

Set #2 - for Thurs April 19

Read Ohanian Ch. 3, Sects. 3.4-3.6, Ch. 4

From Ohanian:

Ch. 3 Problems 35, 37, 40, 48, 53.

Ch. 4 Problems 10, 18, 21.

1. A Compton scattering experiment is arranged so that the scattered photon and the recoil electron are detected only when their paths are at right angles to one another. Take the incident photon energy to be 2.0 MeV and the electron rest energy 0.5 MeV. Find: (a) the wavelength and energy of the scattered photon. (b) The scattering angle of the photon. (c) The kinetic energy of the recoil electron.

2. A π^0 meson decays into two photons. (a) If the π^0 is at rest when it decays, what are the frequencies of the two photons? (b) If the π^0 is moving with velocity v and it decays into one photon which goes directly forward and another which goes directly backward, what is the frequency of the two photons?

3. An alpha particle of energy 5.5 MeV is incident on a silver nucleus with an impact parameter 8.0×10^{-15} m. Neglect the recoil of the silver nucleus and find the distance of closest approach of the particle.

4. Photons vs. molecules For an ideal gas of monoatomic molecules, the average energy is $E = \frac{3}{2}Nk_B T$, where N is the number of molecules. Now consider a gas of photons in a blackbody cavity of volume V . The energy density per unit of frequency interval within such a cavity is

$$u_\nu = \frac{8\pi h}{c^3} \frac{\nu^3}{(e^{h\nu/k_B T} - 1)}$$

a) Show that the average number of photons in the volume V is *not* a constant, but depends on the temperature according to

$$N = V \int_0^\infty \frac{8\pi}{c^3} \frac{\nu^2 d\nu}{(e^{h\nu/k_B T} - 1)}$$

and that the average energy of this gas of photons is

$$E = V \int_0^\infty \frac{8\pi}{c^3} \frac{h\nu^3 d\nu}{(e^{h\nu/k_B T} - 1)}.$$

This helps illustrate that the number of photons in a system is *not* a conserved quantity.

b) Express the integral for the number of photons in dimensionless form and show that the number of photons is proportional to T^3 . Then show that E is proportional to $Nk_B T$, just as is the case for an ideal gas of molecules.

c) Determine the proportionality constant between E and $Nk_B T$ for a gas of photons by using the following result:

$$\int_0^\infty \frac{x^{n-1}}{e^x - 1} dx = \Gamma(n)\zeta(n) \quad \text{for } n > 1,$$

where $\Gamma(n)$, the *gamma function*, is equal to $(n-1)!$ for $n > 1$ and $\zeta(n)$ the *Riemann zeta function*, has the following selected values:

$$\zeta(2) = \pi^2/6, \zeta(3) = 1.202, \zeta(4) = \pi^4/90.$$

Compare to the ideal gas of monoatomic molecules, for which $E = \frac{3}{2}Nk_B T$.