Physics CS 36 **Spring 2012**

Set #8 - for Thurs May 31

<u>**Read Ohanian**</u> Ch. 6 Sects 6.4-6.7 (*particularly the WKB approximation, p. 190-197*)

<u>**Read Feynman Vol. III**</u> Ch. 16

From Ohanian:

Ch. 6 Problems 23, 24, 28, 32, 51

Extra Credit Problems 25, 26, 33

1. A small bead of mass 2.0 g slides around a circular wire of radius 20 cm. Treat the bead as a quantum mechanical particle. a) What are the allowed speeds and energies of the bead (in eV)? b) What is the quantum number that corresponds to the condition in which the bead revolves at 1 rps? c) At what speed does the bead travel around the wire when the bead is in its lowest energy state?

2. Suppose that the wavefunction at $t = 0$ for a particle in a one-dimensional box (∞-well) is given by

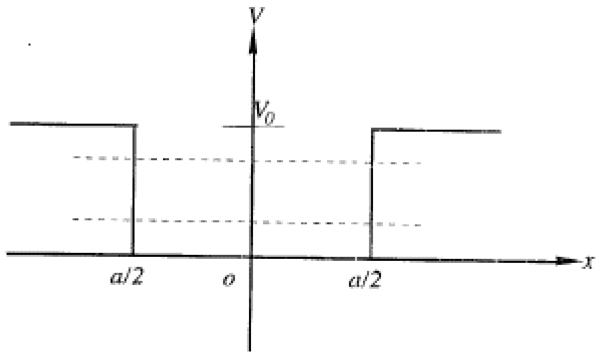
$$\psi(x, 0) = \frac{1}{\sqrt{a}}(\sin\frac{2\pi x}{a} + \cos\frac{3\pi x}{a}).$$

What is the subsequent form of the wavefunction $\psi(x, t)$? Use this form of the wavefunction to compute the probability density, and interpret the time-dependence of the result.

3. Assume that the square-well below is capable of producing at least a ground state and a first excited state, as shown by the dotted lines.

a) Sketch the wavefunction for the first excited state.

b) Obtain the transcendental equation that gives the energy eigenvalue for the first excited state. (*don't need to solve,*)



4. Consider the potential $V(x) = -V_o\delta(x)$ (Dirac δ-function), with $V_o > 0$ a constant.

a) Is the wavefunction continuous at $x = 0$?

b) Is the first derivative continuous at $x = 0$? If not, find the discontinuity by integrating the Schrödinger equation from $-\epsilon$ to $+\epsilon$ and let $\epsilon \to 0$ afterwards. Assume $\psi(x) \to 0$ for $x \to \pm\infty$.

c) How many bound states are there? Find the normalized wavefunctions of any bound state you may be able to find.

5. Given $\psi_n(x)$ the energy eigenfunctions for the infinite square-well, find $\phi_n(p)$ the corresponding wavefunctions in momentum space. If you measure the momentum in the state described by $\psi_n(x)$, do you get a sharp value? Sketch $|\phi_n(p)|^2$.

6. A one-dimensional simple harmonic oscillator is constructed in such a way that the spring constant may be adjusted. The oscillator is in its lowest energy state when the spring constant is reduced to zero. What is the subsequent behavior of the wavefunction? That is, what is $\psi(x, t)$, and the probability density $P(x, t)$?

7. At $t = 0$ the wavefunction of a simple harmonic oscillator is

$$\psi(x, 0) = a\psi_0(x) + b\psi_1(x),$$

where ψ_0 and ψ_1 are the real normalized wavefunctions of the ground state and the first excited states respectively. $\psi(x,0)$ is normalized. a) What is $\psi(x,t)$? b) Measurements on a large number of identical systems yield an average value of the energy of $\frac{5}{4}\hbar\omega$. What are the magnitudes of the constants a and b? c) Another series of measurements on the system shows that the probability of finding the particle in the region $x > 0$ is a maximum whenever $t = (2n+1)\frac{\pi}{\omega}$ with n a positive integer. Assume a is a real positive number, and determine the phase of b (that is, $b = |b|e^{i\phi}$, where ϕ is the phase of the complex number b). d) Determine the expectation value of x as a function of time for this state.

8. Consider the one-dimensional Schrödinger equation with the potential

$$V(x) = \begin{cases} \frac{m}{2}\omega^2 x^2 \text{ for} & x > 0, \\ +\infty \text{ for} & x < 0 \end{cases}$$

Find the energy eigenvalues and eigenfunctions.

Extra Credit: A particle is in the lowest energy state of an infinite square well of width a, when suddenly the width of the well is changed to $2a$, by moving both walls $a/2$. Assume the wavefunction does not have a chance to change during the motion of the walls. What is the probability a) the particle has lost energy? b) that its energy is unchanged?