Physics CS 36 **Spring 2012**

Set #7 - for Thurs May 24

<u>Read Ohanian</u> Ch. 6 Sects 6.2-6.6
<u>Read Feynman Vol. III</u> Ch. 7, Ch. 8

From Ohanian:

Ch. 6 Problems 14, 15, 16, 19

1. The wavefunction of a free particle at $t = 0$ is

$$\psi(x, 0) = A \sin^2 kx.$$

Show that the time-dependent free particle state is given by a superposition of momentum eigenstates with energies $E = 0$, and $E = 2\hbar^2 k^2/m$ and with the probability to measure $E = 0$ twice the probability to measure $E \neq 0$.

2. a) Show directly from the time-independent Schrödinger equation that

$$\langle p^2 \rangle = \langle 2m|E - V(x)| \rangle$$

in general for any potential $V(x)$, and that $\langle p^2 \rangle = \langle 2mE \rangle$ for the infinite square well.

b) Use the result from part (a) to compute $\langle p^2 \rangle$ for the ground-state of the infinite square well.

c) Find $\Delta x = \sqrt{\langle x^2 \rangle - \langle x \rangle^2}$ and $\Delta p = \sqrt{\langle p^2 \rangle - \langle p \rangle^2}$ for the ground state of an infinite square well. Evaluate $\Delta x \Delta p$.

3. The wavefunction of a particle is $\psi(x) = Ae^{ix - x^2}$.

a) Find the constant A.

b) Find the expectation value of the position and the momentum.

c) What is the probability of finding the particle in the region $x > 0$? Could this wavefunction be the solution for an infinite square well?

4. A free particle of momentum p is represented by a plane wave. A measurement determines that the particle lies somewhere inside a region of length l. The interaction between the measuring apparatus and the particle leaves the wavefunction unchanged for a length l but reduces it to zero outside this region. What are the average momentum and kinetic energy of this particle after the measurement has been made?

5. a) In class we saw that the conservation of probability can be expressed in 1-D as

$$\frac{\partial P(x,t)}{\partial t} + \frac{\partial j(x,t)}{\partial x} = 0$$

where $P(x,t) = |\psi(x,t)|^2$ and $j(x,t) = \frac{i\hbar}{2m}[\psi \frac{\partial \psi^*}{\partial x} - \psi^* \frac{\partial \psi}{\partial x}]$.

Redo the derivation of the continuity equation for the case of a <u>complex</u> potential $V = -V_0 - iW_0$.

b) If this complex potential is constant and if $W_0 << V_0$ show that there are stationary state solutions which are harmonic waves with exponentially decaying amplitude.

6. Find the eigenvalues of an operator $\hat{\sigma}$ satisfying the equation:

$$\hat{\sigma}^2 - 1 = 0.$$

Repeat for the operator $\hat{P}$ satisfying the equation

$$\hat{P}^2 + \hat{P} - 2 = 0.$$

7. Show that the momentum operator $\hat{p} = -i\hbar\frac{\partial}{\partial x}$ is Hermitian. Find the eigenvalues and eigenfunctions.

8. Start with the expression derived in class for the expectation value of the momentum,

$$\langle p \rangle = \int_{-\infty}^{\infty} \psi^*(x,t)| - i\hbar \frac{\partial \psi(x,t)}{\partial x} | dx.$$

Evaluate $\frac{d}{dt}\langle p \rangle$ and replace $\frac{\partial \psi}{\partial t}$ and $\frac{\partial \psi^*}{\partial t}$ as given in the Schrödinger equation. Do some appropriate integrations by parts and show that the expectation value of the force acting on a particle satisfies the equation $\langle F(x) \rangle = \frac{d\langle p \rangle}{dt}$ where $F(x) = -\frac{\partial V(x)}{\partial x}$.

This is Newton's 2^{nd} law as it applies to expectation values, instead of instantaneous values, and it shows that the center of a wavepacket moves like a classical particle. This result is known as **Ehrenfest's Theorem**.

Extra Credit A perfectly elastic ball is bouncing between two plane parallel walls. Apply classical mechanics and compute the change in the energy of the ball as the walls are slowly and uniformly moved closer together. Show that the change in energy is just what one has quantum mechanically if the ball's quantum number n does not change.