Physics CS 36 **Spring 2012**

Set #6 - for Thurs May 17

Read Ohanian Ch. 6 Sects 6.1-6.5
Read Feynman Vol. III Ch. 7

From Ohanian:

Ch. 6 Problems 3, 5, 8, 9, 11, 17.

1. A 20-cm-long pencil is balanced on its point. Classically, this is a configuration of (unstable) equilibrium, so the pencil could remain there forever if it were perfectly placed. A quantum mechanical analysis shows that the pencil must fall. (a) Why is this the case? (b) Estimate how long it will take the pencil to hit the table if it is positioned initially as well as possible?

2. The wavefunction of a free non-relativistic particle is at $t = 0$:

$$\psi(x,0) = \begin{cases} 0\ ; & |x| > a/2, \\ C\ ; & |x| \leq a/2 \end{cases}$$

a) Find the constant C.

b) Find $\psi(x,t)$. Leave your answer in integral form.

c) How long does it take for this wavepacket to double in size?

3. Consider the Schrödinger equation for a free particle. Find the constant A so that

$$\psi(x,t) = \cos(kx - \omega t) + A\sin(kx - \omega t)$$

is a solution. What can you conclude from your answer in regards to the reality and normalizability of $\psi(x,t)$?

4. Show that the Schrödinger equation for a particle of mass m in a potential $V(x)$ is *linear*. If $\psi(x,t)$ satisfies the Schrödinger equation for a particle of mass m in a potential

$V(x)$, what is the equation satisfied by $\psi^*(x,t)$, the complex conjugate of ψ? Does $Re\ \psi$ satisfy the Schrödinger equation?

5. Suppose that a particle is described by the wavefunction $\psi = Ae^{i(kx-\omega t)}$. Calculate the probability current density

$$j(x,t) = \frac{i\hbar}{2m}[\psi\frac{\partial\psi^*}{\partial x} - \psi^*\frac{\partial\psi}{\partial x}]$$

for this state and interpret the result. Replace ψ by $\psi = A\cos kx\ e^{-i\omega t}$ and repeat the calculation. Give a physical interpretation of your result.