{-10})^2} \\ \lambda &= 129.5\ m \end{aligned}$$

HRK 24.24

We use the Maxwell-Boltzmann energy distribution

$$n(E) = \frac{2N}{\sqrt{\pi}} \frac{1}{(kT)^{\frac{3}{2}}} E^{\frac{1}{2}} e^{-\frac{E}{kT}}$$

and the fraction of particles with energy between .01 kT and .03 kT is

$$\begin{aligned} \text{fraction} &= \frac{1}{N} \int_{.01kT}^{.03kT} n(E) dE \\ \text{fraction} &= \frac{2}{\sqrt{\pi}(kT)^{\frac{3}{2}}} \int_{.01kT}^{.03kT} E^{\frac{1}{2}} e^{-\frac{E}{kT}} dE \\ \text{fraction} &\approx \frac{2}{\sqrt{\pi}(kT)^{\frac{3}{2}}} \int_{.01kT}^{.03kT} E^{\frac{1}{2}} \left(1 - \frac{E}{kT}\right) dE \\ \text{fraction} &\approx \frac{2}{\sqrt{\pi}(kT)^{\frac{3}{2}}} \int_{.01kT}^{.03kT} E^{\frac{1}{2}} - \frac{E^{\frac{3}{2}}}{kT} dE \\ \text{fraction} &\approx \frac{2}{\sqrt{\pi}(kT)^{\frac{3}{2}}} \left[\frac{2E^{\frac{3}{2}}}{3} - \frac{2E^{\frac{5}{2}}}{5kT} \right]_{.01kT}^{.03kT} \\ \text{fraction} &\approx \frac{2}{\sqrt{\pi}(kT)^{\frac{3}{2}}} \Bigg[\left(\frac{2(.03kT)^{\frac{3}{2}}}{3} - \frac{2(.03kT)^{\frac{5}{2}}}{5kT} \right) \\ &- \left(\frac{2(.01kT)^{\frac{3}{2}}}{3} - \frac{2(.01kT)^{\frac{5}{2}}}{5kT} \right) \Bigg] \\ \text{fraction} &\approx \frac{2}{\sqrt{\pi}} \Bigg[\left(\frac{2(.03)^{\frac{3}{2}}}{3} - \frac{2(.03)^{\frac{5}{2}}}{5} \right) \\ &- \left(\frac{2(.01)^{\frac{3}{2}}}{3} - \frac{2(.01)^{\frac{5}{2}}}{5} \right) \Bigg] \\ \text{fraction} &\approx 0.0031 \qquad \text{i.e. } 31\% \end{aligned}$$

HRK 24.31

We have 6 degrees of freedom, 3 translational and 3 rotational.

24.31 (a)

$$\begin{aligned}
E_{trans} = \frac{1}{2}m\langle v_x^2 + v_y^2 + v_z^2\rangle &= \frac{3}{2}k_B T \\
\langle\frac{1}{2}mv^2\rangle &= \frac{3}{2}k_B T \\
\Rightarrow \langle v^2\rangle &= \frac{3k_B T}{m} \\
\text{Now use: } m &= \rho V = \frac{4}{3}\pi r^3 \rho \\
m &= 3.35 \times 10^{-20}\ kg \\
\text{so that } \langle v^2\rangle &= \frac{3(1.38\times10^{-23})(100)}{3.35\times10^{-20}} \\
\langle v^2\rangle &= 0.12354 \\
\Rightarrow v_{rms} &= \sqrt{\langle v^2\rangle} = 0.3515\ m/s
\end{aligned}$$

24.31 (b)

Because this shape has non-zero length in all 3 dimensions, we have contributions to the rotational energy from rotations about all three axes

$$\begin{aligned}
E_{rot} = \frac{1}{2}I\langle\omega_x^2 + \omega_y^2 + \omega_z^2\rangle &= \frac{3}{2}k_B T \\
\langle\frac{1}{2}I\omega^2\rangle &= \frac{3}{2}k_B T \\
\Rightarrow \langle\omega^2\rangle &= \frac{3k_B T}{I} \\
\langle\omega^2\rangle &= \frac{(3)(5)k_B T}{2mr^2} \qquad \text{using } I_{sphere} = \frac{2mr^2}{5} \\
\langle\omega^2\rangle &= \frac{5\langle v^2\rangle}{2r^2} \\
\langle\omega^2\rangle &= \frac{5(0.12354)}{2(2\times10^{-8}\ m)^2} \\
\langle\omega^2\rangle &= 7.72\times10^{14} \\
\omega_{rms} = \sqrt{\langle\omega^2\rangle} &= 2.778\times10^{7}\ rad/s \\
\nu_{rms} = \frac{\omega_{rms}}{2\pi} = \sqrt{\langle\omega^2\rangle} &= 4.42\times10^{6}\ rev/s
\end{aligned}$$

which is an amazingly high rotation rate. Let's see what the inward acceleration is at the surface of a grain

$$a_{in} = \frac{v^2}{r} = \omega^2 r = 1.54\times10^{7}\ m/s^2 = 1.58\times10^{6} g \qquad (\text{where } g = 9.81\ m/s^2)$$

It seems miraculous that they can hold themselves together.

HRK 25.3

The relevant expression for heat transferred in a phase change is

$$Q = Lm$$

We are interested in the ice/water transition so $L = L_f$ (fusion) and this only affects an unknown portion of the 258 g of water.

$$\begin{aligned} Q &= L_f m \\ 50.4\ kJ &= (333\ kJ/kg)m \\ \Rightarrow m &= 0.151\ kg \text{ of ice forms} \end{aligned}$$

The amount that is still liquid water is

$$0.258 - 0.151 = 0.107\ kg$$

HRK 25.7

We will be using specific heat capacities c of two different materials (water: $c_w = 4190\ J/(kg\ K)$, copper: $c_c = 387\ J/(kg\ K)$) where the general expression for heat (energy) entering/leaving an object is

$$Q = cm\Delta T$$

We also need to account for the heat that is "used up" in turning $4.7g$ of water into steam (so we will need the latent heat of vaporization $Q = mL_v$ – referring to the table in your textbook we see L_v for water is $2256 \times 10^3\ J/kg$)

$$\begin{aligned} \text{Heat flow into water } Q_{water} &= c_w m_{water}\Delta T + m_{w\to s}L_v \\ \text{Heat flow into bowl } Q_{bowl} &= c_c m_{bowl}\Delta T \\ \text{Heat flow out of cylinder } Q_{cylinder} &= -c_c m_{cylinder}\Delta T \end{aligned}$$

We must have that the total amount of heat is conserved. Physically we know that Q_{water} and Q_{bowl} will be greater than zero (because they heat up), and $Q_{cylinder}$ will be negative because it cools down.

$$Q^{into}_{water} + Q^{into}_{bowl} = Q^{out\ of}_{cylinder}$$

Be careful with g vs. kg and J vs kJ etc. Since ΔT is the same on the Kelvin or Celsius scale we will stick with $^\circ C$

25.7 (a)

$$\begin{aligned} \text{Heat flow into water } Q_{water} &= c_w m_{water}\Delta T + m_{w\to s}L_v \\ \text{Heat flow into water } Q_{water} &= (4190)(0.223)(100-21) + (4.7\times10^{-3})(2256\times10^3) \\ \text{Heat flow into water } Q_{water} &= 84,418\ J \end{aligned}$$

25.7 (b)

$$\begin{aligned} \text{Heat flow into bowl } Q_{bowl} &= c_c m_{bowl}\Delta T \\ \text{Heat flow into bowl } Q_{bowl} &= (387)(0.146)(100-21) \\ \text{Heat flow into bowl } Q_{bowl} &= 4,464\ J \end{aligned}$$

25.7 (c)

$$\text{Heat flow out of cylinder } Q_{cylinder} = -c_c m_{cylinder}\Delta T$$

We must have that

$$\begin{aligned} 84,418\ J + 4463\ J &= -(387)(0.314)(100 - T_i) \\ 88881 &= -(121.52)(100 - T_i) \\ 731.4 &= -100 + T_i \\ 831.4^\circ C &= T_i \text{ Mistake in book} \end{aligned}$$

HRK 25.10

When using the heat capacity C' of *a particular object* we have

$$Q = C'\Delta T$$

Physically we know that the heat gained by the thermometer will be equal to the heat lost by the water so

$$Q_{therm} + Q_{water} = 0$$

where

$$\begin{aligned} \text{Heat into therm} \quad Q_{therm} &= C'_{therm}\Delta T \\ Q_{therm} &= (46.1\ J/K)(44.4 - 15) \\ Q_{therm} &= 1355\ J \end{aligned}$$

and

$$\begin{aligned} \text{Heat into water} \quad Q_{water} &= c_w m_w \Delta T \quad \text{(we know this will be negative since water cools)} \\ Q_{water} &= (4190 J/(kg\ K))(0.3\ kg)(44.4 - T_i) \\ Q_{water} &= 1257(44.4 - T_i) \end{aligned}$$

Solving for T_i we get

$$\begin{aligned} Q_{therm} + Q_{water} &= 0 \\ 1355 + 1257(44.4 - T_i) &= 0 \\ 57165.8 &= 1257 T_i \\ T_i &= 45.5\ ^\circ \end{aligned}$$

HRK 25.14

We are told that the volume passing through the tube every second is

$$\text{Vol/sec } = 8.2 cm^3/s$$

Which means that the mass of liquid passing every second is

$$\text{mass/sec } = (\rho)(\text{Vol/sec }) = (8.2 cm^3/s)(0.85 \times 10^{-3}\ kg/cm^3) = 6.97 \times 10^{-3} kg/s$$

The amount of heat entering the liquid (with specific heat capacity c_{liq}) every second is

$$\begin{aligned} \text{Heat/sec} &= \text{(mass/sec) } c_{liq}\Delta T \\ \text{Heat/sec} &= (6.97 \times 10^{-3}\ kg/s)(c_{liq})(15\ K) \\ 250\ J/s &= (6.97 \times 10^{-3}\ kg/s)(c_{liq})(15\ K) \\ \Rightarrow c_{liq} &= 2391\ J/(kg\ K) \end{aligned}$$

HRK 25.19

This question uses the notion of linear expansion we encountered a couple of chapters ago. Recall

$$\Delta L = \alpha L \Delta T \quad \text{or} \quad L_f = L_i(1+\alpha\Delta T) \quad \text{(equivalent)}$$

The relevant lengths in this question are the diameters of the copper ring and aluminum sphere. Let us denote the coefficients of linear expansion for copper and aluminum as α_c and α_{Al} respectively. The sphere will drop through when $d^{(sphere)} = d^{(ring)}$. This happens when both objects reach the same temperature T_{eqm}. Note that the sphere will shrink as it cools down and the inner and outer diameters of the ring will both increase as it heats up

$$\begin{aligned} d_f^{(sphere)} &= d_i^{(sphere)}\left(1+\alpha_{Al}(T_{eqm}-T_i^{(sphere)})\right) \\ d_f^{(ring)} &= d_i^{(ring))}\left(1+\alpha_c(T_{eqm}-T_i^{(ring)})\right) \end{aligned}$$

Let us also take into account the flow of heat in this question. We have

$$Q_{ring}^{in} = Q_{sphere}^{out}$$

where

$$\begin{aligned} Q_{ring}^{in} &= m_{ring}\ c_c(T_{eqm}-T_i^{(ring)}) \\ Q_{sphere}^{out} &= -m_{sphere}\ c_{Al}(T_{eqm}-T_i^{(sphere)}) \end{aligned}$$

Since $Q_{ring}^{in} = Q_{sphere}^{out}$ we have

$$m_{sphere}\ c_{Al}(T_{eqm}-T_i^{(sphere)}) = -m_{ring}\ c_c(T_{eqm}-T_i^{(ring)})$$

$$\begin{aligned} \Rightarrow \qquad m_{sphere} &= -m_{ring}\left(\frac{c_c}{c_{Al}}\right)\frac{(T_{eqm}-T_i^{(ring)})}{(T_{eqm}-T_i^{(sphere)})} \\ m_{sphere} &= -(21.6\times10^{-3})\left(\frac{387}{900}\right)\frac{(T_{eqm}-0)}{(T_{eqm}-100)} \\ m_{sphere} &= -(9.3\times10^{-3})\frac{(T_{eqm}-0)}{(T_{eqm}-100)} \end{aligned}$$

We are half-way there; now use the expression $d_f^{(sphere)} = d_f^{(ring)}$ to finish out

$$\begin{aligned} d_i^{(sphere)}\left(1+\alpha_{Al}(T_{eqm}-T_i^{(sphere)})\right) &= d_i^{(ring))}\left(1+\alpha_c(T_{eqm}-T_i^{(ring)})\right) \\ (2.545333\times10^{-2})\left(1+(17\times10^{-6})(T_{eqm}-100)\right) &= (2.54\times10^{-2})\left(1+(23\times10^{-6})(T_{eqm}-0)\right) \end{aligned}$$

which if solved carefully gives

$$T_{eqm} = 34\ ^\circ C$$

and so, finally,

$$m_{sphere} = -(9.3\times10^{-3})\frac{(34-0)}{(34-100)} = 4.8\times10^{-3}\ kg$$

HRK 25.22

Using molar heat capacity, C, the amount of heat transferred looks like

$$\begin{aligned} Q &= nC\Delta T \qquad \text{if } C \text{ constant} \\ \text{or } Q &= \int_{T_i}^{T_f} nC(T)\ dt \qquad \text{if } C = C(T) \end{aligned}$$

Our case is the latter so

$$\begin{aligned} Q &= n\int_{50}^{90}\left(0.318T - 0.00109T^2 - 0.628\right)\ dt \\ Q &= n\left[0.318\frac{T^2}{2} - 0.00109\frac{T^3}{3} - 0.628T\right]_{50}^{90} \\ Q &= n(645.8) \end{aligned}$$

The number of moles is given by

$$n = \frac{316\ g}{107.8\ g/mol} = 2.93\ mol$$

so that

$$Q = 1893\ J$$

HRK 25.25

The heat transferred in a constant-pressure process is

$$Q = nC_p\Delta T$$

where C_p is the molar heat capacity at a constant temperature. Table 3 give the value for O_2 gas, $C_p = 29.4\ J/(mol\ K)$. To find ΔT we use

$$\begin{aligned} \frac{V_1}{T_1} &= \frac{V_2}{T_2} \qquad \text{constant pressure} \\ \frac{V_1}{T_1} &= \frac{2V_1}{T_2} \\ \Rightarrow T_2 &= 2T_1 \\ \Rightarrow \Delta T &= 2T_1 - T_1 = 284\ K \end{aligned}$$

$$\begin{aligned} Q &= nC_p\Delta T \\ Q &= (1.35)(29.4)(284) = 11,272\ J \end{aligned}$$