Physics 23

Assignment 3 Solutions

T. Tao[1]

[1]*Department of Physics, University of California, Santa Barbara, CA 93106*

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E25-4

a) The electro-static forces on the two masses are opposite and equal, so by Newton's Law we have

$$m_1 a_1 = m_2 a_2, \tag{1}$$

where a_1 and a_2 the magnitudes of m_1's and m_2's accelerations, respectively. Taking $m_1 \approx 6.31 \times 10^7 kg$, we find $m_2 \approx 4.97 \times 10^-7kg$.

b) Since the only force acting on the particles are electro-static, for force on one of the masses, for instance m_1, is

$$F_{on1} = m_1 a_1 = \frac{q^2}{4\pi\epsilon_0 r^2}, \tag{2}$$

where q is the common charge of the particles and r is the distance between them. Here all quantities are written as magnitudes of vectors since we have a simple 1-D problem. Putting in the numbers given to solve for q, we obtain $q = 7.2 \times 10^-11C$.

E25-8

a) Define a Cartesian coordinate system with the +2q charge at the origin. The forces on +2q due to the three other charges are

$$\mathbf{F}_{\mathbf{+q\ on\ +2q}} = \frac{(2q)(q)(-\hat{\mathbf{j}})}{4\pi\epsilon_0 r^2_{+q,+2q}} = \frac{-\hat{j}2q^2}{4\pi\epsilon_0 a^2} \tag{3}$$

$$\mathbf{F}_{\mathbf{-2q\ on\ +2q}} = \frac{(2q)(-2q)(-\hat{\mathbf{i}})}{4\pi\epsilon_0 r^2_{-2q,+2q}} = \frac{\hat{i}4q^2}{4\pi\epsilon_0 a^2} \tag{4}$$

and

$$\mathbf{F}_{\mathbf{-q\ on\ +2q}} = \frac{(2q)(-q)(-\hat{\mathbf{i}}cos(\pi/4) - \hat{\mathbf{j}}sin(\pi/4))}{4\pi\epsilon_0 r^2_{-q,+2q}} = \frac{2^{1/2}(\hat{\mathbf{i}} + \hat{\mathbf{j}})q^2}{4\pi\epsilon_0 2a^2}. \tag{5}$$

Here the bold face denotes vectors while $\hat{i}$ and $\hat{j}$ are unit vectors in the x and y directions, respectively. The sine and cosine factors come from resolving the force due to -q into x and y components. The angle between the force vector in question and the x-axis is $\pi/4$ radian.

We can then sum all the forces in the x-direction to obtain the horizontal component of the total force on +2q. We have

$$F_{on\ +2q,x} = \frac{4q^2}{4\pi\epsilon_0 a^2} + \frac{2^{1/2}q^2}{4\pi\epsilon_0 2a^2} \approx 2.34N. \tag{6}$$

Based on our coordinate system, this force is in the positive x-direction.

b) The vertical force on +2q has contributions from the y-components of the forces from +q and -q,

$$F_{on\ +2q,y} = -\frac{2q^2}{4\pi\epsilon_0 a^2} + \frac{2^{1/2}q^2}{4\pi\epsilon_0 2a^2} \approx -0.642N. \tag{7}$$

Note that +q charge applies a force on the +2q charge only in the y-direction, and the sign of its contribution is negative given our coordinate system.

E25-16

First, define a coordinate system in which the rod lies completely on the x-axis with its ends at x=0 and x=L. Let the distance between the right end of the rod and the charge +q be d. (the book used x which is confusing) We divide the rod into infinitely many differential (or very small) segments of length dx each and calculate the force on +q due to each bit of the rod. Finally, we obtain the total force on +q by summing up the contributions of all infinitesimal segments via integration across the length of the rod. The force on +q due to the differential amount of charge contained in length dx on the rod is

$$dF_{on+q} = \frac{qdQ}{4\pi\epsilon_0(L+d-x)^2} = \frac{dxQ/L}{4\pi\epsilon_0(L+d-x)^2}. \tag{8}$$

Here $dQ = (Q/L)dx$ is the infinitesimal amount of charge carried on the small length dx. Note that the linear charge density of the rod is simply Q/L since the charge is uniformly distributed. With out coordinate system, L+d-x gives the distance between the differential bit of charge dQ and the observing charge, +q. We did not include vector symbols in the above expression since the problem is 1-D and it is clear that the total force on +q is in the positive x-direction.

Finally, we integrate Eqn.(8) across the length of the rod from 0 to L to obtain

$$F_{on+q} = \int dF = \int_0^L \frac{dxQ/L}{4\pi\epsilon_0(L+d-x)^2} = \frac{qQ}{4\pi\epsilon_0 L}\left(\frac{1}{d} - \frac{1}{d+L}\right). \tag{9}$$

E25-22

a) The ^4He particle carries charge $+2e \approx 2 \times 1.6 \times 10^-19C$ while the ^234Th nuclei has charge $+90e \approx 90 \times 1.6 \times 10^-19C$. (since it has 90 protons) The magnitude of the repulsiveforce between them is given by Coloumb's Law,

$$F_{He,Th} = \frac{Q_{He}Q_{Th}}{4\pi\epsilon_0 R_{He,Th}} \approx 290N. \tag{10}$$

b) The only force on the He particle is the electro-static force, we thus have

$$F_{He,Th} = m_{He}a_{He}, \tag{11}$$

where $m_{He} = 4m_{proton} \approx 4 \times 1.66 \times 10^-27kg$. We find $a_{He} \approx 4.4 \times 10^{28}m/s^2$.

P25-4

a) In equilibrium, the net force on each ball is 0. In particular, the horizontal component of the tension pulling on each ball must cancel with the electro-static force (which is purely horizontal and opposite to the horizontal component of tension for a given ball). The electro-static force on each ball has magnitude

$$F_E = \frac{q^2}{4\pi\epsilon_0 x^2}. \tag{12}$$

The vertical component of the tension on each ball must balance gravity, we therefore have

$$T_y = Tcos(\theta) = mg. \tag{13}$$

Solving for the magnitude of the tension T and then computing its x-component, we obtain

$$T_x = Tsin(\theta) = mgtan(\theta) \approx mgsin(\theta) = \frac{mgx}{2L}. \tag{14}$$

Here we used the fact that $tan(\theta) \approx sin(\theta)$ for small θ. Equating T_x to F_E and solving for x yields the desired result.

b) Once again equate T_x to F_E but now solve for q using a given x. We obtain $q \approx 2.28 \times 10^-8C$.

P25-8

Initially, the charge -q experiences no net force since it is exactly half way between two charges of +Q at distance d apart. Now we move -q upwards a small distance y perpendicular to the line joining the +Q charges (the x-axis, or y=0). There will then be a net electrostatic force on -q in the negative y-direction (vertical), as the horizontal components of the forces due to the two +Q charges still cancel exactly. We now have a restoring net force that pulls -q back towards the line connecting the +Q charges. However, when -q reaches its initial position, it will have some finite velocity from being accelerated for some time by the restoring net force. We then expect -q to 'over shoot' and begin to experience a net force in the opposite (+y) direction. The new net force will eventually stop the particle and turn it around back towards the positive y-direction. In effect, -q will undergo harmonic oscillation about the line joining the +Q charges. When -q is at a vertical distance y from the x-axis, the total electro static force on it is

$$F_{net} = -\frac{2qQsin(\phi)}{4\pi\epsilon_0 R_{-q,+Q}} = -\frac{2qQsin(\phi)}{4\pi\epsilon_0[(d/2)^2 + y^2]} = -\frac{2qQy}{4\pi\epsilon_0[(d/2)^2 + y^2)]^{3/2}} \quad (15)$$

in the y direction. (+ or - y don't really matter here) Here ϕ is the angle that position vector of -q makes with the x-axis when displaced vertically by y and we used

$$R_{-q,+Q} = [(d/2)^2 + y^2)]^{1/2}, \quad (16)$$

so that

$$sin(\phi) = \frac{y}{R_{-q,+Q}} = \frac{y}{[(d/2)^2 + y^2)]^{1/2}}. \quad (17)$$

Next, note that y is assumed to be a very small number compared to d and so y^2 is vanishingly small compared to $(d/2)^2$. We can then approximate F_{net} by dropping the y^2 in the denominator. We then have

$$F_{net} \approx -\frac{2qQy}{4\pi\epsilon_0(d/2)^3}. \quad (18)$$

Rewriting Eqn.(18) in a more enlightening form, we find

$$\frac{d^2y}{dt^2} \approx -\frac{4qQ}{m\pi\epsilon_0 d^3}y, \quad (19)$$

which is the differential equation for a simple harmonic oscillator with angular frequency

$$\omega^2 = \frac{4qQ}{m\pi\epsilon_0 d^3}. \quad (20)$$

Finally, taking the period $T = 2\pi/\omega$ gives the desired result.

P25-10

a) Since all the Cs^+ ions are arranged symmetrically at the same distance around the Cl^-, the net force on the Chlorine ion is zero.

b) After removing one Cs^+, note that there are still four Cs^+ ions in symmetric positions around the Chlorine and so the forces due to those still cancel. Thus the net force on the Cl^- comes only from the remaining Cesium ion at $d = (0.2^2 + 0.2^2 + 0.2^2)^{1/2} nm$ away. We then find

$$|\ \mathbf{F}_{net\ on\ Cl^-}\ | = \frac{e^2}{4\pi\epsilon d^2} \approx 1.92 \times 10^-9 N. \tag{21}$$

Here e is the electron charge and the double vertical lines denote the magnitude of the net force vector.

P25-11

Let the line joining the two +q charges be on the x-axis. Without loss of generality, we can assume that the test charge Q is at a distance y=R above the x-axis. The net force on Q is in the y-direction (+ or - does not matter here) since the horizontal components of the forces exerted by the +q charges cancel. The total force on Q is therefore

$$|\ \mathbf{F_{on\ Q}}\ | = \frac{2qQ sin(\phi)}{4\pi\epsilon_0 d}, \tag{22}$$

where d is the distance between Q and either q and ϕ is the angle between the position vector of Q and the x-axis. Next we use

$$d = (a^2 + R^2)^{1/2} \tag{23}$$

and

$$sin(\phi) = \frac{R}{d} = \frac{R}{(a^2 + R^2)^{1/2}} \tag{24}$$

to obtain

$$|\ \mathbf{F_{on\ Q}}\ | = \frac{2qQR}{4\pi\epsilon_0 (a^2 + R^2)^{3/2}}. \tag{25}$$

The value of R that maximizes $|\ \mathbf{F_{on\ Q}}\ |$ is given by

$$\frac{d\ |\ \mathbf{F_{on\ Q}}\ |}{dR} = 0 = \frac{2qQ}{4\pi\epsilon_0}\left[\frac{1}{(a^2 + R^2)^{3/2}} - \frac{3R^2}{(a^2 + R^2)^{5/2}}\right]. \tag{26}$$

The result $R = \pm a/2^{1/2}$ immediately follows.