HRK 25.34

The key piece of insight you need is that

$$\Delta E_{int} = 0$$

for a closed path in a pV diagram of a reversible process. The first law of thermodynamics $\Delta E_{int} = Q + W$ then implies

$$Q = -W$$

where Q is the heat (energy) that flows into the system and W is the work done on the gas. If $122g$ of ice melts that means

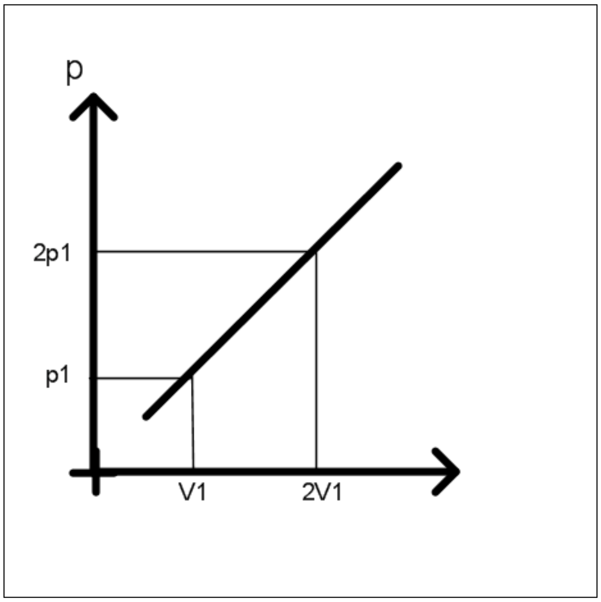
$$\begin{aligned} Q &= L_f m \\ &= (333\ kJ/kg)(0.122\ kg) \\ &= 40.6\ J. \end{aligned}$$

of heat has flowed out of the cylinder, and into the ice water. Using the convention that positive Q corresponds to heat entering the system of interest, this means that

$$Q_{cylinder} = -40.6\ J$$

Finally, work done *on the gas* is 40.6 J

HRK 25.37



25.37 (a)

Normally, to find the work done in a process like this, we would need to integrate

$$W = -\int p\ dV$$

Since the function is so simple we can just get the area under the line $(p_1, V_1) \rightarrow (p_2, V_2)$ to get the work *done by the gas*. The work done on the gas is the negative of this.

$$\begin{aligned} W &= -\frac{1}{2}(2p_1 - p_1)(2V_1 - V_1) + (2p_1 - p_1)(2V_1 - V_1) \\ W &= -\frac{3}{2}p_1 V_1 \\ \Rightarrow W &= -\frac{3}{2}nRT_1 \qquad \text{(ideal gas law)} \end{aligned}$$

The negative sign means the *gas did work on the piston.*

25.37 (b)

An ideal monatomic gas at temperature T has internal energy $E_{int} = \frac{3}{2}k_BT = \frac{3}{2}nRT$. The change in internal energy for a process that goes from $T_1 \to T_2$ is

$$\begin{aligned} \Delta E_{int} &= \frac{3}{2}nR(T_2 - T_1) \\ \Delta E_{int} &= \frac{3}{2}nR(4T_1 - T_1) \\ \Delta E_{int} &= \frac{9}{2}nRT_1 \end{aligned}$$

where we used the ideal gas law to find T_2

$$(2p_1)(2V_1) = p_2V_2 = nRT_2 = nR(4T_1)$$

25.37 (c)

Using the first law of thermodynamics

$$\Delta E_{int} = Q + W$$

we have enough information to solve for Q for this process. Using results from (a) and (b)

$$\begin{aligned} \frac{9}{2}nRT_1 &= Q - \frac{3}{2}nRT_1 \\ \Rightarrow Q &= 6nRT_1 \end{aligned}$$

flows into the gas.

25.37 (d)

Using the definition of molar specific heat

$$\begin{aligned} Q &= nC\Delta T \\ \Rightarrow C &= \frac{Q}{n\Delta T} \\ C &= \frac{6nRT_1}{n(4T_1 - T_1)} \\ C &= 2R \end{aligned}$$

HRK 25.43

We will repeatedly use the first law

$$\Delta E_{int} = Q + W$$

with various simplifications arising depending on the type of process undergone along a particular path. The sign convention used is **positive** quantities means **into** the system, and negative quantities mean out of the system.

25.43 (a)

$$\begin{aligned} \text{Const. Volume:} \quad W_{AB} &= 0 \\ \Rightarrow \Delta E_{int}^{AB} &= Q_{AB} \\ \\ Q_{AB} &= nC_V \Delta T \\ &= (1)(\frac{3}{2}R)(300) \\ &= 3740\ J \\ E_{int}^{AB} &= 3740\ J \end{aligned}$$

$$\begin{aligned} \text{Adiabatic:} \quad Q_{BC} &= 0 \\ \Rightarrow \Delta E_{int}^{BC} &= W_{BC} \\ \\ W_{BC} &= \frac{p_C V_C - p_B V_B}{\gamma - 1} \qquad \text{See sec. 23-5 for derivation} \\ W_{BC} &= \frac{p_C V_C - p_B V_B}{\frac{5}{3} - 1} \\ W_{BC} &= \frac{nRT_C - nRT_B}{\frac{2}{3}} \\ W_{BC} &= \frac{3(8.31)(455 - 600)}{2} \\ W_{BC} &= -1807\ J \\ \\ \Rightarrow \Delta E_{int}^{BC} &= -1807\ J \end{aligned}$$

$$\begin{aligned} \text{Const. Pressure:} \quad W_{CA} &= p_A(V_A - V_C) \\ W_{CA} &= p_A V_A - p_C V_C \\ W_{CA} &= nR(T_A - T_C) \\ W_{CA} &= 8.31(155) \\ W_{CA} &= 1288\ J \\ \\ Q_{CA} &= nC_p \Delta T \\ Q_{CA} &= (1)(\frac{5}{2}R)(-155) \\ Q_{CA} &= -3220\ J \\ \\ \Delta E_{int}^{CA} &= Q_{CA} + W_{CA} \\ \Delta E_{int}^{CA} &= -1932\ J \end{aligned}$$

As a direct check of this last result use the fact that $E_{int} = nC_v T$ for an ideal gas. For a monatomic ideal gas $C_v = \frac{3}{2}R$, so we have

$$\begin{aligned} \Delta E_{int}^{CA} &= \frac{3}{2} R \Delta T \\ \Delta E_{int}^{CA} &= \frac{3}{2}(8.31)(-155) \\ \Delta E_{int}^{CA} &= -1932\ J \end{aligned}$$

We should have $\Delta E_{int} = 0$ over the cycle so check

$$3740 - 1807 - 1932 = 1 \approx 0$$

25.43 (b)

We will use the ideal gas law along with information obtained from the diagram

$$\begin{aligned}
\text{To find } p_B: \qquad p_A V_A &= nRT_A \\
\text{and } p_B V_A &= nRT_B \\
\Rightarrow \frac{p_A}{p_B} &= \frac{T_A}{T_B} \\
\Rightarrow p_B &= p_A \frac{T_B}{T_A} \\
p_B &= (1.0\ atm)\frac{600}{300} \\
p_B &= 2.0\ atm
\end{aligned}$$

(we didn't really need math to see that doubling T, holding V constant, we must double p)

$$\begin{aligned}
\text{To find } V_B: \qquad p_B V_B &= nRT_B \\
V_B &= \frac{(8.31)(600)}{2.026 \times 10^5\ Pa} \\
V_A = V_B &= 0.0246\ m^3
\end{aligned}$$

$$\text{To find } p_C: \qquad p_C = p_A = 1.0\ atm$$

$$\begin{aligned}
\text{To find } V_C: \qquad p_A V_C &= nRT_C \\
\Rightarrow V_C &= \frac{(8.31)(455)}{1.01 \times 10^5} \\
V_C &= 0.0373\ m^3
\end{aligned}$$

HRK 25.44

The piston falls under the force of gravity (and also atmospheric pressure), but at a constant speed because of the steady rate of conversion of steam into water (due to steady rate of heat flow out)

$$\begin{aligned}
\text{Net Force} = 0: \qquad pA &= p_0 A + mg \\
\Rightarrow p &= \frac{(1.01 \times 10^5)(2 \times 10^{-4}) + (2)(9.81)}{2 \times 10^{-4}} \\
p &= 199,100\ Pa
\end{aligned}$$

25.44 (a)

Since the piston falls at a constant speed we have

$$\begin{aligned}
\frac{\Delta V}{\Delta t} &= -(0.3\ cm/s)(2\ cm^2) \\
\frac{\Delta V}{\Delta t} &= -0.6\ cm^3/s = -6 \times 10^{-7} m^3/s
\end{aligned}$$

The density of steam is given as $\rho = 6 \times 10^{-4}\ g/cm^3$ so that

$$\begin{aligned}
\frac{\Delta m}{\Delta t} &= \rho \frac{\Delta V}{\Delta t} \\
\frac{\Delta m}{\Delta t} &= (6 \times 10^{-4}\ g/cm^3)(0.6\ cm^3/s) = 3.6 \times 10^{-7} kg/s \qquad \text{converted}
\end{aligned}$$

This is the rate at which steam gets converted to water, ignoring the slight volume occupied by the condensed steam.

25.44 (b)

To condense steam into water you must remove heat in the amount

$$Q = (2256\ kJ/kg)(\text{mass of steam converted to water})$$

Since the water + steam mixture stays at the same temperature, the heat that is released in condensation must leave the chamber. The result from (a) gives us

$$\begin{aligned}\frac{\Delta Q}{\Delta t} &= -(2256 \times 10^3 J/kg)(3.6 \times 10^{-7} kg/s)\\ \frac{\Delta Q}{\Delta t} &= -0.812\ J/s = -0.812W\end{aligned}$$

where the negative sign means heat leaving the system.

25.44 (c)

There is working being done on the gas (because of the change in volume), and also there is heat flowing out of the gas, so use the first law (being sure to note that Q is negative because heat is leaving our system (the steam + water)

$$\begin{aligned}\Delta E_{int} &= Q + W\\ \frac{\Delta E_{int}}{\Delta t} &= \frac{\Delta Q}{\Delta t} + \frac{\Delta W}{\Delta t}\\ \frac{\Delta E_{int}}{\Delta t} &= -0.812\ J/s - p\frac{\Delta V}{\Delta t}\\ \frac{\Delta E_{int}}{\Delta t} &= -0.812\ J/s - (199,100\ Pa)(-6 \times 10^{-7} m^3/s)\\ \frac{\Delta E_{int}}{\Delta t} &= -0.812\ J/s + 0.12\ J/s\\ \frac{\Delta E_{int}}{\Delta t} &= -0.693\ J/s\end{aligned}$$

Note: If you assumed a vacuum above the piston you should have gotten

$$\frac{\Delta E_{int}}{\Delta t} = -0.753\ J/s$$

HRK 25.50

25.50 (a)

$$H = \frac{Q}{\Delta t} = kA\left(\frac{T_H - T_L}{L}\right)$$

where k for silver is 428 $W/(m\ K)$.

$$\begin{aligned}H &= (428\ W/(m\ K))(4.76 \times 10^{-4}\ m^2)\left(\frac{100\ K}{1.17\ m}\right)\\ H &= 17.4\ W\end{aligned}$$

25.50 (b)

Using

$$\begin{aligned}
Q &= L_f m \\
\frac{Q}{\Delta t} &= L_f \frac{m}{\Delta t} \\
\Rightarrow \frac{m}{\Delta t} &= \frac{H}{L_f} \\
\frac{m}{\Delta t} &= \frac{17.4\ J/s}{333 \times 10^3\ J/kg} \\
\frac{m}{\Delta t} &= 5.22 \times 10^{-5}\ kg/s
\end{aligned}$$

HRK 25.61

Use

$$H = -kA\frac{dT}{dr}$$

where A is the area of a spherical surface between the inner and outer spheres, i.e.

$$H = -k(4\pi r^2)\frac{dT}{dr}$$

We can rearrange and integrate from the surface of the inner sphere to the outer sphere. We will use the fact that H must be independent of r, when the system is in steady state, because if it were not then net heat would be entering or leaving different regions, which would mean the temperature was changing.

$$\begin{aligned}
\int_{r_1}^{r_2} \frac{H}{4\pi r^2}\ dr &= -\int_{T_1}^{T_2} k\ dT \\
\frac{H}{4\pi}\int_{r_1}^{r_2} \frac{1}{r^2}\ dr &= -k\int_{T_1}^{T_2} dT \\
\frac{H}{4\pi}\left(-r_2^{-1} + r_1^{-1}\right) &= k(T_1 - T_2) \\
H\left(\frac{1}{r_1} - \frac{1}{r_2}\right) &= 4\pi k(T_1 - T_2) \\
H &= \frac{4\pi k r_1 r_2 (T_1 - T_2)}{r_2 - r_1} \qquad \text{using } \left(\frac{1}{r_1} - \frac{1}{r_2}\right) = \left(\frac{r_2 - r_1}{r_1 r_2}\right)
\end{aligned}$$

HRK 25.63

Note that the rod has uniform cross-sectional area A and that little a is just a constant that has nothing to do with area. Using

$$H = -kA\frac{dT}{dx}$$

and plugging in the given expression $k = aT$, we get

$$H = -aAT\frac{dT}{dx}$$

Rearrange and integrate along the length of the rod

$$\begin{aligned}
H\int_{x=0}^{x=L} dx &= -aA\int_{T_1}^{T_2} T\ dT \\
HL &= -\frac{aA}{2}\left(T_2^2 - T_1^2\right) \\
H &= \frac{aA}{2L}\left(T_1^2 - T_2^2\right)
\end{aligned}$$

HRK 26.3

Since 1 metric ton is 1000 kg, the amount of heat generated (in one hour) by burning 382 tons of coal is

$$Q/\text{hour} = (382,000\ kg)(28 \times 10^6\ J/kg)/\text{hour} = 1.07 \times 10^{13}\ J/\text{hour}$$

The work rate can be used to find the amount of useful work done in one hour

$$\begin{aligned} W/\text{sec} &= 755 \times 10^6 J/\text{sec} \\ W/\text{hour} &= (755 \times 10^6\ J/\text{sec})(3600\ \text{sec/hour}) \\ W/\text{hour} &= 2.718 \times 10^{12}\ J/\text{hour} \end{aligned}$$

Finally, the efficiency is given by

$$\begin{aligned} e &= \frac{|W|}{|Q_{in}|} \\ e &= \frac{2.718 \times 10^{12}\ J/\text{hour}}{1.07 \times 10^{13}\ J/\text{hour}} \\ e &= 0.254 = 25.4\% \end{aligned}$$

HRK 26.6

Along the way we will need to make use of the fact that, for an ideal diatomic gas, $\gamma = \frac{7}{5} = 1.4$, provided the vibrational modes are not excited (which they are not below thousands of Kelvin).

26.6 (a)

By the ideal gas law

$$p_a V_a = nRT_a$$

so we have

$$\text{vertex a:} \qquad (p_a, T_a) = (p_a, T_a)$$

$$\begin{aligned} \text{vertex b:} \qquad V_b &= V_a \\ p_b V_b &= nRT_b \\ (3p_a)(V_a) &= nRT_b \\ \Rightarrow (p_b, T_b) &= (3p_a, 3T_a) \end{aligned}$$

$$\begin{aligned} \text{vertex c:} \qquad V_c &= V_d = 4V_a \\ p_b V_b^{\frac{7}{5}} &= p_c V_c^{\frac{7}{5}} \\ (3p_a)(V_a)^{\frac{7}{5}} &= p_c (4V_a)^{\frac{7}{5}} \\ \Rightarrow p_c &= \frac{3}{4^{\frac{7}{5}}} p_a \\ p_c &= 0.431 p_a \\ p_b V_b^{\frac{7}{5}} &= p_c V_c^{\frac{7}{5}} \\ T_b V_b^{\frac{2}{5}} &= T_c V_c^{\frac{2}{5}} \\ T_c &= (3T_a)\left(\frac{V_b}{V_c}\right)^{\frac{2}{5}} \\ T_c &= (3T_a)\left(\frac{1}{4}\right)^{\frac{2}{5}} \\ T_c &= 1.72 T_a \\ \Rightarrow (p_c, T_c) &= (0.43 p_a, 1.72 T_a) \end{aligned}$$

so we notice that the diagram in the book that shows $P_c > P_a$ is misleading!

$$\begin{aligned}
\text{vertex d:} \quad V_d &= 4V_a \\
p_a V_a^{\frac{7}{5}} &= p_d V_d^{\frac{7}{5}} \\
\Rightarrow p_d &= p_a \left(\frac{1}{4}\right)^{\frac{7}{5}} = 0.144 p_a \\
p_d V_d &= nRT_d \\
(0.144 p_a)(4V_a) &= nRT_d \\
\text{and since} \quad (0.144)(4) p_a V_a &= (0.144)(4) nRT_a \\
T_d &= (0.144)(4) T_a = 0.574 T_a \\
\Rightarrow (p_d, T_d) &= (0.144 p_a, 0.574 T_a)
\end{aligned}$$

26.6 (b)

No heat enters or leaves the system during the adiabatic parts of the cycle (by definition). For the constant volume processes, the work done is zero so the change in E_{int} corresponds to the heat Q entering/leaving the system. For a diatomic ideal gas

$$E_{int} = nC_v T = \frac{5}{2} nRT$$

and so along path $a \rightarrow b$ we have

$$\begin{aligned}
\Delta E_{int}^{ab} &= \frac{5}{2} nR(T_b - T_a) \\
\Delta E_{int}^{ab} &= \frac{5}{2} nR(2T_a) = Q_{in}
\end{aligned}$$

Along path $c \rightarrow d$ we have

$$\begin{aligned}
\Delta E_{int}^{cd} &= \frac{5}{2} nR(T_d - T_c) \\
\Delta E_{int}^{cd} &= \frac{5}{2} nRT_a(0.574 - 1.72) \\
\Delta E_{int}^{cd} &= \frac{5}{2} nRTa(-1.146) = -Q_{out}
\end{aligned}$$

The efficiency can be written as

$$\begin{aligned}
e &= 1 - \frac{|Q_{out}|}{|Q_{in}|} \\
e &= 1 - \frac{\frac{5}{2} nRTa(1.146)}{\frac{5}{2} nRT_a(2)} \\
e &= 1 - \frac{1.146}{2} \\
e &= 0.427 = 42.7\%
\end{aligned}$$

Alternatively, we could calculate the work directly

$$\begin{aligned}
W_{BC} &= \frac{1}{\gamma - 1}(p_c V_c - p_b V_b) \\
&= \frac{1}{0.4} p_a V_a((0.43)(4) - (3)(1)) \\
&= -3.192 \; p_a V_a
\end{aligned}$$

i.e. the gas does this much work during the expansion. To compress it, the piston does work

$$\begin{aligned}
W_{DA} &= \frac{1}{0.4} p_a V_a(1 - (0.144)(4)) \\
W_{DA} &= 1.06 \; p_a V_a
\end{aligned}$$

Net work out of the sytem is 2.13 $p_a V_a$ per cycle. The heat **we** put in (not the net heat) is $\frac{5}{2}2(nRT_a) = 5p_a V_a$ so that the efficiency is

$$\begin{aligned} e &= \frac{W_{out}}{Q_{in}} \\ e &= \frac{2.132 p_a V_a}{5 p_a V_a} \\ e &= 0.426 \end{aligned}$$

HRK 26.10

The coefficient of performance of a refrigerator is

$$K = \frac{|Q_L|}{|W|}$$

where W is the work supplied, and where Q_L is defined as the heat *extracted* from the cold reservoir, so in this question that corresponds to $|Q_L| = 10.0\ J$. For a Carnot refrigerator K can be shown to be

$$K = \frac{T_L}{T_H - T_L}$$

where the T's correspond to the temperatures of the hot and cold reservoirs. Putting it together

$$\begin{aligned} |W| &= \frac{|Q_L|}{K} \\ |W| &= \frac{|Q_L|(T_H - T_L)}{T_L} \end{aligned}$$

26.10 (a)

$$\begin{aligned} |W| &= \frac{(10\ J)(20\ K)}{280\ K} \\ |W| &= 0.714\ J \end{aligned}$$

26.10 (b)

$$\begin{aligned} |W| &= \frac{(10\ J)(100\ K)}{200\ K} \\ |W| &= 5\ J \end{aligned}$$

26.10 (c)

$$\begin{aligned} |W| &= \frac{(10\ J)(200\ K)}{100\ K} \\ |W| &= 20\ J \end{aligned}$$

26.10 (d)

$$\begin{aligned} |W| &= \frac{(10\ J)(250\ K)}{50\ K} \\ |W| &= 50\ J \end{aligned}$$

So, as T_L approaches zero Kelvin, it takes ever more work to extract a given amount of heat. For this reason one can not actually reach 0K, although you can get awfully close. The current record is $100pK = 10^{-10}K$!