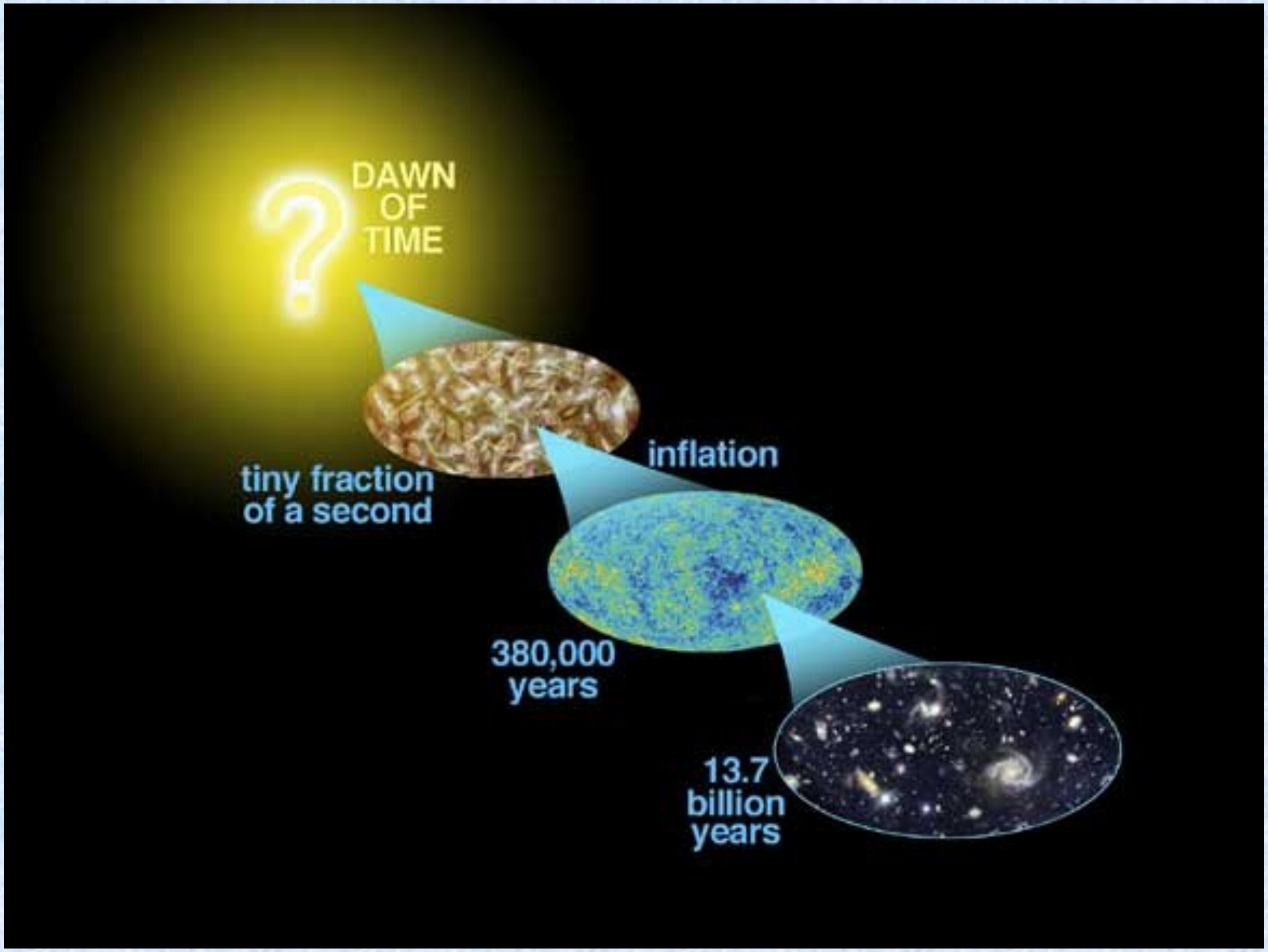
Physics 133: Extragalactic Astronomy and Cosmology



Lecture 12; February 24 2014

Previously:

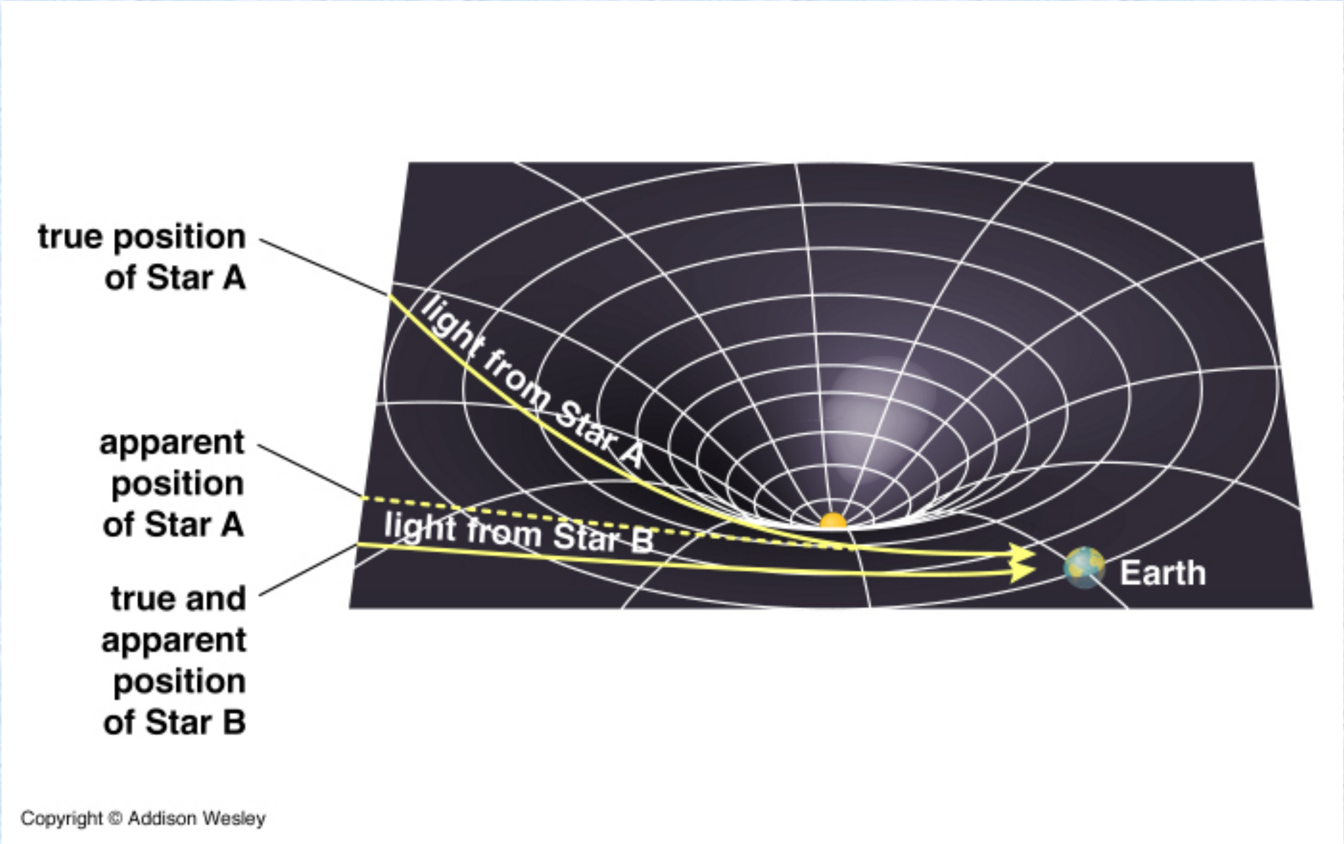
- There is dark matter
  - Galaxies – rotation curves
  - Clusters – virial theorem and hydrostatic equilibrium
- We do not know what it is:
  - It cannot be hidden baryons
  - It could be new exotic particles..

Outline:

- Gravitational lensing (intro)
- Gravitational lensing (theory):
  - Strong
  - Weak
  - Micro (Ryden 8.4)
- Cool things you can do with lensing (applications):
  - Detect dark matter
  - Test gravity
  - Cross section of dark matter

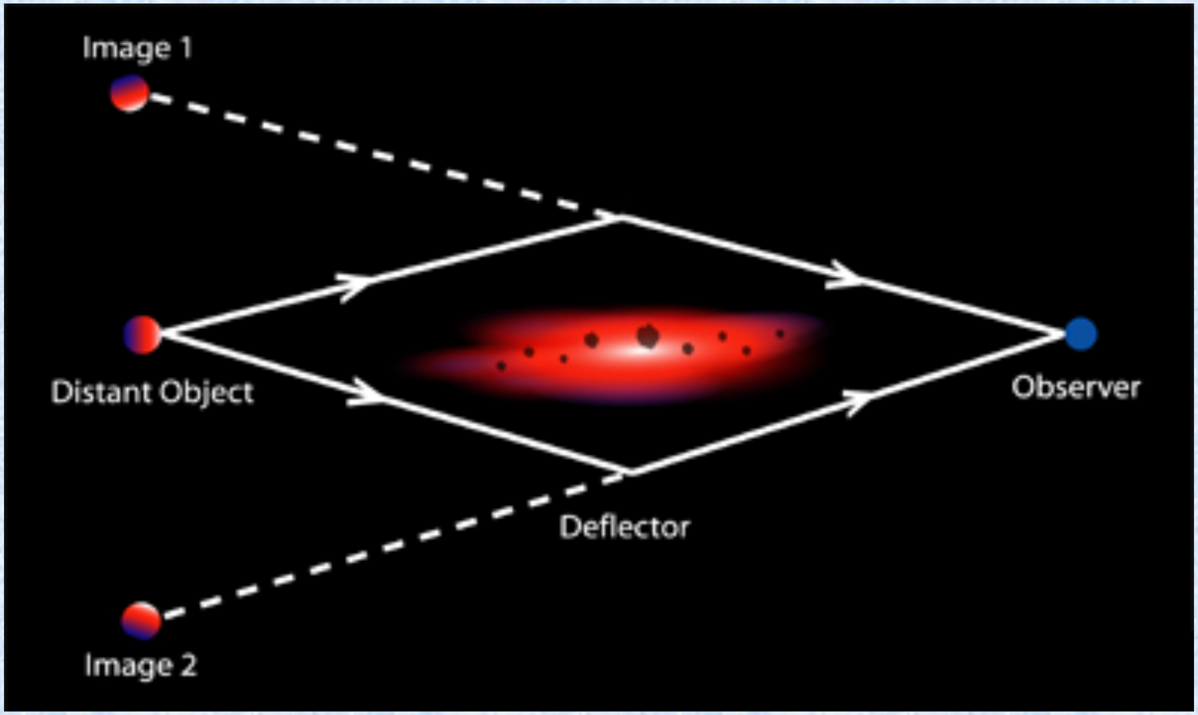
Detecting dark matter. Gravitational lensing!

- Mass concentrations perturb spacetime, altering the propagation of light

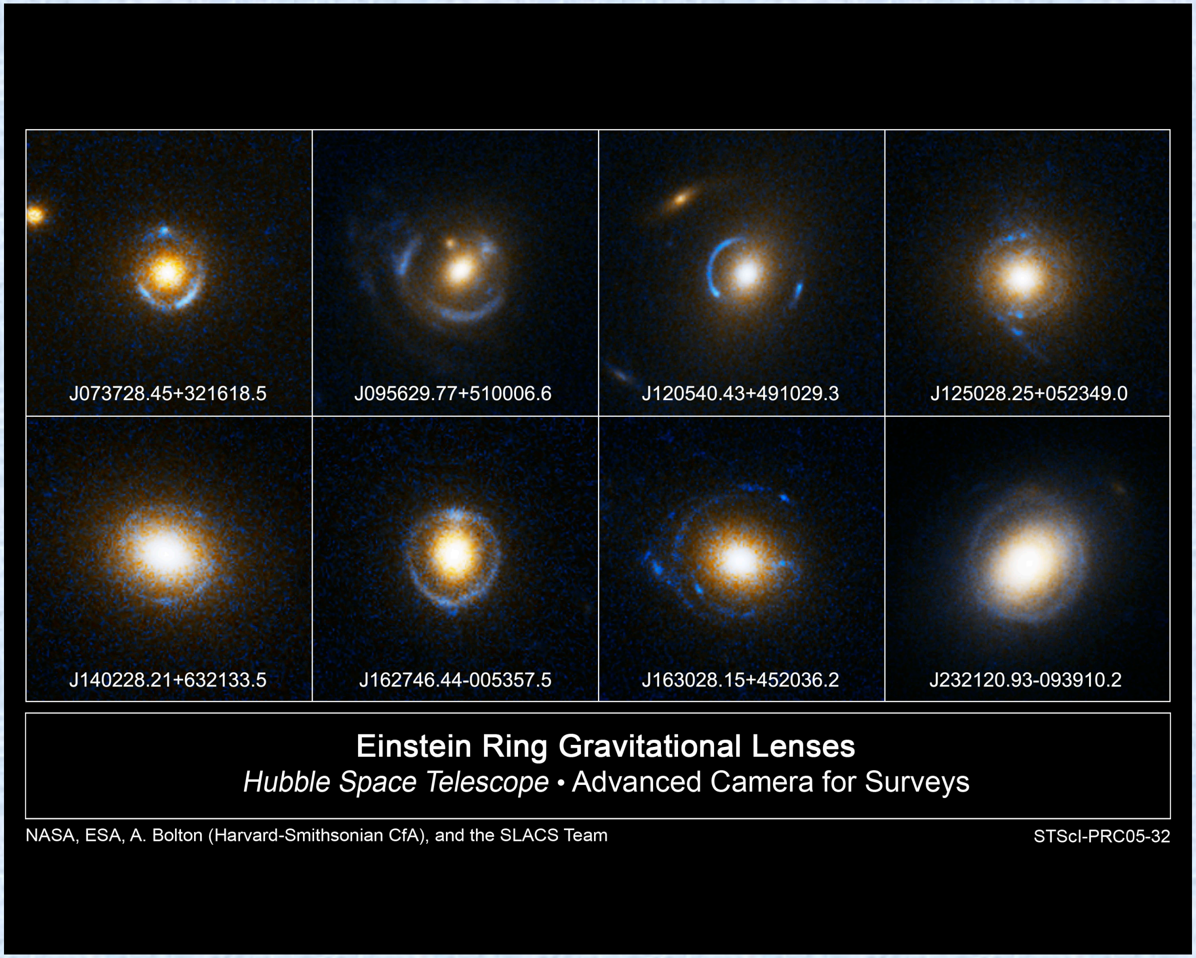


Detecting dark matter. Strong gravitational lensing!

- Under special circumstances the distortion is so strong that creates two images of a background object. This is called strong lensing



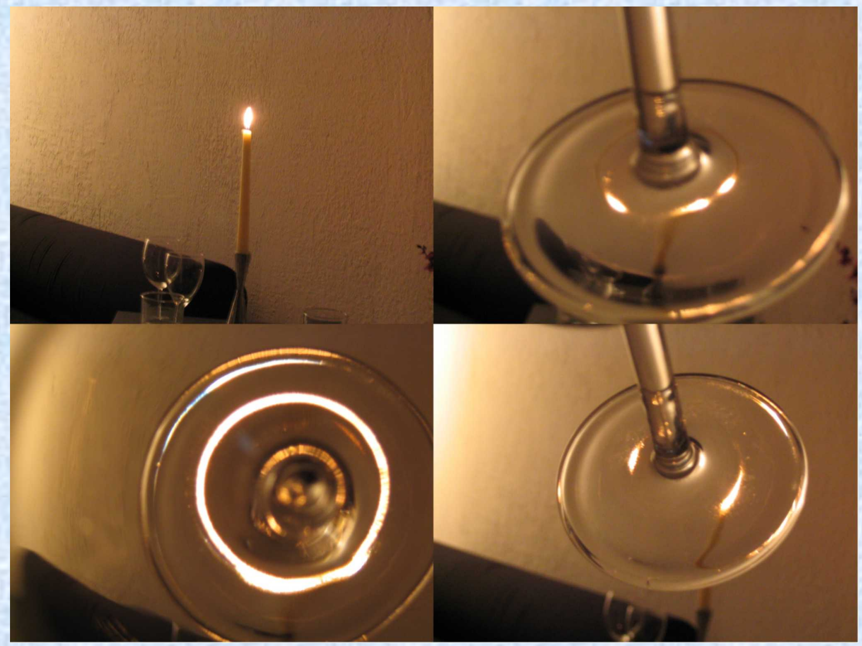
Detecting dark matter. Examples of strong lenses



Detecting dark matter. Why is it called lensing?

- The physics is very similar to that of common optical lenses
- In fact many of the features of gravitational lensing can be reproduced by common optical devices.

Source Quadruple



Einstein Ring Double

Figure courtesy of Phil Marshall

Detecting dark matter. Strong lensing by clusters

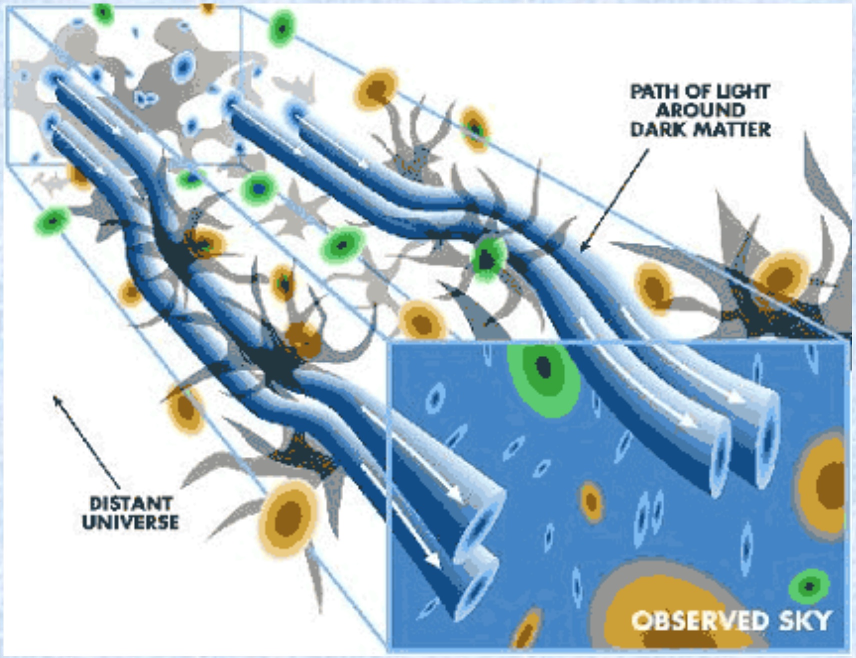
- Clusters are also strong lenses
- The blue objects here are distorted images of the same object

Detecting dark matter. Why do we care about lensing?

- The image separation gives us a direct measurement of the mass enclosed by the images.
- It is arguably the most precise measurement of mass that we can make.
- And there are other applications too (we' ll see later..)

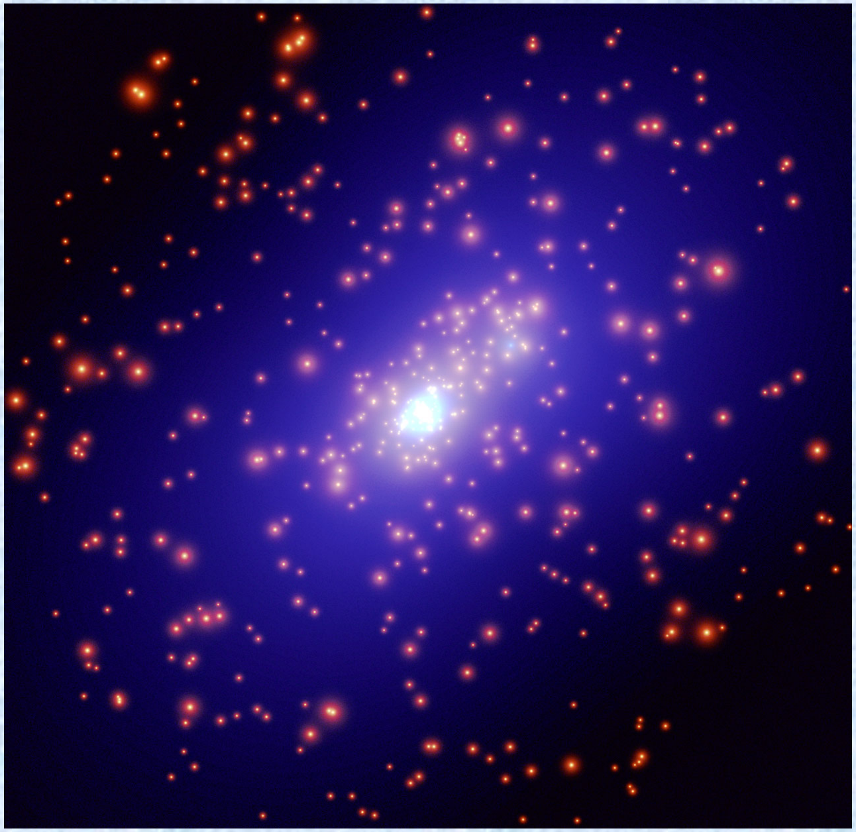
Detecting dark matter. Weak lensing

- Even when the gravitational field is not strong enough to produce multiple images, the large scale structure perturbs space time
- This alters the shape of observed galaxies in the sky, shearing and magnifying them in a measurable way



Detecting dark matter. Weak lensing mass maps

Optical image



Dark matter mass

Lensing… a little math

Lensing Basics.
I: Thin Screen Approximation

Surface mass density

$$\Sigma(\vec{\xi}) = \int \rho(\vec{\xi}, z)\, dz$$

Deflection angle

$$\vec{\hat{\alpha}}(\vec{\xi}) = \frac{4G}{c^2} \int \frac{(\vec{\xi} - \vec{\xi}')\Sigma(\vec{\xi}')}{|\vec{\xi} - \vec{\xi}'|^2}\, d^2\xi'$$

$$\hat{\alpha}(\xi) = \frac{4GM(\xi)}{c^2 \xi}$$

$$M(\xi) = 2\pi \int_0^{\xi} \Sigma(\xi')\xi'\, d\xi'$$

$$\vec{\alpha} = \frac{D_{\rm ds}}{D_{\rm s}}\, \vec{\hat{\alpha}}$$

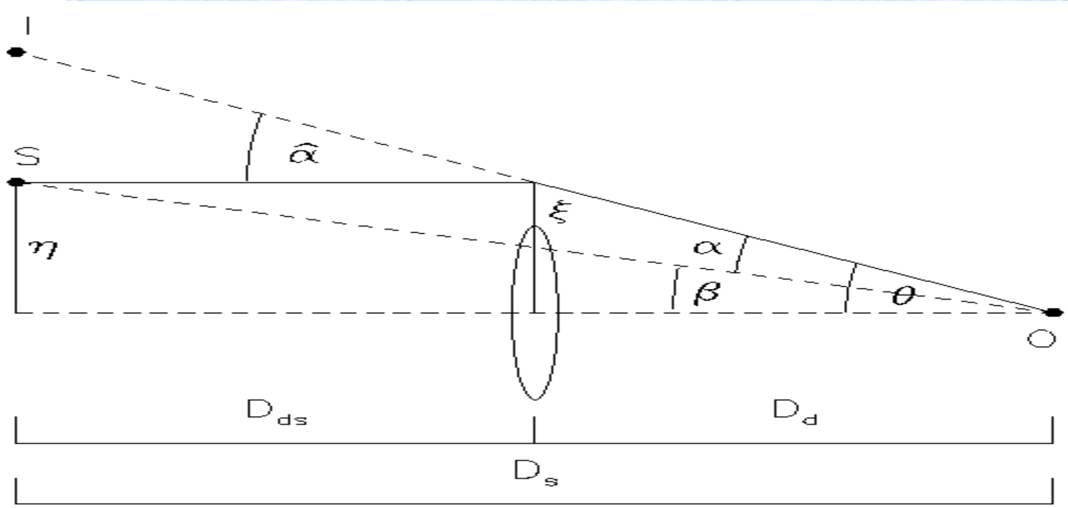
Reduced deflection angle

$$\vec{\beta} = \vec{\theta} - \vec{\alpha}(\vec{\theta})$$

Lens equation

$$\Sigma_{\rm cr} = \frac{c^2}{4\pi G} \frac{D_{\rm s}}{D_{\rm d} D_{\rm ds}} = 0.35\,{\rm g\,cm^{-2}} \left(\frac{D}{1\,{\rm Gpc}}\right)^{-1}$$

Critical density



Lensing Basics.
II: useful general relations

2D potential →

$$\psi(\vec{\theta}) = \frac{D_{\rm ds}}{D_{\rm d} D_{\rm s}} \frac{2}{c^2} \int \Phi(D_{\rm d}\vec{\theta}, z)\, dz$$

$$\vec{\nabla}_\theta \psi = D_{\rm d} \vec{\nabla}_\xi \psi = \frac{2}{c^2} \frac{D_{\rm ds}}{D_{\rm s}} \int \vec{\nabla}_\perp \Phi\, dz = \vec{\alpha}$$

2D Poisson Equation

$$\nabla^2_\theta \psi = \frac{2}{c^2} \frac{D_{\rm d} D_{\rm ds}}{D_{\rm s}} \int \nabla^2_\xi \Phi\, dz = \frac{2}{c^2} \frac{D_{\rm d} D_{\rm ds}}{D_{\rm s}} \cdot 4\pi G \Sigma = 2 \frac{\Sigma(\vec{\theta})}{\Sigma_{\rm cr}} \equiv 2\kappa(\vec{\theta})$$

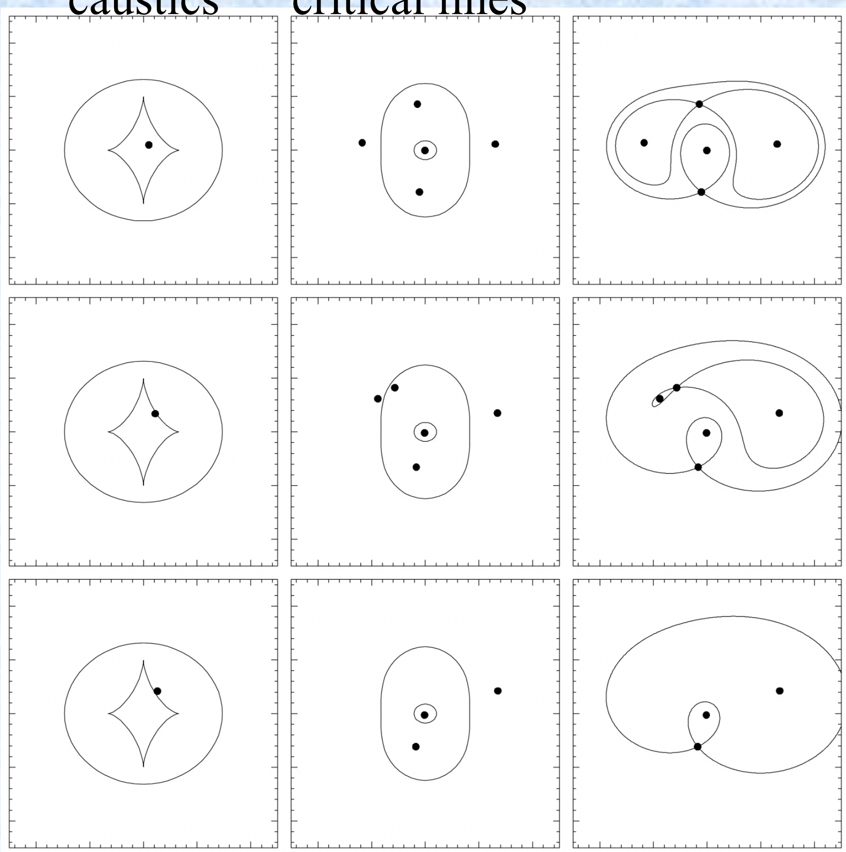
$$\vec{\alpha}(\vec{\theta}) = \vec{\nabla}\psi = \frac{1}{\pi} \int \kappa(\vec{\theta}') \frac{\vec{\theta} - \vec{\theta}'}{|\vec{\theta} - \vec{\theta}'|^2}\, d^2\theta'$$

Jacobian matrix

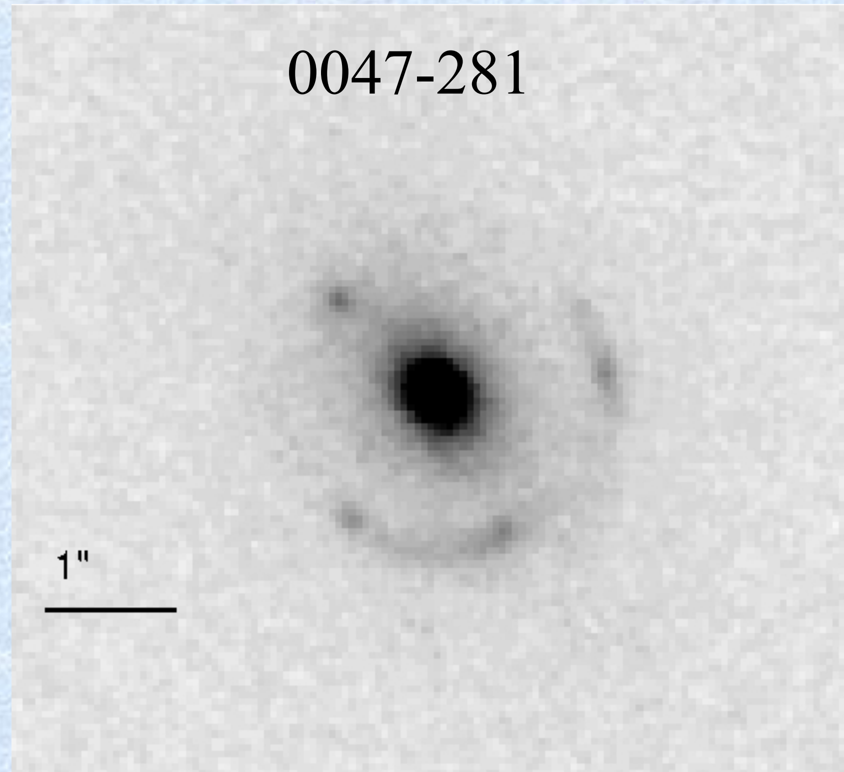
$$\mathcal{A} \equiv \frac{\partial \vec{\beta}}{\partial \vec{\theta}} = \left(\delta_{ij} - \frac{\partial \alpha_i(\vec{\theta})}{\partial \theta_j} \right) = \left(\delta_{ij} - \frac{\partial^2 \psi(\vec{\theta})}{\partial \theta_i \partial \theta_j} \right) = \mathcal{M}^{-1}$$

Lensing Basics.
III: Caustics and Critical Lines

caustics critical lines



Saha & Williams 2003



Koopmans & Treu 2003

The time delay surface and Fermat's principle

The time delay surface and Fermat's principle

$$(\vec{\theta} - \vec{\beta}) - \vec{\nabla}_\theta \psi = 0$$

Lens equation

$$\vec{\nabla}_\theta \left[\frac{1}{2} (\vec{\theta} - \vec{\beta})^2 - \psi \right] = 0$$

Extrema of the time delay surface

Time delay surface

$$t(\vec{\theta}) = \frac{(1 + z_\mathrm{d})}{c} \frac{D_\mathrm{d} D_\mathrm{s}}{D_\mathrm{ds}} \left[\frac{1}{2} (\vec{\theta} - \vec{\beta})^2 - \psi(\vec{\theta}) \right] = t_\mathrm{geom} + t_\mathrm{grav}$$

adimensional

$$\mathcal{T} = \frac{\partial^2 t(\vec{\theta})}{\partial \theta_i \partial \theta_j} \propto (\delta_{ij} - \psi_{ij}) = \mathcal{A}$$

Multiple images form at the extrema of the time delay surface

Gravitational lensing.
Meaning of the time delay surface

- Gravitational fields not only bend light, but the also “slow” down time
- For this reason as light travels close to a lens it takes longer than it should in normal geometry
- As we will see, we can use this effect to measure distances and the Hubble constant

Summary. Lensing basics

- Mass concentrations distort the images of objects in on the sky in a way similar to that of optical images.
- Strong and weak lensing provide very accurate mass maps of objects (in projection)
- Gravitational fields also “slow down” light. Strong lensing can be formulated in terms of Fermat’s principle

Dark Matter. Smooth…

A simple example. Singular isothermal sphere

- The singular isothermal sphere is the simplest model that provides a decent description of galaxies
- Let's see its properties and how it can be used to measure masses. [Blackboard]

$$\rho_{\mathrm{SIS}} = \frac{\sigma_{\mathrm{v}}^2}{2\pi G r^2}$$

$$\Sigma(\xi) = \int_{-\infty}^{+\infty} dr_3 \rho_{\mathrm{SIS}} = \frac{\sigma_{\mathrm{v}}^2}{2G\xi}$$

$$\theta_{\mathrm{E}} \equiv = 4\pi \left(\frac{\sigma_{\mathrm{v}}}{c}\right)^2 \frac{D_{\mathrm{ds}}}{D_{\mathrm{s}}}$$

$$\Delta\theta = 2\theta_{\mathrm{E}}$$

$$M_{\mathrm{E}} = \pi\theta_{\mathrm{E}}^2\Sigma_{\mathrm{crit}}$$

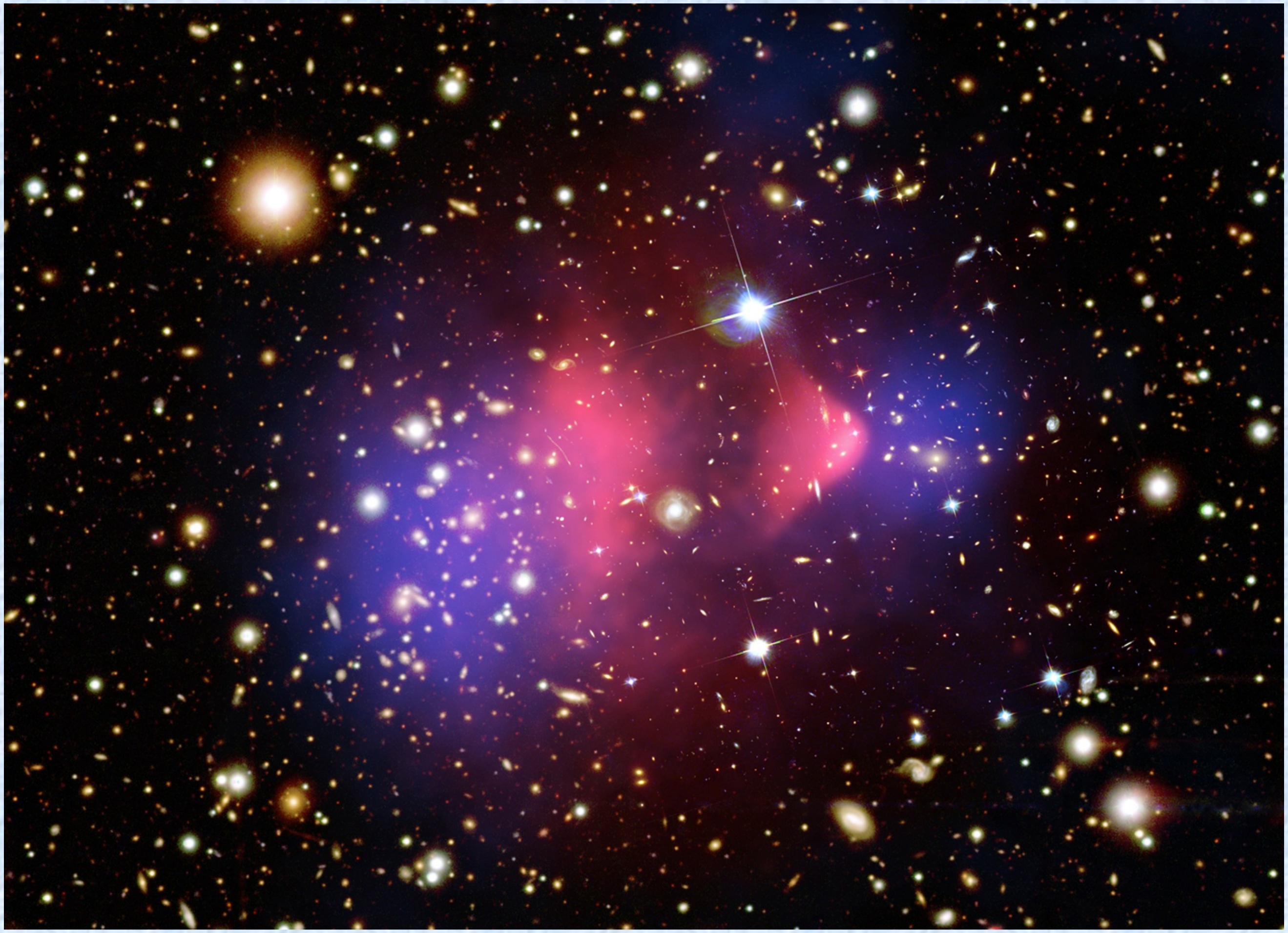
…and clumpy

Gravitational lensing. Detecting substructure..

- Substructure problem
- Two alternatives:
  - 1) Cold dark matter is wrong
  - 2) Satellites are present but not visible
- Lensing can detect them through their effects on multiple images.. There have been claims that this has been detected.. The jury is still out..

Lensing and dark matter

A case study: the bullet cluster



MOVIE!

Clowe et al. 2006; Bradac et al. 2006

Inferences from the bullet cluster

- There is mass where there are no baryons
  - Non baryonic dark matter
  - Mond is wrong
- Dark matter is collisionless
  - Limits on self interaction cross section $<0.7 cm^2/g$

The End

See you on wednesday!